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New methods of testing nonlinear hypothesis using iterative NLLS estimator

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Abstract. This research paper discusses the method of testing nonlinear hypothesis using iterative Nonlinear Least Squares (NLLS) estimator. Takeshi Amemiya [1] explained this method. However in the present research paper, a modified Wald test statistic due to Engle, Robert [6] is proposed to test the nonlinear hypothesis using iterative NLLS estimator. An alternative method for testing nonlinear hypothesis using iterative NLLS estimator based on nonlinear hypothesis using iterative NLLS estimator based on nonlinear studentized residuals has been proposed. In this research article an innovative method of testing nonlinear hypothesis using iterative restricted NLLS estimator is derived. Pesaran and Deaton [10] explained the methods of testing nonlinear hypothesis. This paper uses asymptotic properties of nonlinear least squares estimator proposed by Jenrich [8]. The main purpose of this paper is to provide very innovative methods of testing nonlinear hypothesis using iterative NLLS estimator, iterative NLLS estimator based on nonlinear studentized residuals and iterative restricted NLLS estimator. Eakambaram et al. [12] discussed least absolute deviation estimations versus nonlinear regression model with heteroscedastic errors and also they studied the problem of heteroscedasticity with reference to nonlinear regression models with suitable illustration. William Grene [13] examined the interaction effect in nonlinear models disused by Ai and Norton [14] and suggested ways to examine the effects that do not involve statistical testing. Peter [15] provided guidelines for identifying composite hypothesis and addressing the probability of false rejection for multiple hypotheses.

1. Introduction

In estimating linear regression models the least squares method of estimation is often applied to estimate the unknown parameters of the linear regression model and the errors are usually assumed to be independent and identically distributed normal random variables with zero mean and unknown constant variance. The violations of these crucial assumptions on error variables can have several consequences on estimates of parameters and test statistics. The estimation procedures for nonlinear regression models and error assumptions are usually analogous to those made for linear regression models. The tests for the hypotheses on parameters of nonlinear regression models are usually based on nonlinear least squares estimates and the normal assumptions of error variables. In the present research study some new methods of testing nonlinear hypotheses using (i) iterative NLLS estimator,

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(ii) iterative NLLS estimator based on nonlinear student- zed residuals (iii) iterative restricted NLLS estimator has been proposed.

2. Testing of nonlinear hypothesis using iterative nulls estimator

Consider the standard nonlinear regression model with usual assumptions in matrix notation as

$$Y_{n\times 1} = f_{n\times 1}(\beta) + \varepsilon_{n\times 1} \tag{2.1}$$

and $\varepsilon \sim N_n\left(O, \sigma^2 I_n\right)$

Here, β is $(p\times 1)$ vector of unknown parameters.

Where a nonlinear hypothesis for the test procedure as

$$H_0 : h(\beta) = 0 \text{ against } H_1 : h(\beta) \neq 0$$

Where h is a q-vector valued nonlinear function and q < p and $h(\beta)$ is continuously first order differentiable function mapping $\mathbb{R}^p$ into $\mathbb{R}^q$ with Jacobian

$$H(\beta) = \frac{\partial}{\partial \beta'} h(\beta) \tag{2.2}$$

Here, $H(\beta)$ is of order $q\times p$.

By evaluating $H(\beta)$ at $\beta = \hat{\beta}_n$, where $\hat{\beta}_n$ is iterative NLLS estimator for β, one may write $\hat{H}_n = H\left(\hat{\beta}_n\right)$

Under characterizations of least squares estimators,

$$\hat{\beta}_n = \beta_n + \left[F'(\beta_n)F(\beta_n)\right]^{-1} F'(\beta_n)\varepsilon + O_p\left(\frac{1}{\sqrt{n}}\right) \tag{2.3}$$

Where, $F(\beta) = \frac{\partial}{\partial \beta'} f(\beta), h\left(\hat{\beta}_n\right)$ may be characterized as

$$h\left(\hat{\beta}_n\right) = h(\beta) + H(\beta)\left[F'(\beta)F(\beta)\right]^{-1} F'(\beta)\varepsilon + O_p\left(\frac{1}{\sqrt{n}}\right) \tag{2.4}$$

