

MHD micropolar fluid flow over a stretching permeable sheet in the presence of thermal radiation and thermal slip flow: a numerical study

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Abstract. In this manuscript, a mathematical explanation is an attempt at meant for two-dimensional, micropolar fluid flow over a permeable stretching sheet with viscous dissipation in the occurrence of thermal radiation and temperature dependent slip flow. With the similarity transformations, the governing equations have been changed into a scheme of ordinary differential equations. These differential equations are extremely nonlinear which cannot be solved analytically. Thus, bvp4c MATLAB solvers have been used for solving it. Numerical consequences are obtained for the skin-friction coefficient, the couple wall stress and the local Nusselt number with the same as velocity, microrotation and temperature profiles for various values of the governing parameters, namely, material factor, magnetic factor, thermal slip factor, radiation factor, Prandtl number and Eckert number. It is found that magnetic field reduces the fluid velocity and angular velocity, but magnetic field enhances the fluid temperature. Furthermore, fluid temperature increases with increases in thermal slip parameter.

1. Introduction

The essential continuum hypothesis for micropolar fluid was initially presented and defined by Eringen [1-2]. This hypothesis thinks about the infinitesimal impacts coming about because of the nearby structure and small scale movements of the fluid components and has been utilized to contemplate various different stream circumstances, for example, the stream of low focus suspensions, fluid precious stones, blood, colloidal fluids, Ferro-liquids, and so forth, for which the established Navier-Stokes hypothesis is insufficient.

The magnetohydrodynamics (MHD) heat and mass exchange from various geometries implanted in a permeable medium are of enthusiasm for building and geological applications, for example, geothermal repositories, warm protection, cooling of atomic reactors and improved oil recuperation. Numerous concoction designing procedures, for example, metallurgical and polymer expulsion forms include cooling of molten liquid being extended into a cooling framework; the fluid mechanical properties of the penultimate item rely on for the most part on the cooling liquid utilized and the rate of stretching. Some polymer fluids, for example, polyethylene oxide arrangement in cetane, having better



electromagnetic properties, are basically utilized as cooling liquid as their stream can be controlled by outer magnetic fields with a specific end goal to enhance the nature of the conclusive item. Sakiadis ([3– 4]) researched the boundary layer flow incited by a moving plate in a tranquil encompassing fluid. From that point, different parts of the issue have been explored by many creators, such as Fang [5], Fang and Lee [6] and white [7]. Chamkha and Khaled [8] have researched the impacts of magnetic field on normal convection flow over a vertical surface.

Majidiana et al. [9] investigated the joule heating effects on a boundary layer of an MHD micropolar fluid over a non-isothermal permeable stretching sheet with variable electric conductivity and concluded that the velocity boundary layer thickness diminishes as the magnetic parameter coefficient increments however it expands temperature profile thickness. Ahmad and Hussain [10] researched the heat transfer for MHD micropolar fluids flow through a porous medium over a stretching surface in the nearness of magnetic field and a heat source and presumed that the microrotation expands somewhat far from the boundary with increment in the estimations of micropolar parameters. Mishra et al. [11] contemplated the planar flow of an electrically leading incompressible viscous fluid on a vertical plate with variable wall temperature and concentration in a double stratified micropolar fluid within the sight of a transverse magnetic field and inferred that the skin erosion coefficient increments with the expanding estimation of the magnetic parameter M and the couple number N . Mahmood and Waheed [12] have demonstrated the MHD flow and heat exchange of a micropolar fluid over an extending surface with heat generation (absorption) and slip velocity. Chaudhary and Abhay [13] considered the impact of chemical reactions on MHD micropolar fluid flow over a vertical plate in slip-flow administration.

The impacts of radiation on unsteady free convection flow and heat transfer issues have turned out to be more essential in enterprises. At high working temperatures, radiation impacts can be very critical. Many procedures in engineering happen at high temperature and information of radiation heat transfer turns out to be critical for the outline of solid hardware, atomic plants, gas turbines and different impetus gadgets or airplane, rockets, satellites and space vehicles. Sharma et al. [14] have explored the Effects of chemical reaction on magneto-micropolar fluid flow from a radiative surface with variable porousness. Ibrahim et al. [15] examined the instance of mixed convection flow of a micropolar fluid went through a semi-boundless, steadily moving porous plate with differing suction velocity ordinary to the plate within the sight of thermal radiation and viscous dissipation. Oahimire and Olajuwon [16] considered the heat and mass transfer consequences for an unsteady of a micropolar fluid over an endless moving penetrable plate in an immersed porous medium within the sight of a transverse magnetic field, radiation absorption and thermo-diffusion. Modather et al. [17] contemplated MHD heat and mass exchange flow of a micropolar fluid over a vertical penetrable plate in a porous medium without considering the impacts of thermal radiation, heat generation, and thermal diffusion. Abbasi et al. [18] exhibited an examination to show the MHD two-dimensional boundary layer flow of Jeffrey nanofluid over an extending sheet with thermal radiation. Hussain et al. [19] analyzed the flow issue coming about because of the extending of a surface with convective conditions in an MHD third-grade nanofluid within the sight of thermal radiation. Hayat et al. [20] researched the Brownian movement and thermophoresis impacts on two-dimensional boundary layer flow of an Oldroyd-B nanofluid within the sight of thermal radiation and heat generation.

Kameswaran et al. [21] explored the hydromagnetic convective heat and mass transfer in nanofluid flow over an stretching sheet subject to viscous dissipation, chemical reaction, and Soret impacts. Eldabe and Ouaf [22] tackled the issue of heat and mass transfer in a hydro magnetic flow of a micropolar fluid past an extending surface with ohmic heating and viscous dissipation utilizing the Chebyshev finite difference method. Mohanty et al. [23] considered the unsteady heat and mass transfer attributes of a viscous incompressible electrically directing micropolar fluid over a stretching sheet through a porous medium within the sight of viscous dissipation and inferred that the viscous dissipation produces heat because of drag between the fluid particles, which drag causes an expansion in fluid temperature. Pavithra and Gireesha [24] talked about the impact of inside heat generation/absorption on dusty fluid flow over an exponentially extending sheet with viscous

dissipation. Krishnamurthy et al. [25] examined the boundary layer flow and heat transfer of a nanofluid with fluid molecule suspension over an exponentially extending surface within the sight of transverse magnetic field and viscous dissipation. Mabood, Khan and Ismail ([26], [27]) concentrated on the investigation of joined heat and mass transfer of electrically directing nanofluid over a non-linear extending surface within the sight of a first-order chemical response and viscous dissipation.

The present examination explores the viscous incompressible electrically leading micropolar fluid on stretching permeable sheet within the sight of thermal radiation and thermal slip impact. Utilizing the similarity transformations, the governing equations have been changed into an arrangement of ordinary differential equations, which are nonlinear and can't be explained analytically, in this way, bvp4c MATLAB solver has been utilized for solving it. The outcomes for velocity, microrotation, and temperature capacities are completed for the extensive variety of essential parameters to be a specific material parameter, magnetic parameter, Eckert number and first order slip velocity parameter and second order velocity slip parameter. The skin friction, the couple wall stress and the rate of heat transfer have likewise been processed.

