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Flexural Vibration Band Gap Characteristic of Double Periodic Structure Beams

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Abstract. A double-periodic structural beam is designed. The complex band structure of the structure is deduced by spectral element method. The influence of the length of the third element on the band gap characteristics of the double-periodic structural beam is studied. The results show that the double-periodic structure beams can reduce the band gap frequency, but it will affect the bandwidth of the band gap while maintaining the structural characteristics of the mechanical structure. The length of the third element has little effect on the centre frequency of each band gap, but it will affect the band gap bandwidth.

1. Introduction

The periodic structure has the characteristic of band gap, which has wide application prospect in vibration reduction, sound filtering and new sensor [1]. Therefore, the band gap characteristics in periodic structures have been a hotspot and difficult. There are many researches on One-dimensional periodic structure [2], since the preparation is simple, and the effect is obvious. However, most of the researches are on the general binary cycle structure [3]. There are also ternary periodic structure [4] researches. These researches have great significance to improve the periodic structure of the band gap characteristics and apply the band gap to vibration and noise reduction [5]. However, since the composition of the binary periodic structure is simple, change the band gap characteristics mainly by changing the length of the cell, the use of new materials [6]. Although these methods are very effective, but in changing the length of the cell, it will obviously have a greater impact on the mechanical properties of the structure [7]. And the use of new materials, on the one hand will make the design cost increases, on the other hand subject to material constraints [8]. Many designs still need to further change the length of the cell so that the structure of the band gap to meet the design requirements, so these two design methods are very effective, but the actual use of relatively difficult.

On the basis of the binary periodic structure beams, the five-element double-periodic structural beam is designed in this paper. On the one hand, it can ensure that the mechanical properties of the double-periodic structural beams are similar to those of the binary periodic structure. On the other hand, the length of the cell of the five-element periodic structure beam changes will have an impact on the structure of the band gap. And because of the increase in the third component, the mechanical properties and band gap characteristics of the beam are enriched, and there is more tunability in vibration and noise reduction.

2. Band gap calculation theory of double periodic structures

As shown in figure 1, the double periodic structural beam [9] is a periodic structure composed of AB-AB-C cells. There is an internal period AB-AB structure in the cell, so it is called a double-periodic structure. Compared with the single cycle, the double periodic structure introduces the

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third component C, which makes the structure have more adjustable space and makes it easy to adjust the band gap of the periodic structure. In a single cell, the length of the beams of component A is L_1, component B is L_2 and component C is L_3. The length of the beam of each component is much greater than the width and height of the beam. The beam is satisfied with the condition of Bernoulli-Euler beam. The structural model of the beam is established by the spectral element method. Each component beam can be regarded as a spectral unit, and the single cell can be regarded as the five spectral elements.

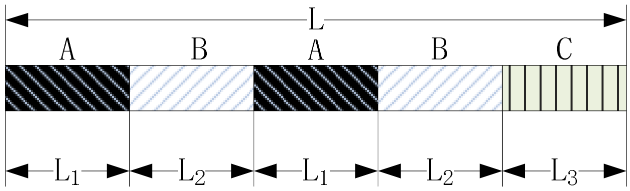

Figure 1. Cell schematic diagram of five-element double periodic structure.

2.1. Connection of spectral unit

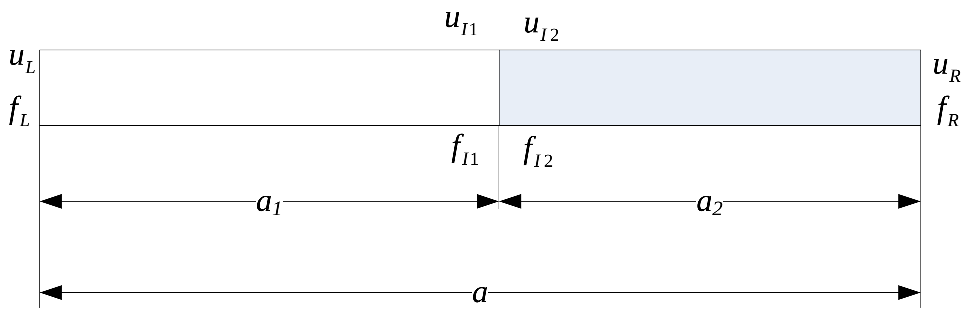

Figure 2. single cell diagram of periodic beam structure.