Ignoring the remainder term, one may obtain,

$$h\left(\hat{\beta}_n\right) \overset{\text{approx}}{\sim} N_q\left[h(\beta),\ \sigma^2 H(\beta)\left[F'(\beta)F(\beta)\right]^{-1} H'(\beta)\right] \tag{2.5}$$

Thus, $$\left[\frac{h'\left(\hat{\beta}_n\right)\left[H(\beta)\left[F'(\beta)F(\beta)\right]^{-1} H'(\beta)\right]^{-1} h\left(\hat{\beta}_n\right)}{\sigma^2}\right] \overset{\text{approx}}{\sim} \text{Noncentral } \chi^2_q \tag{2.6}$$

and noncentrality parameter is given by

$$\lambda = \frac{h'(\beta)\left[H(\beta)\left[F'(\beta)F(\beta)\right]^{-1} H'(\beta)\right]^{-1} h(\beta)}{2\sigma^2} \tag{2.7}$$

Further, within the order of approximation, $\frac{(n-p)S^2}{\sigma^2}$ is distributed independently of $\hat{\beta}_n$ as the χ^2 distribution with $(n-p)$ degrees of freedom. Here, S^2 is an unbiased estimator of unknown error variance σ^2. Hence, the ratio

$$\frac{h'(\hat{\beta}_n)\left[H(\beta)\left[F'(\beta)F(\beta)\right]^{-1}H'(\beta)\right]^{-1}h(\hat{\beta}_n)\Big/(q\sigma^2)}{(n-p)S^2\Big/\left[(n-p)\sigma^2\right]}$$

or $\frac{h'(\hat{\beta}_n)\left[H(\beta)\left[F'(\beta)F(\beta)\right]^{-1}H'(\beta)\right]^{-1}h(\hat{\beta}_n)}{qS^2} \sim \text{Noncentral}\, F_{[q,(n-p)]}$ (2.8)

With noncentrality parameter λ

By substituting the estimates $\hat{H}_n$ and $\hat{D}_n$ for $H(\beta)$ and $\left[F'(\beta)F(\beta)\right]^{-1}$ respectively the Modified Wald test statistic for testing $H_0 : h(\beta)=0$ is given by

$$W^* = \frac{h'(\hat{\beta}_n)\left[\hat{H}_n\hat{D}_n\hat{H}_n^{'}\right]^{-1}h(\hat{\beta}_n)}{qS^2} \sim F_{[q,(n-p),\lambda]} \tag{2.9}$$

3. Alternative method for testing nonlinear hypothesis using iterative nulls estimator based on nonlinear studentized residuals

Consider the Standard Nonlinear Regression Model with usual assumptions in matrix notation as

$$Y_{n\times1} = f_{n\times1}(\beta) + \varepsilon_{n\times1} \tag{3.1}$$

And $\varepsilon \sim N_n(O, \sigma^2 I)$

Suppose that the NLLS residual vector is given by

$$e = \left[Y - \hat{Y}\right] = \left[Y - f(\hat{\beta})\right]$$

Where $\hat{\beta} \simeq \beta + \left[F'(\beta)F(\beta)\right]^{-1}F'(\beta)\varepsilon$ and

$$F(\beta) = \left(\left(\frac{\partial}{\partial\beta_j} f(X_i,\beta)\right)\right)_{n\times p}$$

Here, $\frac{\partial}{\partial\beta_j} f(X_i,\beta)$ is the $(i, j)^{th}$ element of $(n \times p)$ matrix $F(\beta)$.

Write the internally nonlinear studentized residuals as

$$e_i^* = \frac{e_i}{\hat{\sigma}\sqrt{(1-\nu_{ii})}}, \qquad i = 1, 2, ..., n \tag{3.2}$$

Where $\hat{\sigma}^2 = \frac{e'e}{n-p} = \frac{\sum_{i=1}^{n} e_i^2}{n-p}$ (3.3)

and $\nu = \left(\left(\nu_{ii}\right)\right)$ is Hat matrix

Consider the problem of testing nonlinear hypothesis as

$$H_o : H\left(\hat{\beta}_n\right)\delta = h\left(\hat{\beta}_n\right) \sim H_1 : H\left(\hat{\beta}_n\right)\delta \neq h\left(\hat{\beta}_n\right)$$