2. Mathematical formulation

Give us a chance to consider an incompressible laminar two-dimensional MHD and radiative micropolar fluid disregarding a penetrable plane surface. The flow is thought to be in the x-bearing, which is brought the plate the forward way and y-pivot is ordinary to it. A variable magnetic field is connected in the y-bearing that is ordinary to flow direction. No outside electric field is connected. Additionally, the magnetic Reynolds number is small to the point that the magnetic field actuated by the moving fluid is immaterial as for the outer magnetic field. The physical outline of the issue appears in figure 1. The electrical conductivity is accepted to have the form:

$$\sigma = \sigma_0 u \quad (2.1)$$

Where σ_0 is a constant

For the flow under study, it is relevant to assume that the applied magnetic field strength has the form [36]:

$$B(x) = B_0 x^{-1/2} \quad (2.2)$$

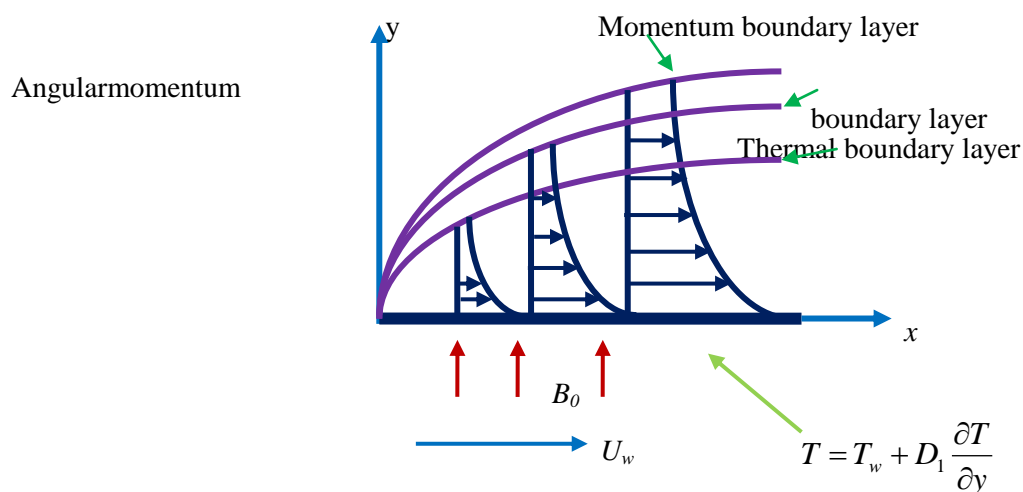


Figure 1. Schematic diagram of the physical problem

Where B_0 is a constant

With regular boundary conditions, the governing equation for the force, temperature and angular velocity field inside the boundary layer are given by [9]:

Continuity equation

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = 0 \quad (2.3)$$

Linear momentum equation

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \left(\nu + \frac{\kappa}{\rho} \right) \frac{\partial^2 u}{\partial y^2} - \frac{\sigma_0 B_0^2}{\rho x} u^2 + \frac{\kappa}{\rho} \frac{\partial N}{\partial y} \quad (2.4)$$

Angular momentum equation

$$\rho j \left(u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} \right) = \gamma \frac{\partial^2 N}{\partial y^2} - \kappa \left(2N + \frac{\partial u}{\partial y} \right) \quad (2.5)$$

Energy equation

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \left(\frac{\mu + \kappa}{\rho c_p} \right) \left(\frac{\partial u}{\partial y} \right)^2 - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} + \frac{\sigma B_0^2}{\rho c_p x} u^3 \quad (2.6)$$

The boundary conditions for the velocity, Angular Velocity and temperature fields are

$$u = U_w = Cx, v = v_w, N = -s \frac{\partial N}{\partial y}, T = T_w + D_1 \frac{\partial T}{\partial y} \text{ at } y = 0$$

$$u \rightarrow 0, N \rightarrow 0, T \rightarrow T_\infty \text{ as } y \rightarrow \infty \quad (2.7)$$

Where u and v are the velocity segment along the x and y axes separately, ν is kinematic viscosity, N is angular velocity, κ is vortex viscosity, ρ is fluid density, γ is spin gradient viscosity, j is the micro inertia per unit mass, $B(x)$ and σ is variable magnetic field and the electrical conductivity, respectively, T is temperature, k is the thermal conductivity, q_r is the radiative heat flux and c_p is the specific heat at constant pressure, C is a positive constant, s is microrotation parameter.. At the point when microrotation parameter, $s = 0$ we get $N(x,0)=0$ which speaks to no-spin condition i.e. the microelements in a concentrated molecule flow near the wall are not ready to turn as was expressed by Jena and Mathur [28]. The case comparing to $s = 0.5$ outcomes in the vanishing of the antisymmetric part of the stress tensor and represents weak concentrations. The case relating to $s = 1.0$ is the agent of turbulent boundary layer flows (see Peddison and McNitt [29]).

By utilizing the Rosseland estimate the radiative heat flux q_r is given by

$$q_r = - \frac{4\sigma_1}{3k^*} \frac{\partial T^4}{\partial y} \quad (2.8)$$

Where σ_1 is the Stefan -Boltzmann constant and k^* is the mean absorption coefficient. It ought to be noticed that by utilizing the Rosseland estimation, the present investigation is restricted to optically thick fluids. In the event that temperature contrasts inside the flow are fundamentally small, at that point condition (2.8) can be linearized by growing T^4 into the Taylor series about T_∞ , which after neglecting the higher order terms takes the form:

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \quad (2.9)$$

In view of equations (2.8) and (2.9), eqn. (2.6) reduces to

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \left(\frac{k}{\rho c_p} + \frac{16\sigma_1 T_\infty^3}{3k^*} \right) \frac{\partial^2 T}{\partial y^2} + \left(\frac{\mu + \kappa}{\rho c_p} \right) \left(\frac{\partial u}{\partial y} \right)^2 + \frac{\sigma B_0^2}{\rho c_p x} u^3 \quad (2.10)$$

The continuity equation (2.3) is satisfied by equations

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x} \quad (2.11)$$

where $\psi(x, y)$ is the stream function.

In order to obtain local similarity solution of the problem, following changes are introduced:

$$\begin{aligned} \eta = \sqrt{C/\nu} y, \psi = \sqrt{C\nu} x f(\eta), N = C\sqrt{C/\nu} x h(\eta) \\ T = T_\infty + \frac{D}{\kappa} \left(\frac{\nu}{C} \right) \left(\frac{x}{L} \right)^2 \theta(\eta), K = \frac{\gamma}{\mu j}, \lambda = \frac{\kappa}{\mu}, B = \frac{\nu}{Cj} \\ M = 1 + \frac{\sigma_0 B_0^2}{\rho}, Pr = \frac{\rho \nu c_p}{k}, Ec = \frac{k}{D} \frac{L^2 C^3}{\sqrt{C\nu} c_p}, R = \frac{16\sigma_1 T_\infty^3 \rho c_p}{3k * k} \end{aligned} \quad (2.12)$$

where f is the dimensionless stream function, θ is the dimensionless temperature, η is the similarity variable, C and D are equation constants, M is the magnetic parameter, Ec is the Eckert number, λ is the vertex viscosity parameter, L is the characteristic length, K and B are the dimensionless material parameters, R is the radiation parameter, Pr is the Prandtl number.

In view of equations (2.11) and (2.12), the equations (2.4), (2.5) and (2.10) transform into

$$(1 + \lambda) f''' + f f'' + K h' - M f'^2 = 0 \quad (2.13)$$

$$K h'' + f h' - f' h - \lambda B (2h + f'') = 0 \quad (2.14)$$

$$(1 + R) \frac{1}{Pr} \theta'' + f \theta' - 2 f' \theta + Ec (1 + \lambda) (f'')^2 + Ec (M - 1) f'^3 = 0 \quad (2.15)$$

The corresponding boundary conditions are

$$\begin{aligned} f(0) = f_w, f'(0) = 1, h(0) = 0, \theta(0) = 1 + \beta \theta'(0) \\ f' = h = \theta = 0 \text{ as } \eta \rightarrow \infty \end{aligned} \quad (2.16)$$

where the primes denote differentiation with respect to η and the $\beta = D_1 \sqrt{C/\nu}$ is the thermal slip parameter

The physical quantities of interest are the skin friction coefficient C_{fx} , the local couple wall stress M_{wx} and the local Nusselt number Nu_x which are defined as

$$C_{fx} = \frac{2}{\rho U_w^2} \left[(\mu + \kappa) \left(\frac{\partial u}{\partial y} \right)_{y=0} + \kappa (N)_{y=0} \right] = 2(1 + K) Re_x^{-1/2} f''(0) \quad (2.17)$$

$$M_{wx} = \gamma \left(\frac{\partial N}{\partial y} \right)_{y=0} = \frac{\gamma C^2}{\nu} x h'(0) \quad (2.18)$$

$$Nu_x = -\frac{x}{T_w - T_\infty} \left(\frac{\partial T}{\partial y} \right)_{y=0} = -Re_x^2 \theta'(0) \quad (2.19)$$

Our main aim is to investigate how the values of $f''(0)$, $h'(0)$ and $-\theta'(0)$ vary in terms of the various parameters.