As shown in figure 2, the beam a is composed of beam a_1 and beam a_2, which are connected Head-to-tail. Each beam can be represented by a spectral unit. The force at the left and right end of beam a_1 are $\boldsymbol{f}_L$ and $\boldsymbol{f}_{I1}$. The displacement at the left and right end of beam a_1 are $\boldsymbol{u}_L$ and $\boldsymbol{u}_{I1}$. The force at the left and right end of beam a_2 are $\boldsymbol{f}_{I2}$ and $\boldsymbol{f}_R$. The displacement at the left and right end of beam a_2 are $\boldsymbol{u}_{I2}$ and $\boldsymbol{u}_R$. The dynamic equation of the beam a_1 is:

$$\begin{Bmatrix} \boldsymbol{f}_L \\ \boldsymbol{f}_{I1} \end{Bmatrix} = \begin{bmatrix} \hat{\boldsymbol{k}}_{11}(a_1) & \hat{\boldsymbol{k}}_{12}(a_1) \\ \hat{\boldsymbol{k}}_{21}(a_1) & \hat{\boldsymbol{k}}_{22}(a_1) \end{bmatrix} \begin{Bmatrix} \boldsymbol{u}_L \\ \boldsymbol{u}_{I1} \end{Bmatrix} \tag{1}$$

For the flexural vibration of the beams, the force vector contains two components of shear force and bending moment, and the displacement vector includes two components of lateral displacement and section angle. As a result the component of the stiffness matrix $\hat{\boldsymbol{k}}_{11}(a_1)$ in the kinetic equation is a second order matrix.

Similarly, the dynamic equation of beam a_2 is:

$$\begin{Bmatrix} \boldsymbol{f}_{I2} \\ \boldsymbol{f}_R \end{Bmatrix} = \begin{bmatrix} \hat{\boldsymbol{k}}_{11}(a_2) & \hat{\boldsymbol{k}}_{12}(a_2) \\ \hat{\boldsymbol{k}}_{21}(a_2) & \hat{\boldsymbol{k}}_{22}(a_2) \end{bmatrix} \begin{Bmatrix} \boldsymbol{u}_{I2} \\ \boldsymbol{u}_R \end{Bmatrix} \tag{2}$$

By the continuous condition of displacement, it is known that $\boldsymbol{u}_{I1} = \boldsymbol{u}_{I2} = \boldsymbol{u}_I$ and $\boldsymbol{f}_{I1} = -\boldsymbol{f}_{I2}$.

Then, the dynamic equation of the cell can be written as follows:

$$\begin{Bmatrix} \boldsymbol{f}_L \\ \boldsymbol{0} \\ \boldsymbol{f}_R \end{Bmatrix} = \begin{bmatrix} \hat{\boldsymbol{k}}_{11}(a_1) & \hat{\boldsymbol{k}}_{12}(a_1) & \boldsymbol{0} \\ \hat{\boldsymbol{k}}_{21}(a_1) & \hat{\boldsymbol{k}}_{22}(a_1)+\hat{\boldsymbol{k}}_{11}(a_2) & \hat{\boldsymbol{k}}_{12}(a_2) \\ \boldsymbol{0} & \hat{\boldsymbol{k}}_{21}(a_2) & \hat{\boldsymbol{k}}_{22}(a_2) \end{bmatrix} \begin{Bmatrix} \boldsymbol{u}_L \\ \boldsymbol{u}_I \\ \boldsymbol{u}_R \end{Bmatrix} \tag{3}$$

Equation (3) can be simplified to get only the left and right ends of the displacement and force of the expression as follows:

$$\begin{Bmatrix} \boldsymbol{f}_L \\ \boldsymbol{f}_R \end{Bmatrix} = \begin{bmatrix} \hat{\boldsymbol{k}}_{11} & \hat{\boldsymbol{k}}_{12} \\ \hat{\boldsymbol{k}}_{21} & \hat{\boldsymbol{k}}_{22} \end{bmatrix} \begin{Bmatrix} \boldsymbol{u}_L \\ \boldsymbol{u}_R \end{Bmatrix} \tag{4}$$

where:

$$\begin{aligned} \hat{\boldsymbol{k}}_{11} &= \hat{\boldsymbol{k}}_{11}(a_1) - \hat{\boldsymbol{k}}_{12}(a_1)\left(\hat{\boldsymbol{k}}_{22}(a_1)+\hat{\boldsymbol{k}}_{11}(a_2)\right)^{-1}\hat{\boldsymbol{k}}_{21}(a_1) \\ \hat{\boldsymbol{k}}_{12} &= -\hat{\boldsymbol{k}}_{12}(a_1)\left(\hat{\boldsymbol{k}}_{22}(a_1)+\hat{\boldsymbol{k}}_{11}(a_2)\right)^{-1}\hat{\boldsymbol{k}}_{12}(a_2) \\ \hat{\boldsymbol{k}}_{21} &= -\hat{\boldsymbol{k}}_{21}(a_2)\left(\hat{\boldsymbol{k}}_{22}(a_1)+\hat{\boldsymbol{k}}_{11}(a_2)\right)^{-1}\hat{\boldsymbol{k}}_{21}(a_1) \\ \hat{\boldsymbol{k}}_{22} &= \hat{\boldsymbol{k}}_{22}(a_2) - \hat{\boldsymbol{k}}_{21}(a_2)\left(\hat{\boldsymbol{k}}_{22}(a_1)+\hat{\boldsymbol{k}}_{11}(a_2)\right)^{-1}\hat{\boldsymbol{k}}_{12}(a_2) \end{aligned} \tag{5}$$

Contrary to equations (1) and (4), it is clear that the dynamic equation after the connection of the two spectral elements is exactly the same as the form of the dynamic equation for a single spectral element. Similarly, the dynamic stiffness equations of a single spectral unit can be obtained by satisfying any of the spectral unit equations connected to each successive sequence under continuous conditions, and the dynamic stiffness matrix after connection can be obtained from the dynamic stiffness of the connected spectral units.

2.2. Band gap characteristics calculation of double periodic structures

According to the above reasoning, we can see that although the cell of double periodic structure contains three components of five components. But by using spectral unit analysis method, the final dynamic equation is as shown:

$$\begin{Bmatrix} \boldsymbol{f}_L \\ \boldsymbol{f}_R \end{Bmatrix} = \begin{bmatrix} \hat{\boldsymbol{k}}_{11} & \hat{\boldsymbol{k}}_{12} \\ \hat{\boldsymbol{k}}_{21} & \hat{\boldsymbol{k}}_{22} \end{bmatrix} \begin{Bmatrix} \boldsymbol{u}_L \\ \boldsymbol{u}_R \end{Bmatrix} \tag{6}$$

According to the periodic structure of the Bloch theorem [10], the boundary displacement and force vectors of the cells satisfy the following relationship:

$$\boldsymbol{u}_R = e^{-iqL}\boldsymbol{u}_L, \quad \boldsymbol{f}_R = -e^{-iqL}\boldsymbol{f}_L \tag{7}$$

Combing equation (6) with (7), we can get the characteristic equation with displacement only:

$$\left(\begin{bmatrix} \hat{\boldsymbol{k}}_{21} & \hat{\boldsymbol{k}}_{22} \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} - e^{-iqL}\begin{bmatrix} -\hat{\boldsymbol{k}}_{11} & -\hat{\boldsymbol{k}}_{12} \\ \boldsymbol{I} & \boldsymbol{0} \end{bmatrix}\right)\begin{Bmatrix} \boldsymbol{u}_L \\ \boldsymbol{u}_R \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \end{Bmatrix} \tag{8}$$

For each frequency point, solving the eigenvalues of equation (11), we can get two pairs of identical Bloch wave solutions $\pm q_1$, $\pm q_2$. The solution of each q represents a kind of intrinsic wave. When the imaginary part of q is not 0, the eigen wave propagates in the process of propagation, so all the frequency points that make the imaginary part of q are not zero is called the band gap of the structure.

3 .Band gap calculation and result analysis

Aiming at the five-element double periodic structure beam, the band gap of the double period structure and the simple binary period structure is compared and the influence of the third component on the double periodic structure band gap is considered.

3.1. Comparison between double periodic and binary period structure

The double periodic structure model is shown in figure 1, the parameters are as follows. The length $L_1 = L_2 = L_3 = 0.1m$. The shape of each component is the same. Section size: $b = 0.01m$, $h = 0.01m$. The material parameters of each component are shown in table 1.

Table 1. Material parameters of component.

