

The Viscous Dissipation Effect in a Regular Polygonal Duct for H2 Boundary Conditions

T J Teleszewski¹

¹Department of HVAC Engineering, Faculty of Civil and Environmental Engineering, Bialystok University of Technology, Wiejska 45A, 15-351 Bialystok, Poland

Abstract. A numerical investigation based on the Boundary Element Method (BEM) was carried out to solve laminar forced convection with viscous dissipation in a straight regular polygonal duct with H2 boundary conditions. The axial heat conduction in the fluid is neglected. The effects of the Brinkman number and Nusselt number for both the wall heating case and the wall cooling case are considered. Nusselt numbers are obtained as a function of the number of sides of a regular polygonal duct and of the Brinkman number. The singular points of the Brinkman number and the Brinkman number for a limited Nusselt number when the total heat flux reached the heat flux that is generated internally by viscous dissipation processes are obtained.

1. Introduction

The flow of Newtonian fluids and heat transfer through a straight non-circular duct has been studied by many researchers in recent years. Laminar forced convection in a duct with constant axial and peripheral wall heat flux (H2 boundary conditions) has been found in many different systems and applications for very low conductive materials, such as electrical resistance heating, nuclear heating and flow through micro-channels [1]. In particular, laminar forced convection heat transfer in ducts with H2 boundary conditions has been studied by neglecting the effects of viscous dissipation [1]. Wang [2] presented results for H2 forced convection in rounded rectangular ducts. Etemad et al. [3], using the Galerkin Finite Element Method solved the steady laminar flow and H2 heat transfer of non-Newtonian fluids in equilateral triangular ducts. Ray and Misra [4] analysed the evaluation of pressure drop and heat transfer characteristics of laminar forced convection through square and equilateral triangular ducts with rounded corners, for H2 boundary conditions. Gao and Hartnett [5] using a Finite Difference Method presented a numerical solution for fully developed laminar flow forced convection of a power law non-Newtonian fluid in a rectangular duct under different combinations of H2 boundary conditions with adiabatic walls. Morini and Spiga [6] studied the velocity and temperature distribution for Newtonian fluid laminar flow and forced convection heat transfer in rectangular ducts for modified H2 thermal boundary conditions for eight versions involving different combinations of heated and adiabatic walls. Shahmardan et al. [7] proposed an analytical solution for convective heat transfer in rectangular ducts for H2 boundary conditions. Dharaiya and Kandlikar [8] performed a numerical analysis of laminar forced convection in rectangular microchannels subject to H2 boundary conditions. Wang [9], using an analytical method, calculated the Nusselt number for H2 heat transfer in rectangular ducts with large aspect ratios. Wang [9] found the limiting value of the Nusselt number for H2 boundary conditions approaches 2.9162 as the aspect ratio tends to infinity. Chung and Zhang [10] numerically determined the Nusselt number from the thermally developing flow of non-Newtonian fluids in rectangular ducts under H2 boundary conditions. Turgut [11] numerically investigated laminar flow and heat transfer in smooth hexagonal ducts subject to H2 boundary



conditions. Shahsavari et al. [12] presented an analytical solution for H2 heat transfer in microchannels of hyperelliptical and regular polygonal cross sections for flow with a negligible viscous dissipation effect.

The effects of viscous dissipation cannot be neglected for some systems, such as flow through ducts, where the viscosity is large or flow in microchannels. In the literature, some analyses of the effects of viscous dissipation of laminar forced convection in channels with H2 boundary conditions are available. Barletta et al. [13] studied the effect of viscous dissipation for laminar flow in stadium-shaped ducts with H2 boundary conditions. Sheikhalipour and Abbassi [14] investigated the viscous dissipation effects in trapezoidal microchannels under H2 boundary conditions using the Finite Difference Method. Rij et al. [15] studied the frictional and convective heat transfer characteristics of rarified flows in rectangular microchannels with H2 boundary conditions. Sayed-Ahmed et al. [16] analysed the Graetz problem for a Bingham plastic fluid in laminar tube flow for H2 boundary conditions by taking account of viscous dissipation.