3. Solution of the problem

The arrangement of conditions (2.13) to (2.15) was decreased to an arrangement of first-order differential equations and explained utilizing a MATLAB boundary value problem solver called

bvp4c. This program takes care of boundary value problems for ordinary differential equations of the form $y' = f(x, y, p)$, $a \leq x \leq b$, by implementing a collocation method subject to general nonlinear, two-point boundary conditions $g(y(a), y(b), p)$. Here p is a vector of obscure parameters. Boundary value problems (BVPs) emerge in most different structures. Pretty much any BVP can be planned for a solution with bvp4c. The initial step is to compose the ODEs as an arrangement of first order ordinary differential equations. The points of interest of the solution technique are introduced in Shampine and Kierzenka [30].

4. Results and discussion

The representing conditions (2.13) - (2.15) subject to the boundary conditions (2.16) are incorporated as described in section 3. So as to get an unmistakable knowledge into the physical issue, the velocity, angular velocity, and temperature have been examined by allotting numerical estimates to the parameters experienced in the issue. So as to approve the strategy utilized as a part of this investigation and to judge the precision of the present examination, correlation with accessible after effects of Gorla and Sidawi [31] relating to the local Nusselt number for different estimations of Pr is made (Table 1) and found in the phenomenal assertion.

Figures 2a, 2b and 2c demonstrates that the impact of viscosity parameter (λ) on velocity, angular velocity, and temperature separately. The velocity of the fluid increments with an expanding λ (Figure 2a). The angular velocity increments from the sheet and reductions far from the sheet (fig.2b). The temperature of the fluid increments with the impact of viscosity parameter this appears in figure 2c.

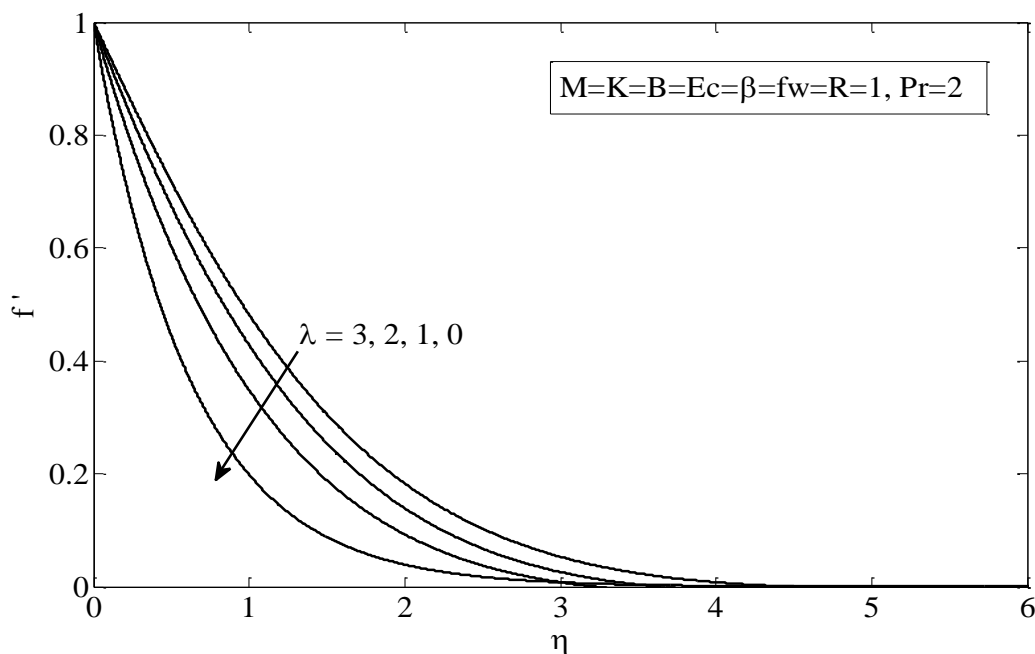


Figure 2a. Velocity for different values of λ

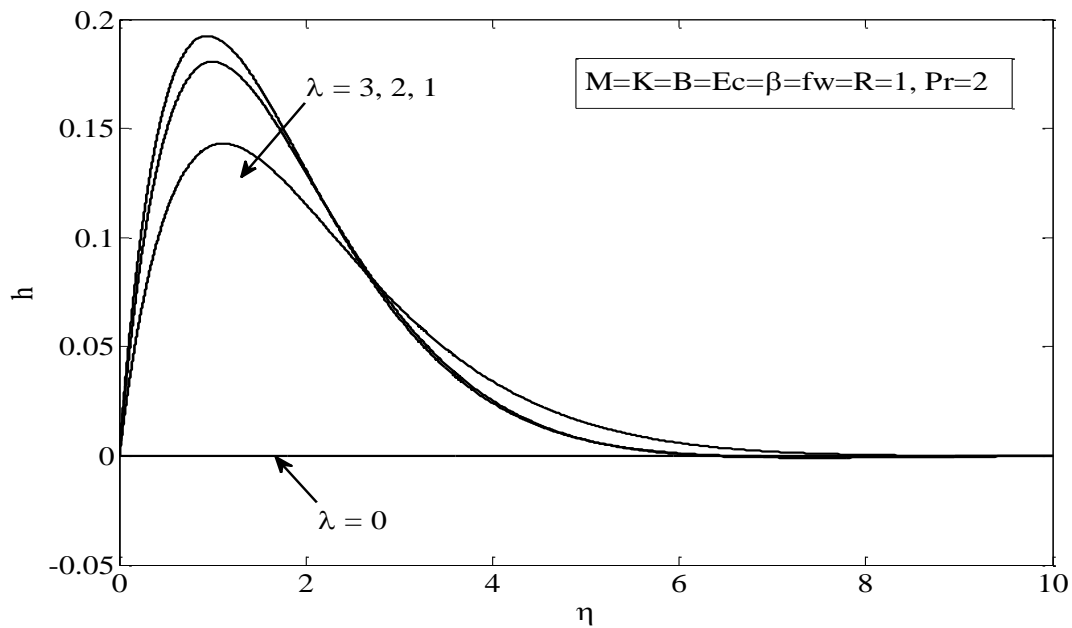


Figure 2b. Angular velocity for different values of λ

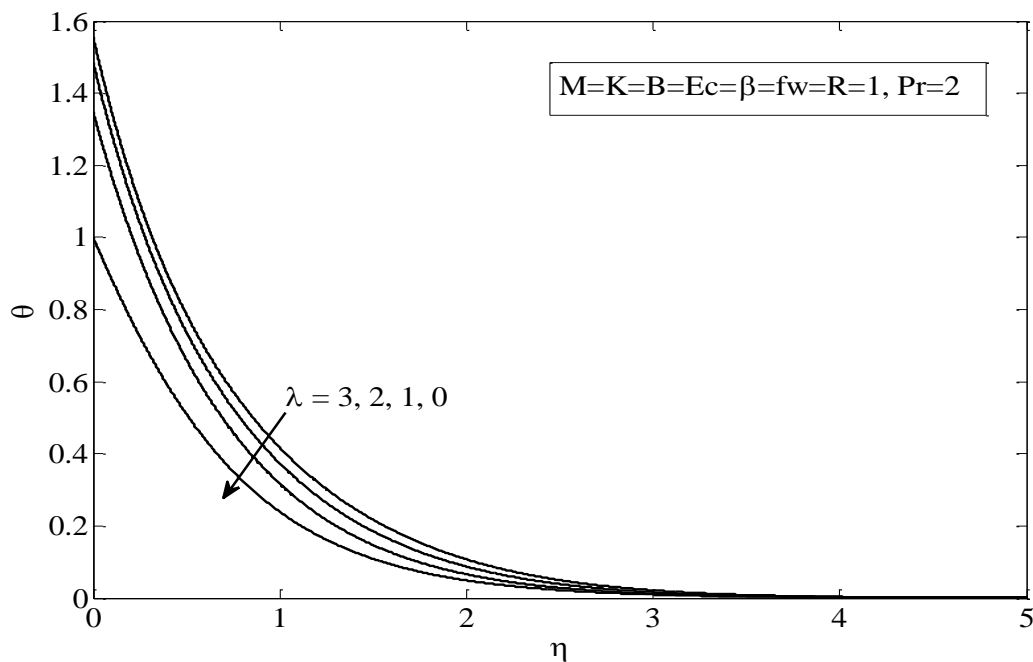


Figure 2c. Temperature for different values of λ

Figures. 3a, 3b and 3c delineate the impact of the magnetic parameter (M) in velocity, angular velocity and temperature profiles individually. It is seen that the velocity of the fluid declines with an expanding magnetic parameter (see figure.2a). The magnetic parameter is found to impede the velocity at all points of the flow field. It is on the grounds that the utilization of transverse magnetic field will bring about a resistive type force (Lorentz force) like drag force which tends to oppose the fluid flow,

therefore, diminishing its velocity. The angular velocity of the fluid reductions close to the sheet and increments far from the sheet with the impact of themagnetic parameter (figure.3b).The temperature of the fluid increments with raising the magnetic parameter (figure.2c). An addition in M produces higher Lorentz force (resistive force) which has the trademark to change over some thermal energy into heat energy.