Code	Material	Density (kg/m^3)	Elastic modulus (GPa)
A	Aluminum	2799	72
B	Plexiglass	1142	2.0
C	Epoxy resin	1180	4.3

In order to compare with the binary periodic structure, two binary periodic structures are designed. The cell length of the first binary periodic structure is equal to the cell length of the double periodic structure. The length of each element is 0.25 meters, The cross-sectional size is consistent with the bending of the double-periodic structure. The cell composition is aluminum and plexiglass. Because the structure is consistent with the characteristic parameters of the double-periodic structure, it is called the characteristic binary periodic structure in this paper. For the second binary periodic structure, the element length is consistent with that of the double-periodic structure. The geometrical parameters of the double periodic structure are close to each other, and this is called geometric binary periodic structure.

In this paper, the complex band structure is calculated. The complex band structure of the double periodic structure, the characteristics binary periodic structure and the geometric binary periodic structure are shows as figure 3, figure 4and figure 5, respectively. The figure 3 shows that the first three band gap frequency of the double periodic structure ranges are 24Hz-30Hz, 102Hz-140Hz, 262Hz-340Hz, and the figure 4 shows that the first three band gap frequency of the characteristics binary periodic structure are 24Hz-38Hz, 108Hz- 212Hz, 308Hz-464Hz. The figure 5 shows that the first two band gap frequency of the geometric binary periodic structure of the first two band gap frequency range of 150Hz-230Hz, 672Hz-1316Hz.

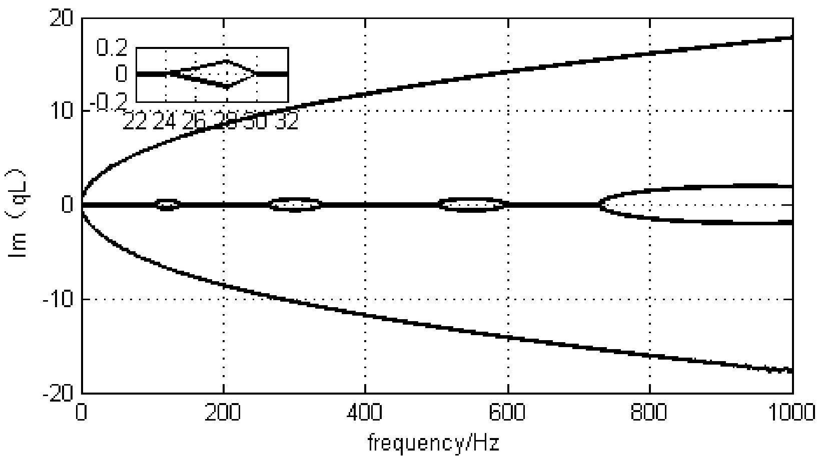

Figure 3. Complex band structure of the double periodic structure.

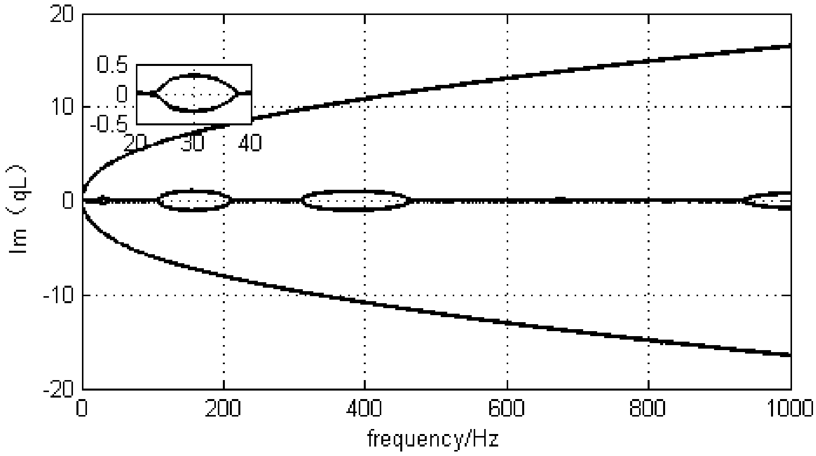

Figure 4. Complex band structure of the characteristics binary periodic structure.