Where $H\left(\hat{\beta}_n\right) = \frac{\partial}{\partial \hat{\beta}_n'} h\left(\hat{\beta}_n\right)$ is a matrix of order $\left(q \times p\right)$. i.e., $H\left(\beta\right)$ is evaluated at $\beta = \hat{\beta}_n$

Now, one may fit the model,

$$e^* = F\left(\hat{\beta}_n\right)\delta + \nu \tag{3.4}$$

and the least squares estimator of δ is given by

$$\hat{\delta} = \left[F'\left(\hat{\beta}_n\right)F\left(\hat{\beta}_n\right)\right]^{-1} F'\left(\hat{\beta}_n\right)e^* \tag{3.5}$$

Since, $\frac{\partial}{\partial \hat{\beta}_n'} R\left(\hat{\beta}_n\right) = -2F'\left(\hat{\beta}_n\right)e^* = 0$

One may have $\hat{\delta} = \left[F'\left(\hat{\beta}_n\right)F\left(\hat{\beta}_n\right)\right]^{-1} F'\left(\hat{\beta}_n\right)e^* = 0$ (3.6)

The F-test statistic for testing H_0 against H_1 is given by

$$F = \frac{\left[H\left(\hat{\beta}_n\right)\hat{\delta} - h\left(\hat{\beta}_n\right)\right]' \left[H\left(\hat{\beta}_n\right)\left\{F'\left(\hat{\beta}_n\right)F\left(\hat{\beta}_n\right)\right\}^{-1} H'\left(\hat{\beta}_n\right)\right]^{-1} \left[H\left(\hat{\beta}_n\right)\hat{\delta} - h\left(\hat{\beta}_n\right)\right] \big/ q}{\left[e^* - F\left(\hat{\beta}_n\right)\hat{\delta}\right]' \left[e^* - F\left(\hat{\beta}\right)\hat{\delta}\right] \big/ n-p} \tag{3.7}$$

or it reduces to Modified Wald test statistic as

$$W^* = \frac{h'\left(\hat{\beta}_n\right)\left[H\left(\hat{\beta}_n\right)\left\{F'\left(\hat{\beta}_n\right)F\left(\hat{\beta}_n\right)\right\}^{-1} H'\left(\hat{\beta}_n\right)\right]^{-1} h\left(\hat{\beta}_n\right)}{qS^2} \sim F_{[q,n-p]} \tag{3.8}$$

Where $S^2 = \frac{R\left(\hat{\beta}_n\right)}{n-p}$ is estimator of the error variance corresponding to the Iterative NLLS Estimator $\hat{\beta}_n$.

4. Testing nonlinear hypothesis using iterative restricted nulls estimator

Consider the standard nonlinear regression model in matrix notation as

$$Y_{n\times 1} = f_{n\times 1}\left(\beta\right) + \varepsilon_{n\times 1} \tag{4.1}$$

and assume that $\varepsilon \sim N_n(O, \sigma^2 I)$. Write the problem of testing nonlinear hypothesis as

$$H_0 : h\left(\beta\right) = 0 \text{ against } H_1 : h(\beta) \neq 0.$$

To test the nonlinear null hypothesis, first, one may compute the iterative NLLS estimator $\hat{\beta}_n$ and hence obtain the unrestricted nonlinear studentized residual sum of squares as $R^*_{UR}\left(\hat{\beta}_n\right)$; secondly, refit the nonlinear model under null hypothesis and compute iterative restricted NLLS estimator $\tilde{\beta}_n$ and hence obtain the restricted nonlinear studentized residual sum of squares as $R^*_R\left(\tilde{\beta}_n\right)$; and finally, the modified likelihood ratio test statistic is given by

$$L^* = \frac{\left[R^*_R\left(\tilde{\beta}_n\right) - R^*_{UR}\left(\hat{\beta}_n\right)\right]/q}{R^*_{UR}\left(\hat{\beta}_n\right)/n-p} \sim F_{[q,n-p]} \tag{4.2}$$

5. Conclusions

In the above research paper two modified Wald test statistics have been proposed to test nonlinear hypothesis in a very different manner. An innovative method of testing nonlinear hypothesis using iterative restricted NLLS estimator has been discussed.

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