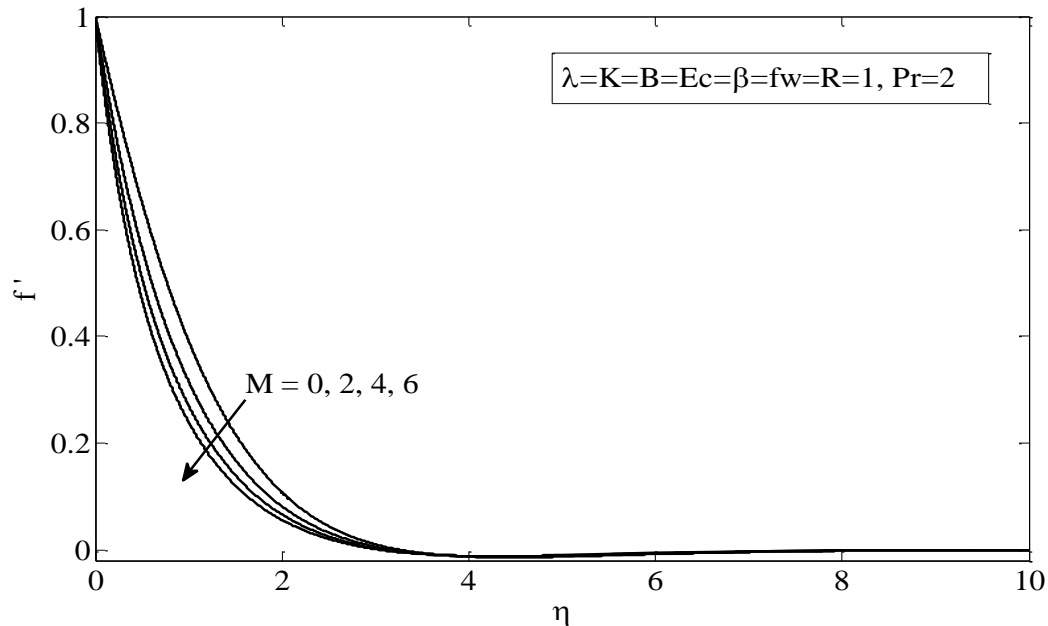


Figure3a.Velocity for different values of M

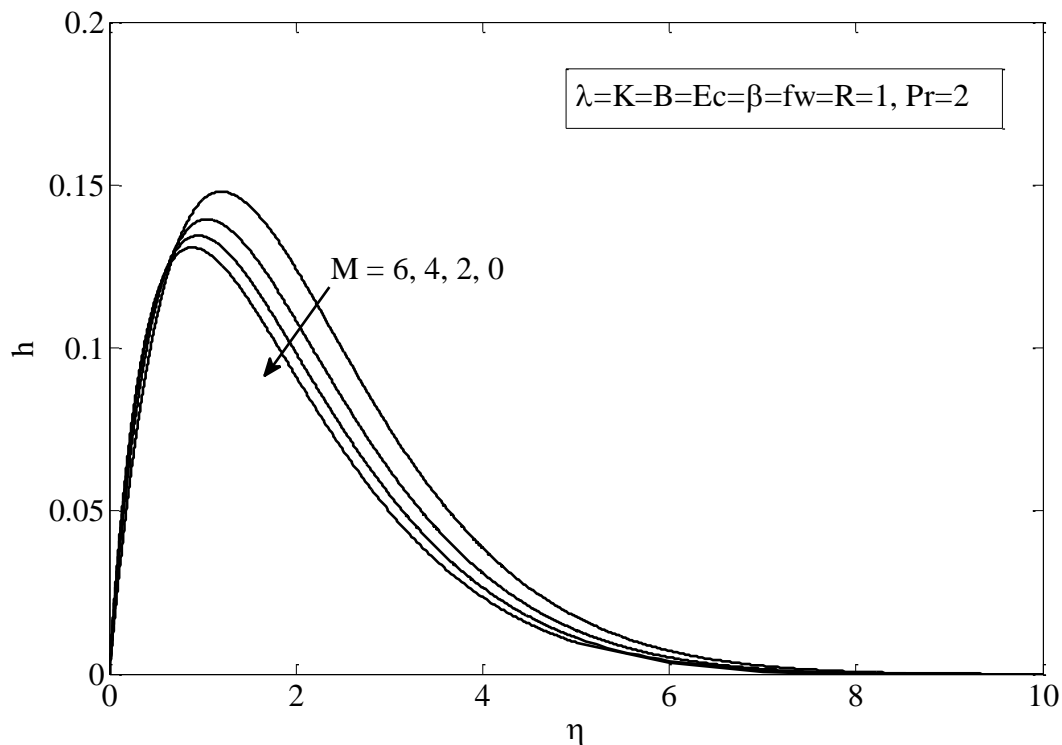


Figure 3b.Angular velocity for different values of M

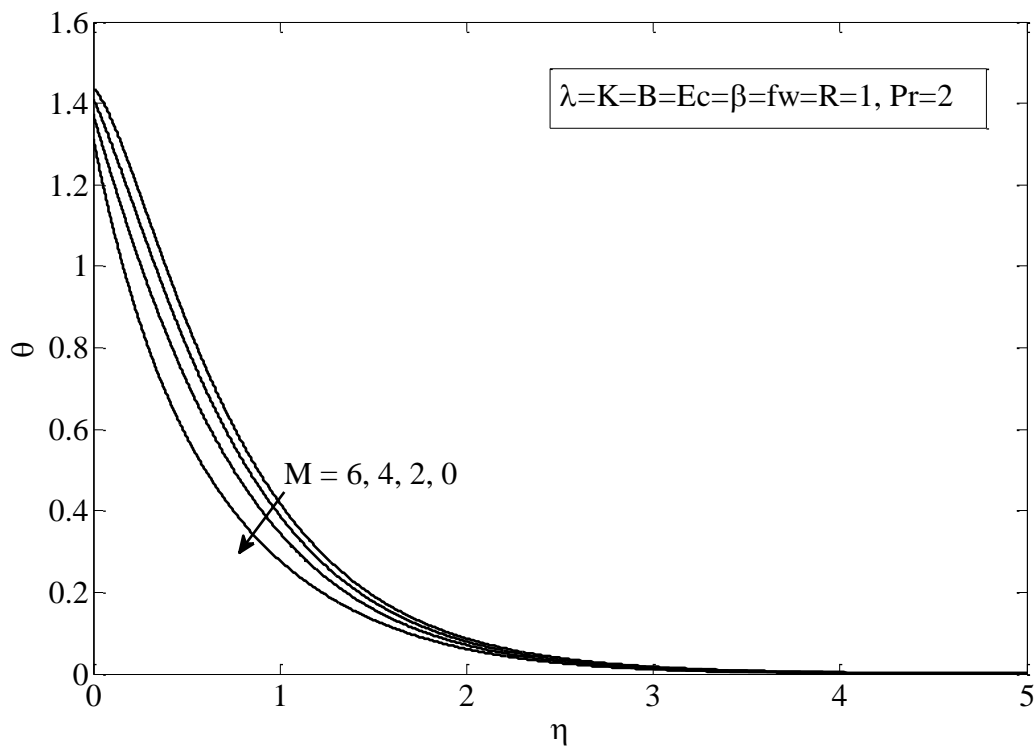


Figure 3c. Temperature for different values of M

Figures.4a, 4b and 4c depict the impacts of the material parameter on velocity, angular velocity, and temperature individually. It is watched that the velocity and angular velocity increment with the impact of the material parameter (figure.4a&4b). The temperature of the fluid reductions with raising the material parameter (4c). Accordingly, micropolar fluids indicate drag lessening contrasted with viscous fluids.