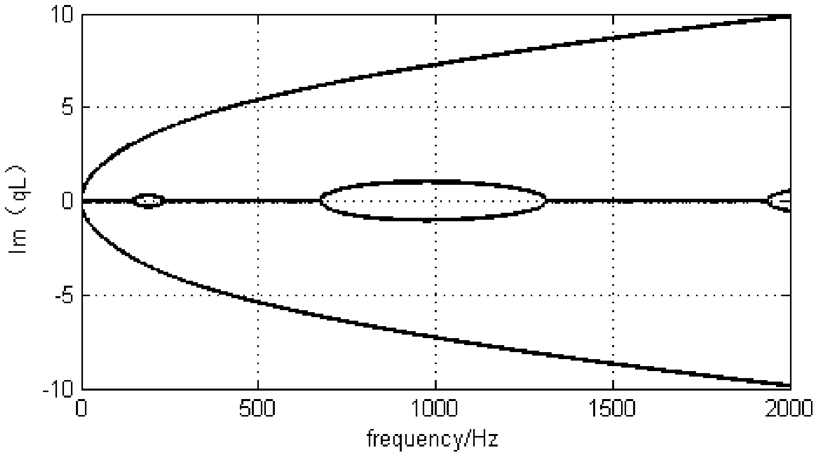

Figure 5. Complex band structure of the geometric binary periodic structure.

Compared with the double periodic structure, the starting frequency of the band gap is slightly lower than the starting frequency of the characteristic binary structure, but the frequency range is narrower. Compared with the double periodic structure and the geometric binary structure, the band gap start frequency of the double periodic structure is obviously reduced in the case of the geometrical feature size of the structure, and the change of the strength and stiffness of the structure due to the small geometric parameters is not large, thus realizing the band gap frequency of the structure without affecting the use function of the structure.

3.2. The influence on the band gap of the third group

For the double periodic structural beam, due to the presence of the third component, the element obviously affects the band gap characteristics of the structure. In this paper, we study the influence of the length of the third element on the gap in the case of a constant cell length. The composition of the calculated model and the cross-section of the structure are consistent with the double periodic structural model in table 2. The length of the cell is 0.5m, and the length of each element of aluminum and plexiglass is equal, the length of the epoxy resin is changed, the first three frequencies obtained band gap in table 1 under different epoxy length is shown.

As it shown in table 2, the frequency band of the first band gap of the double periodic structure is slightly widened with the increase of length of the third element, but the change is not obvious. The starting frequency of the second band gap rises, the termination frequency decreases. The frequency of the third band gap band is reduced, the termination frequency is increased, and the frequency band is widened. For the whole, the center frequency of each band gap does not change much, but the effect of each band gap is changed due to the change in length Therefore, in the actual engineering application, should be based on the actual needs, determine the target attenuation band, select the optimal length of the third component, making the structure to produce the greatest attenuation effect.

Table 2. The first three frequencies obtained band gap under different epoxy length.

Length of epoxy (m)	The first band gap	The second band gap	The third band gap
0.18	24Hz-32Hz	112Hz-128Hz	246Hz-336Hz
0.14	24Hz-32Hz	106Hz-134Hz	250Hz -344Hz
0.1	24Hz-30Hz	104Hz-140Hz	262Hz-340Hz
0.06	24Hz-30Hz	98Hz-148Hz	278Hz -328Hz
0.02	24Hz-28Hz	96Hz-150Hz	292Hz -308Hz

4. Conclusions

Based on the combination of spectral unit theory and Bloch theory, the band gap calculation formula of bi-periodic structural beam is introduced, and the band gap characteristics of the double periodic structural beam are calculated. The periodic structure beam with the same characteristic parameters and the periodic structure beam with several parameters And the effect of the third component on the band gap characteristics is studied. The main conclusions are as follows:

(1)In the case where the cell length is constant, the starting frequency of the band gap is slightly lower than the starting frequency of the characteristic binary structure, but the band is narrower.

(2)In the case of the geometrical feature size of the structure, the band gap start frequency of the double periodic structure is obviously reduced, and the geometrical parameters change little, so that the strength and stiffness of the structure are not changed greatly, the use of the function of the case, reducing the structure of the band gap frequency.

(3) The change of the length of the third element almost does not affect the center frequency of each band gap of the double periodic structural beam, but with the increase of the length, the frequency band of the first band gap of the bi-periodic structure is slightly widened, but the change is not obvious; The starting frequency of the two band gap increases, the termination frequency decreases, the frequency band becomes narrow, the starting frequency of the third band gap band decreases, the termination frequency increases, and the frequency band becomes wider.

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