The aim of this study is analysis of the viscous dissipation effects on the laminar forced convection in regular polygonal channels subject to H2 thermal boundary conditions. Teleszewski and Sorko [17] provide a compact relationship for the Nusselt and Brinkman numbers for different values of sides in regular polygonal channels under H1 boundary conditions with laminar viscous dissipation. No compact solutions were found for the Nusselt number in the H2 case with viscous dissipation effects. The influence of the Brinkman number on the Nusselt number for different values of sides of a regular polygonal duct is obtained for the H2 thermal boundary conditions.

2. Mathematical model

The present analysis is based on the fully developed, laminar, steady, incompressible flow with a constant dynamic viscosity μ and constant thermal conductivity k . Under the aforementioned assumptions, the continuity (1), momentum (2) and energy (3) equations of a Newtonian fluid in a straight duct with uniform heat flux are given as the form:

$$\frac{\partial u_z}{\partial z} = 0 \quad (1)$$

$$\mu \left(\frac{\partial^2 u_z}{\partial x^2} + \frac{\partial^2 u_z}{\partial y^2} \right) = \frac{\partial p}{\partial z}, \quad \frac{\partial p}{\partial x} = \frac{\partial p}{\partial y} = 0 \quad (2)$$

$$k \nabla^2 T = \rho c_p \frac{\partial T}{\partial z} u_z(x, y) - \Phi(x, y), \quad \Phi(x, y) = -\mu \left[\left(\frac{\partial u_z}{\partial x} \right)^2 + \left(\frac{\partial u_z}{\partial y} \right)^2 \right] \quad (3)$$

where u_z is the axial velocity, p is the pressure and Φ is the viscous dissipation term. Figure 1 shows the duct configuration and coordinate system.

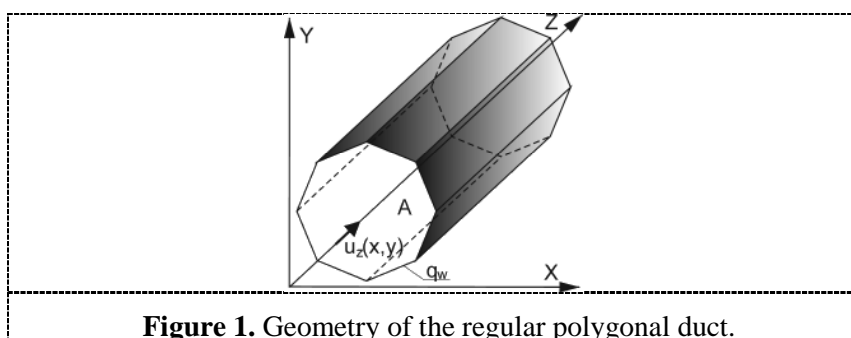


Figure 1. Geometry of the regular polygonal duct.

In the momentum equation (2), the fully developed velocity profile $u_z(x, y)$ is obtained from the boundary element method (BEM) with no-slip conditions on the duct wall $u_z=0$. The BEM method was also employed for the energy equation (3) with H2 boundary conditions. The H2 thermal boundary conditions are defined as uniform peripheral wall heat flux ($q_w=\text{const}$) with constant axial

wall heat flux. The BEM numerical method is used to solve the momentum (2) and energy equations (3) with 1000 constant elements and 40602 triangular cells. To determine the relative differences between the numerical and analytical values, the numerical results obtained by BEM were compared to the analytical values of the Nusselt number with the Brinkman number equal to zero for a circular tube and square duct. For 1000 constant boundary elements and 40602 triangular cells in the duct cross-section, the maximum relative differences in the Nusselt number were found to be $\Delta Nu = 0.005\%$.

3. Results and discussion

The Brinkman number is commonly used to describe the ratio between the heats generated by viscous forces to the heat exchanged by conduction at the walls. The Brinkman referred to the wall heat flux density q_w is given by

$$Br_q = \frac{\mu u_m^2}{D_h q_w}, \quad u_m = \frac{1}{A} \iint_A u_z(x, y) dA, \quad D_h = \frac{4A}{L} \quad (4)$$

where u_m is the mean velocity, L is the wetted perimeter of the cross-section, D_h is the hydraulic diameter and A is the cross-sectional area. The wall heat transfer q_w is positive wall heat transfer, indicating that the fluid is heated by the wall and q_w is negative, showing that the fluid is cooled by the wall. The Nusselt number is obtained as

$$Nu = \frac{q_w D_h}{k(T_w - T_b)}, \quad T_b = \frac{1}{Au_m} \iint_A u_z(x, y) T(x, y) dA \quad (5)$$

where T_b is a bulk temperature.