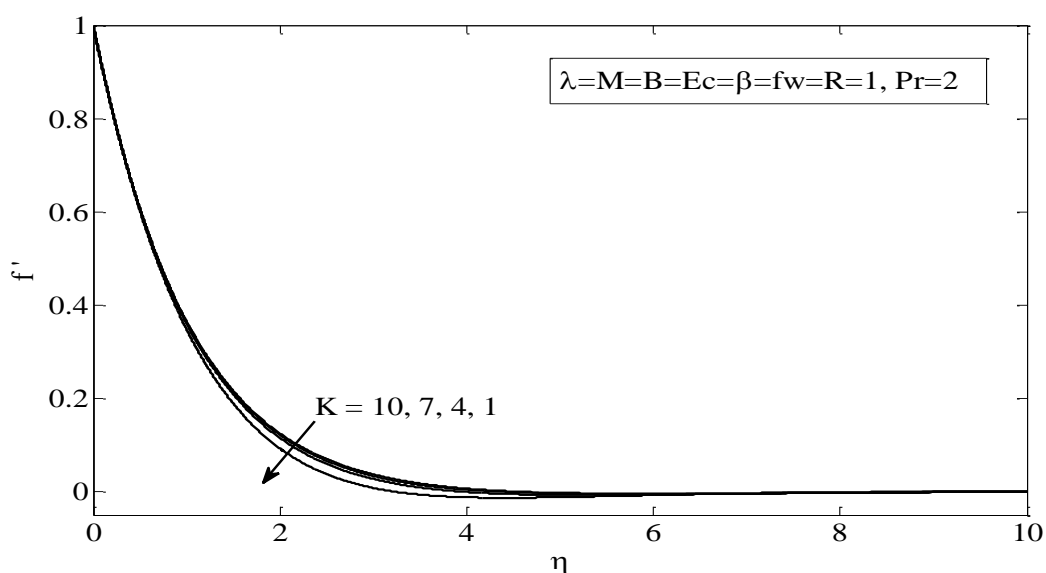


Figure 4a. Velocity for different values of K

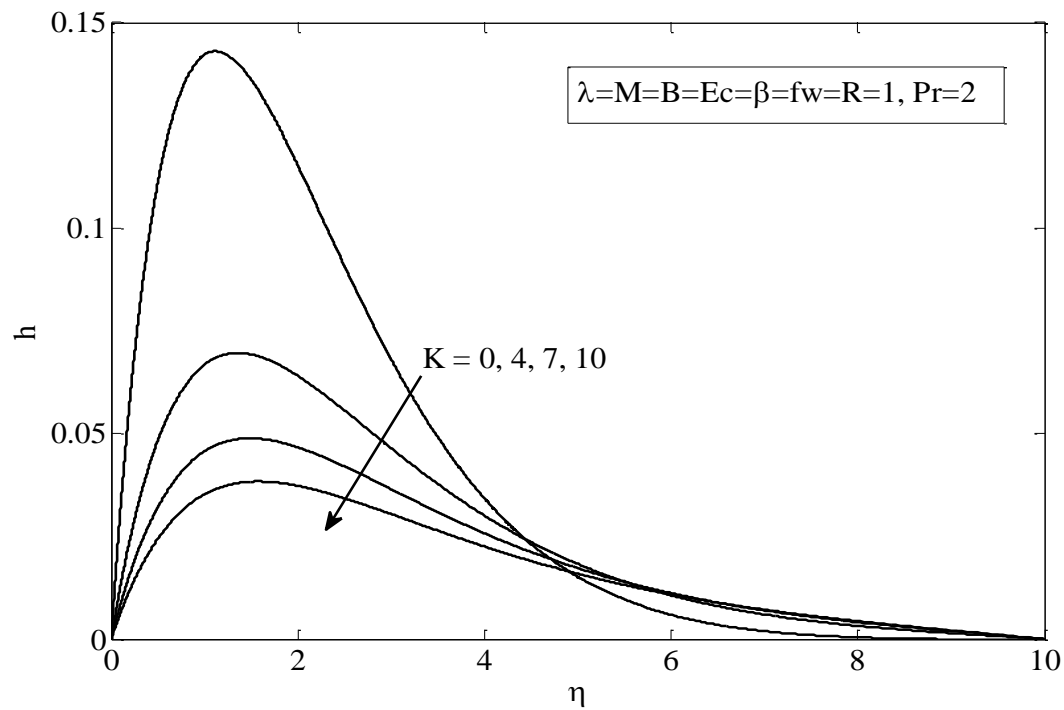


Figure 4b. Angular velocity for different values of K

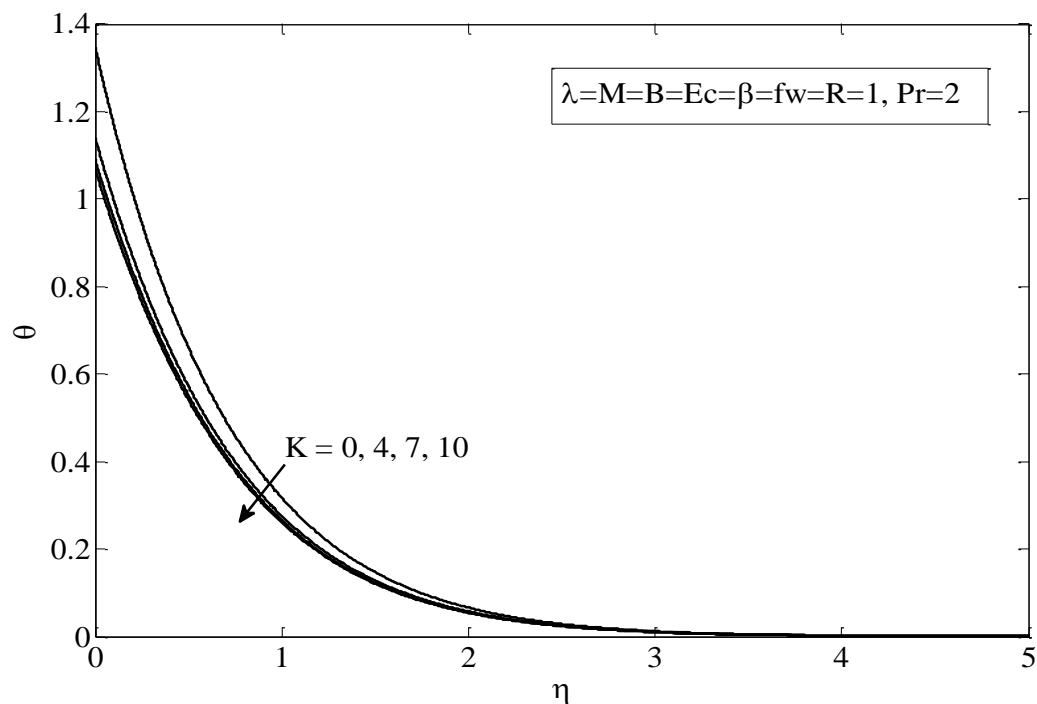


Figure 4c. Temperature for different values of K

Figures.5a, 5b and 5c demonstrates that the impact of suction/injection on velocity, angular velocity, and temperature individually. It is seen that velocity, angular velocity, and temperature diminishes with raising suction/injection parameter. The fluid velocity and angular velocity is higher

on account of injection than that on account of suction. The thermal boundary layer is thicker on account of injection than that on account of suction

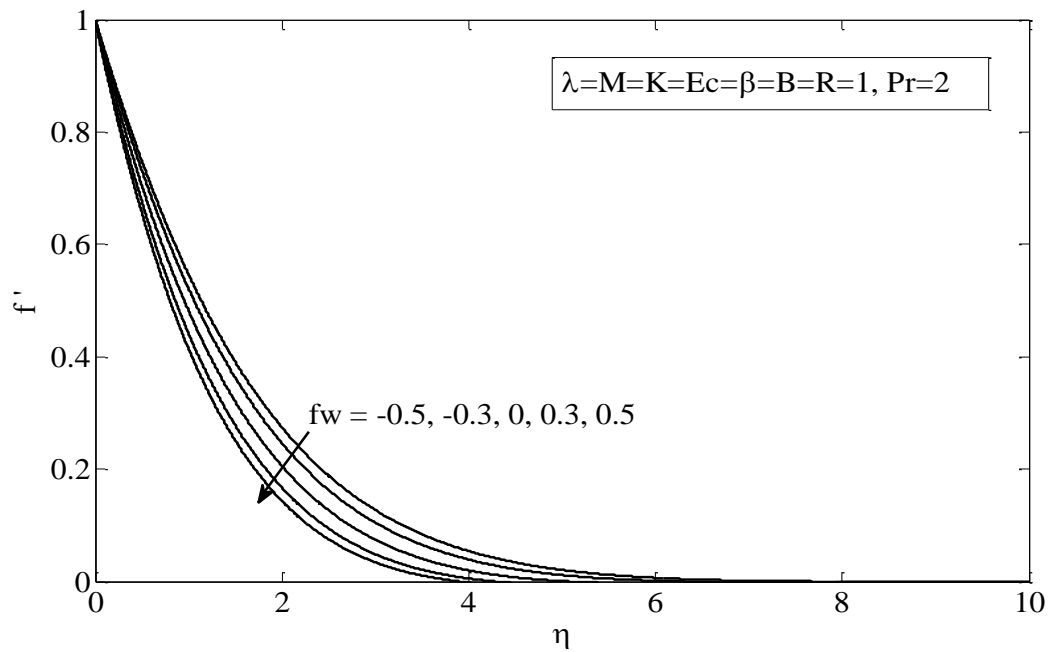


Figure 5a. Velocity for different values of fw

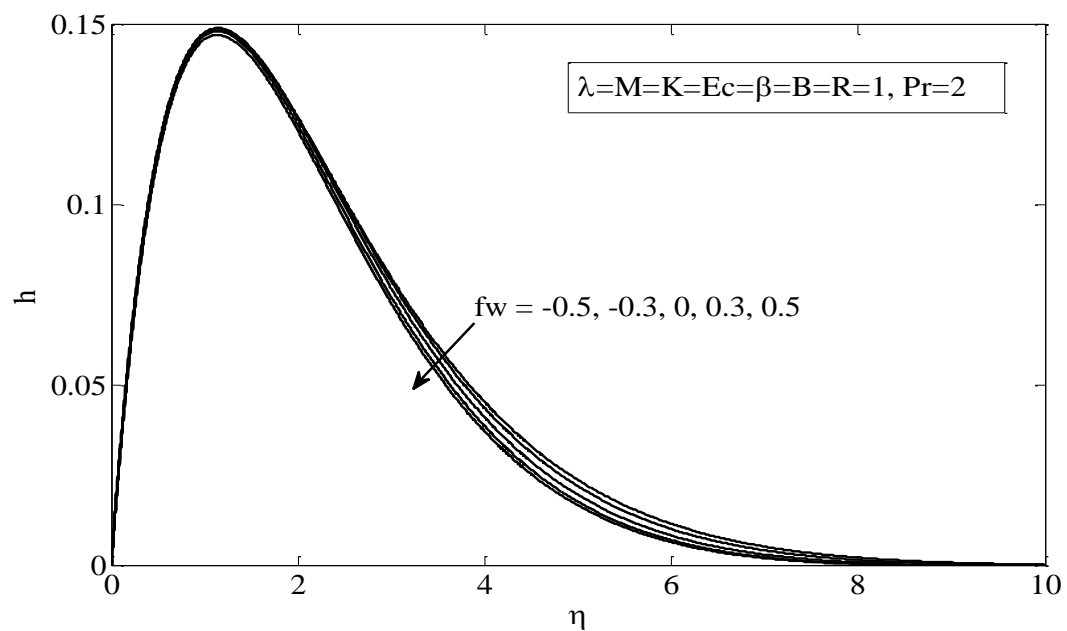


Figure 5b. Angular velocity for different values of fw

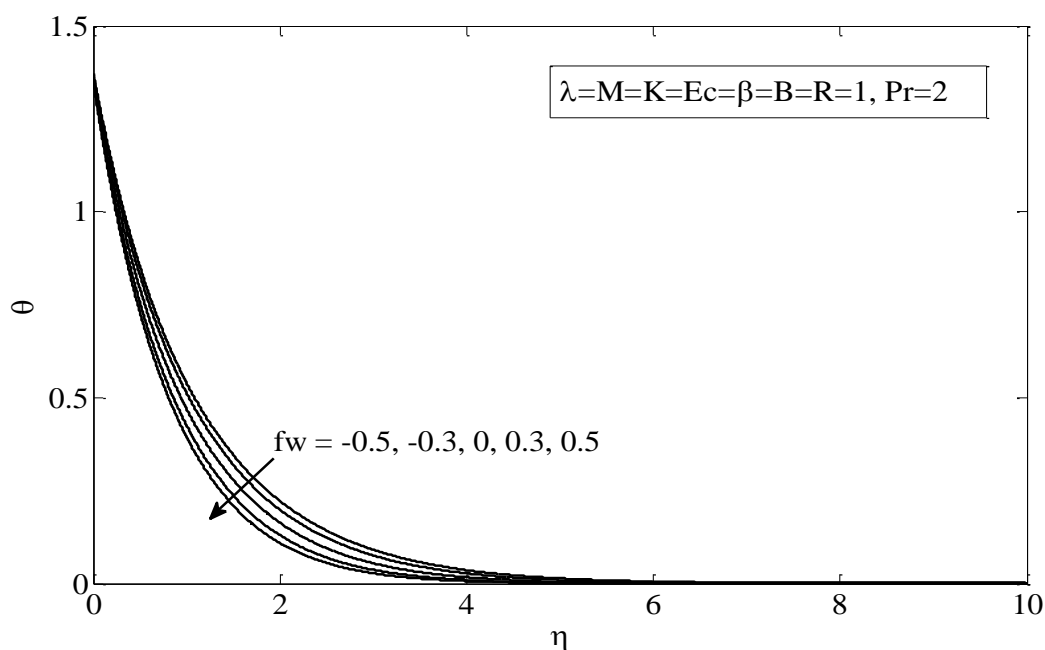


Figure 5c. Temperature for different values of f_w

The impact of thermal slip parameter on temperature appears in the figure. 6. It is watched that the temperature of the fluid increments with expanding β .

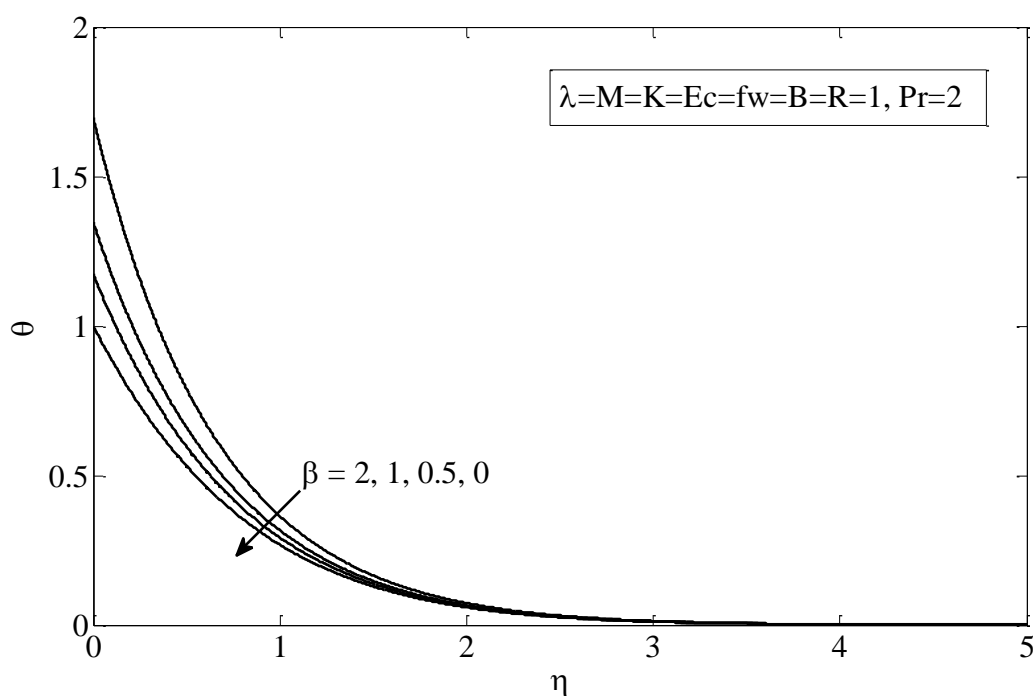


Figure 6. Temperature for different values of β

The impact of Eckert number on temperature profile appears in figure.7. It is seen that temperature increments with raising Ec . The Eckert number Ec communicates the connection between the kinetic energy in the flow and the enthalpy. It exemplifies the change of Kinetic energy into internal energy

by work done against the viscous fluid stresses. More noteworthy viscous dissipative heat causes an ascent in the temperature profile. As Ec expands, the temperature profile additionally increments. This is an adjustment to the way that the energy is put away in the fluid area as a result of scattering because of viscosity and versatile distortion.