In the literature [18-21], the relationship between the Brinkman number and the Nusselt number in flows through a straight duct is described by the formula:

$$Nu = \frac{a_1}{a_2 + a_3 Br_q} \quad (6)$$

where a_1 , a_2 , a_3 are coefficients that depend on the geometrical shape of the cross sections. Similarly, the Nusselt's relation to the Brinkman number is proposed in regular polygonal channels subject to H2 conditions

$$Nu = \frac{Nu_0}{1 + \sigma_n Nu_0 Br_q} \quad (7)$$

where Nu_0 is the Nusselt number with H2 boundary conditions on the assumption of negligible effects due to viscous dissipation ($Br_q = 0$) and the shape factor σ_n . Nu_0 and σ_n depend on the number of n -sided regular polygonal ducts. For a circular pipe the analytical value of σ_n is equal to 1 [21] and the fully developed average Nusselt number Nu_0 is equal to 48/11 [21]. The Nusselt number Nu_0 for H2 boundary conditions, the shape factor σ_n for regular polygonal ducts with the correlation coefficient $R^2 \approx 1$, is proposed:

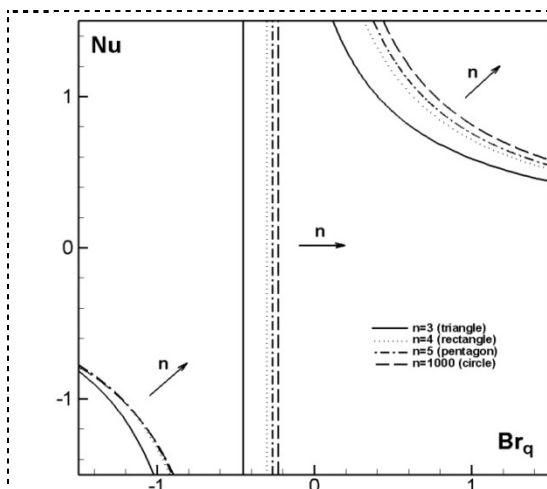
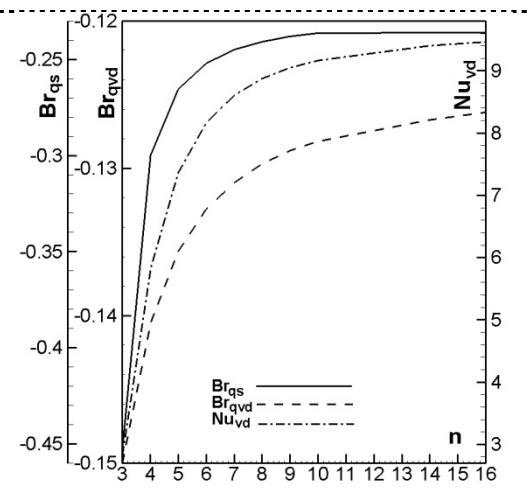
$$Nu_0(n), \sigma_n(n) = \frac{a + bn + cn^2 + dn^3}{e + fn + gn^2 + hn^3} \quad (8)$$

where a , b , c , d , e , f , g , h coefficients are reported in Table 1. Note that the coefficients of shape factor σ_n are calculated in recent paper [17].

Table 1. Coefficients of the Eq. (8) to determine the Nusselt Nu_0 , and shape coefficients in regular polygonal ducts.