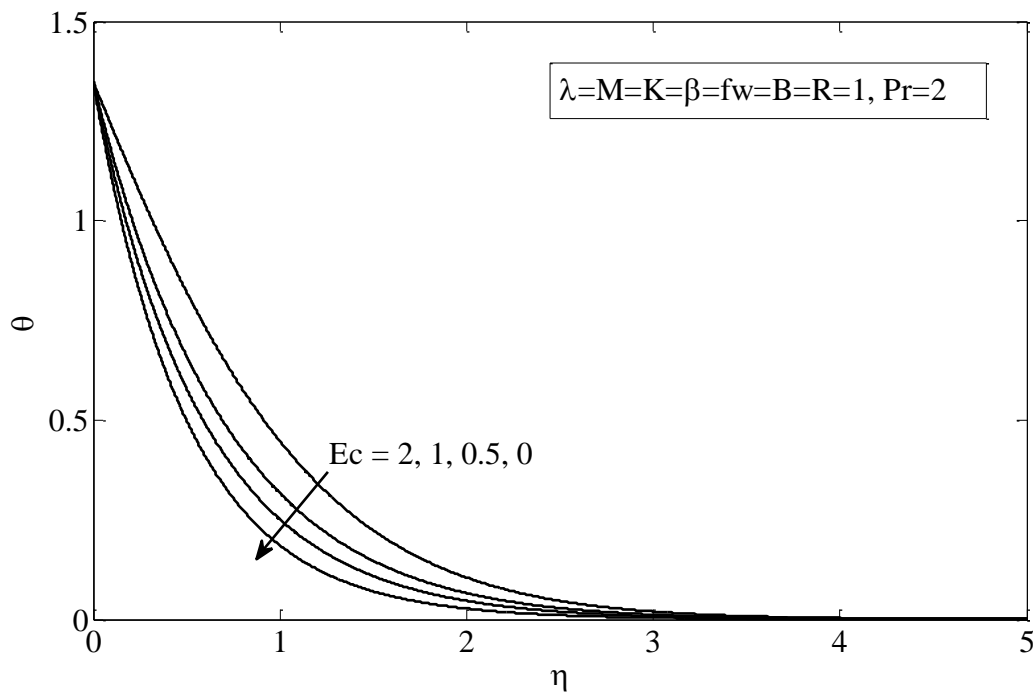


Figure 7. Temperature for different values of Ec

Figure.8 portrays that the impact of radiation on temperature. It is watched that the temperature of the fluid increments with an expanding R . The radiation parameter R is capable to the thickening of the thermal boundary. This empowers the fluid to discharge the heat energy from the flow area and makes the framework cool. This is genuine on the grounds that the Rossel and estimation bring about an expansion in temperature.

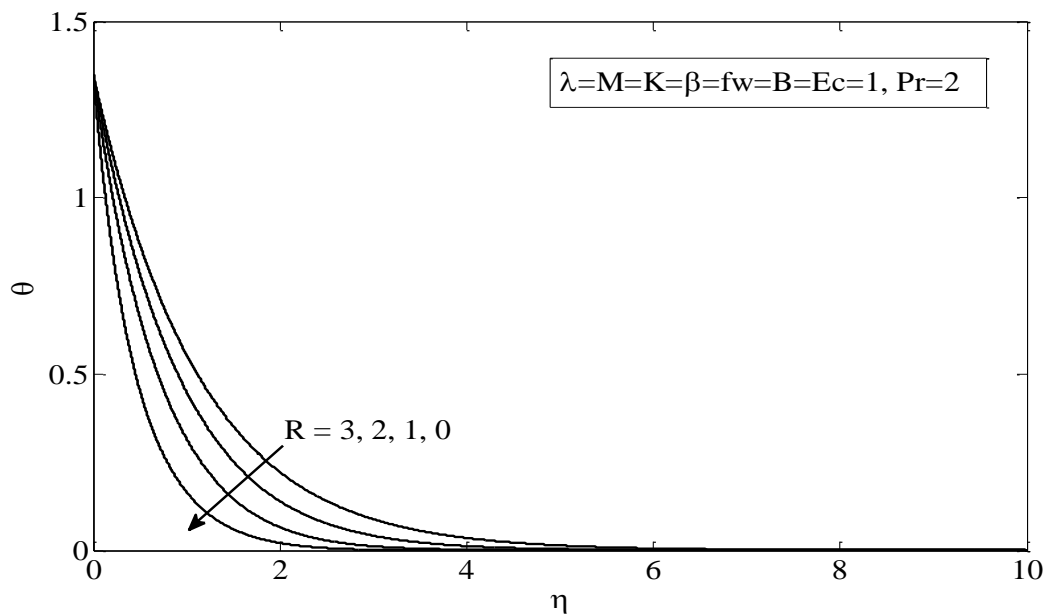


Figure 8.Temperature for different values of R

The impact of Prandtl number on temperature appears in figure.9. It is seen that the temperature of the fluid declines with an expanding Pr. Prandtl number is contrarily relative to the thermal diffusivity of fluid. Thermal diffusivity is weaker for higher Prandtl fluids and more grounded for bringing down Prandtl fluids. Weaker thermal diffusivity compares to bring down the temperature and more grounded thermal diffusivity indicates higher temperature.

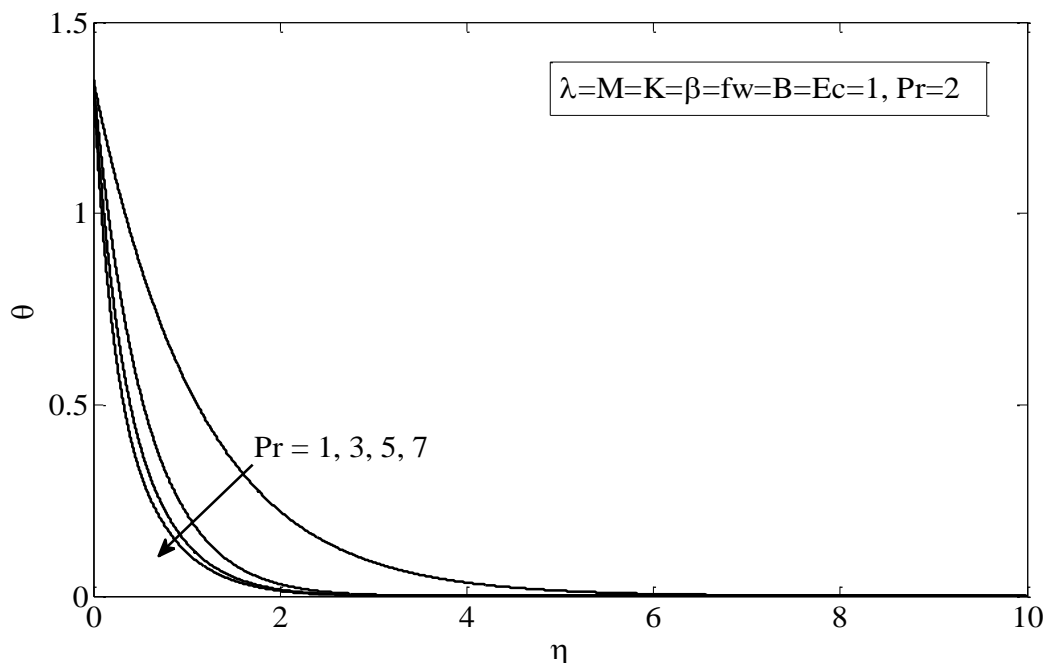


Figure 9.Temperature for different values of Pr

The impacts of the material parameter and suction/injection parameter on skin friction appear in the figure. 10. It is watched that skin friction declines with expanding K or fw .