Coefficients of the Eq. (45)	a	b	c	d	e	f	g	h
Nu_{0_H2}	1.076	-3.7656	1.36446	0.03714	1.000	-0.8052	0.31154	0.00851
$\sigma_n[17]$	0.9193	-0.6928	0.12666	-0.010517	1.000	-0.7082	0.1277	-0.010518

The total wall heat flux q_w contains the positive thermal heat flux transferred to the walls q_f and negative heat flux generated by viscous dissipation q_{vd} . In fact, when the total heat flux q_w is positive, then its direction is from the wall to the fluid and the fluid is heated by the wall, otherwise negative total heat flux goes from the fluid to the wall and the fluid is cooled by the wall. Figure 2 shows the variation of the Nusselt with the Brinkman number for different n side numbers of polygonal ducts: $n=3$ (triangular shape), $n=4$ (square shape), $n=5$ (pentagon shape) and $n=1000$ (circular tube) for H2 boundary conditions. The trend of the Nusselt number and the Brinkman number with the number of sides of a regular polygonal duct in the case of the boundary condition H2 is similar to that of the H1 condition [17]. When the heat flux generated by viscous dissipation and the Brinkman number equal zero ($q_f/q_w=1$), then the Nusselt number corresponds to the fully developed flow in ducts without viscous dissipation and $Nu=Nu_0$. When the Brinkman number goes from zero to infinity, then the total wall heat flux, the Brinkman number, the Nusselt number and the temperature difference between the wall temperature and the fluid bulk temperature are positive. In this case, the value of the Nusselt number decreases with the increase of the Brinkman number and the Nusselt number goes to zero because the total wall heat flux goes to zero. Zanchini [21] proved that, whenever the wall heat flux tends to zero, the asymptotic Nusselt number is zero. After this point total heat flux changes its sign from positive to negative and the Nusselt number decreases with the increase in the Brinkman number. When the temperature difference between wall temperature and fluid bulk temperature changes its sign from positive to negative, then the singularity point is observed as shown in figure 2. In fact, at the singularity point the power generated by the shear rate is balanced with the power transferred to the wall [19, 20]. The singular points of the Brinkman number for H2 boundary conditions can be obtained by the general solutions (7): $Br_{qs}=-1/(\sigma_n Nu_0)$. The function of the singular points of the Brinkman number versus the number of sides of a regular polygonal duct ($3 \leq n \leq 16$) with H2 boundary conditions are reported in figure 3. A similar increasing trend is observed for singular points of the Brinkman number versus number of sides of a regular polygonal duct in the case of the boundary condition H2, as well as for the H1 boundary conditions [17].

**Figure 2.** Nusselt numbers versus the Brinkman for different n side numbers of polygonal ducts for H2 boundary conditions.**Figure 3.** Variation of Br_{qs} , Br_{qvd} , Nu_{vd} with n for H2 boundary conditions.

After the singularity point, the Nusselt number decreases with the increase in the Brinkman number until the total heat flux reaches the heat flux generated internally by viscous dissipation processes and the heat flux supplied by the wall into the fluid approaches zero. For a circular pipe with H1 boundary conditions the analytical value of a limited Brinkman number Br_{qvd} is equal to $1/8$ [21] and the limited Nusselt number Nu_{qvd} is equal to 9.6 [22]. For a circular tube with H2 boundary conditions, this value is also the same. When $Br=Br_{qvd}$ and $Nu=Nu_{qvd}$ then the ratio of heat flux generated by viscous dissipation to the total wall heat flux is equal to one ($q_{vd}/q_w=1$). In figure 3, the limited Brinkman number Br_{qvd} and the limited Nusselt number Nu_{vd} versus the number of sides of a regular polygonal duct ($3 \leq n \leq 16$) are given for H2 boundary conditions. The Brinkman number in singularity points, the limited Brinkman and Nusselt number increase in the range of $3 \leq n < \infty$ for polygonal ducts, with the increasing value of n .

4. Conclusions

The BEM method has been used to numerically solve the momentum and the energy equations for the laminar viscous dissipation of flow through a straight regular polygonal duct with H2 boundary conditions under the assumption that the axial condition in the fluid is neglected. The effects of the Brinkman number, the number of sides of a regular polygonal duct and the Nusselt number are studied. A compact correlation between the Nusselt number and the Brinkman number has been proposed for regular polygons with a different number of sides. The singular points of the Brinkman number and the Brinkman number for a limited Nusselt number when the total heat flux reached heat flux is generated by internally by viscous dissipation processes and the heat flux supplied by wall into the fluid approaching zero is obtained. The value of the Brinkman number in a singularity, and the Brinkman number for a limited Nusselt number increased with the number of sides increases for H2 boundary conditions.

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