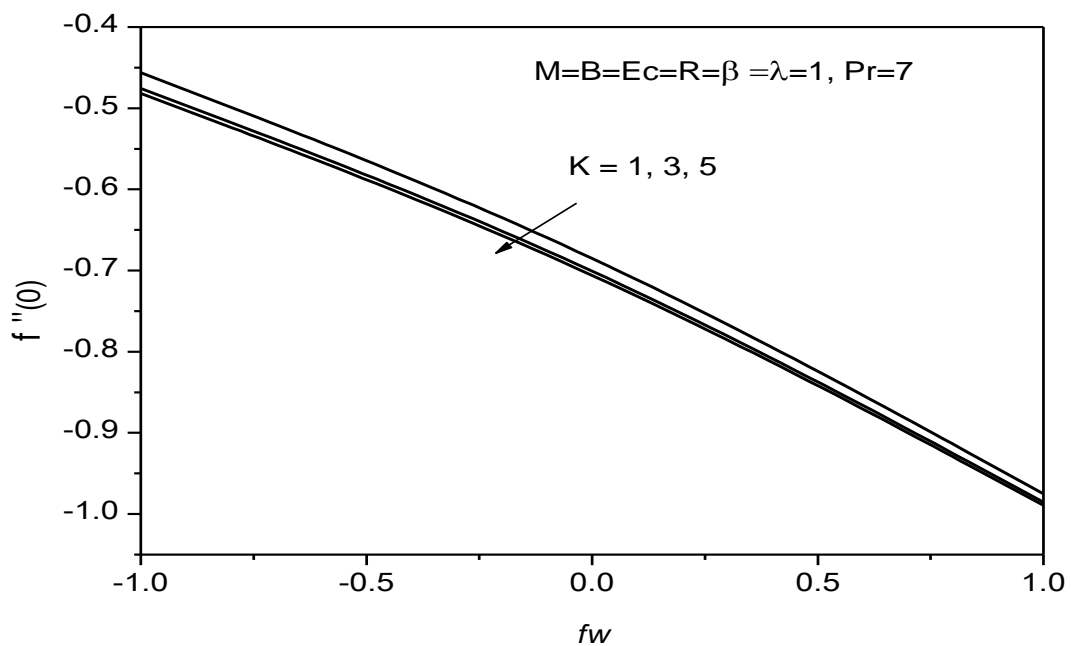


Figure 10. Skin friction for different values of fw and K

Figure.11 demonstrates that the impacts of the material parameter and viscosity parameter on couple wall stress. It is seen that the couple wall stress increments with raising K while diminishes with an expanding λ .

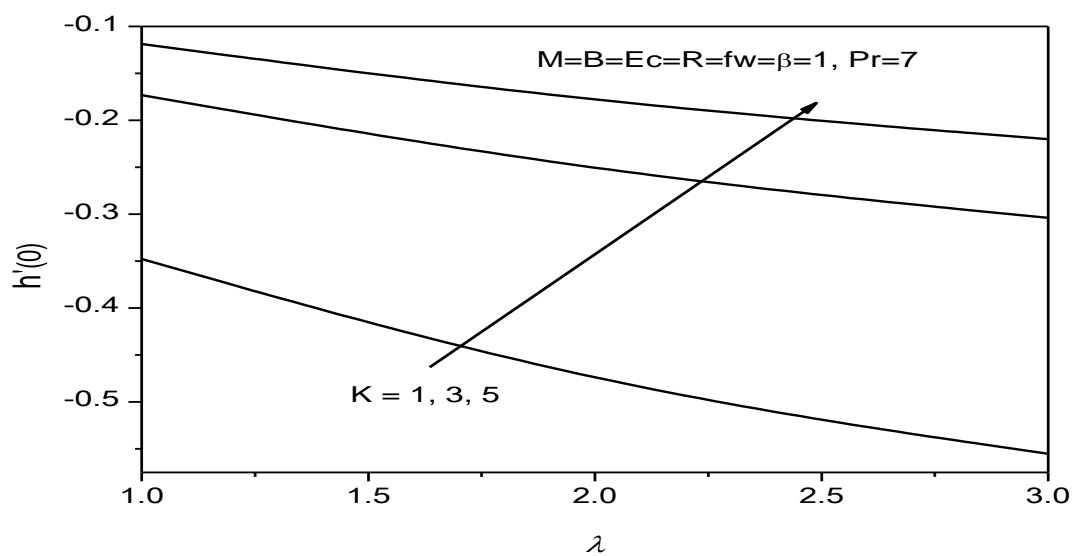


Figure 11. Couple wall stress for different values of λ and K

The impacts of K and R on the local Nusselt number are shown in fig. 12. It is watched that local Nusselt number increments with the impact of K , though it diminishes with raising R

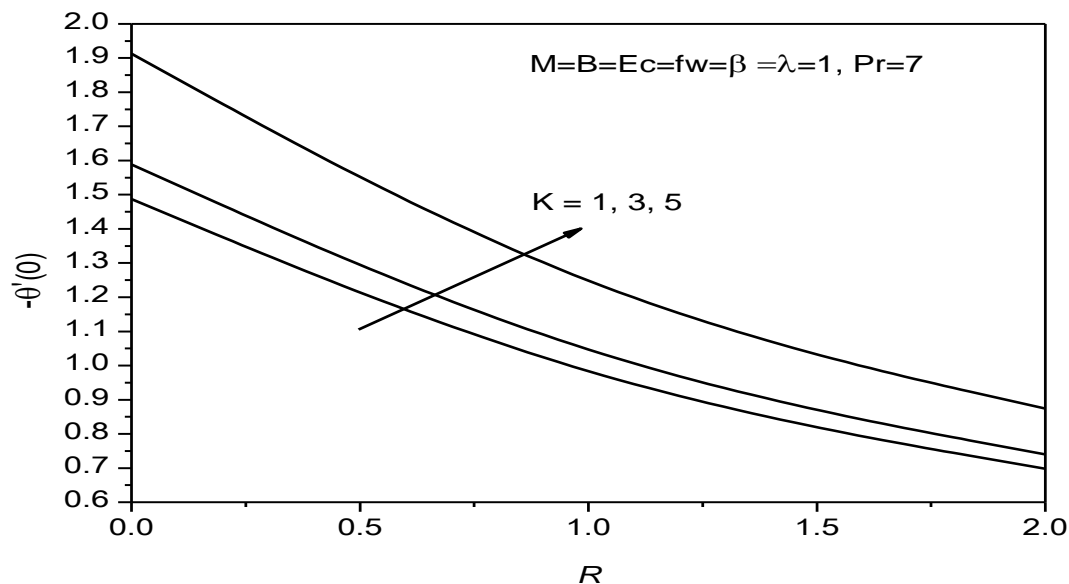


Figure 12. Nusselt number for different values of R and K

Table 1. Comparison of the present results of local Nusselt number $-\theta'(0)$ for the case of Newtonian fluid for $M = \lambda = Ec = B = fw = 0$, $\beta \rightarrow \infty$ with the results of Gorla and Sidawi [31]

Pr	$-\theta'(0)$	
	Present Study	Gorla and Sidawi [31]
0.7	0.4544	0.5349
2	0.9114	0.9114
7	1.8954	1.8905
20	3.3539	3.3539

5. Conclusions

In the present paper, the unsteady mixed convection flow of a viscous incompressible electrically conducting micropolar fluid on a vertical and impermeable extending surface by producing radiation and thermal slip results into account, are analyzed. The governing equations are approximated to an arrangement of non-linear ordinary differential equations by a similarity transformation. Numerical calculations are completed for different estimations of the dimensionless parameters of the problem. It has been discovered that

1. The velocity and angular velocity diminish temperature increments with an expansion in the magnetic parameter.
2. The velocity and in addition temperature increments with an expanding in the viscosity parameter.
3. The angular velocity of the fluid increments close to the sheet and declines far from the sheet with an expanding viscosity parameter.
4. The temperature of the fluid increments with an expansion in the Eckert number and thermal radiation parameter.

5. The skin friction decreases the suction/injection or material parameter (K).
6. The couple wall stress increments with raising material parameter (K) though it diminishes with an expanding viscosity parameter.
7. Local Nusselt number declines with the impact of thermal radiation parameter and increments with the expanding material parameter (K).

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