

# MHD mass transfer flow of an Eyring-Powell fluid over a stretching sheet

**D Harish Babu<sup>1</sup>, M Sudheer Babu<sup>1</sup>, and P V Satya Narayana<sup>2</sup>**

<sup>1</sup>Department of Mathematics, Sree Vidyanikethan Eng. College, Tirupati, A.P, India

<sup>2</sup>Department of Mathematics <sup>2</sup>Fluid dynamics division, School of Advanced Sciences, VIT University, Vellore-632014, India

E-mail: pvsatya8@yahoo.co.in

**Abstract.** This study investigates the mass transfer flow of Powell-Eyring fluid due to the porous stretching sheet with magnetic field. A second-order approximation of the Eyring-Powell fluid model is used to obtain the flow equations. Using usual similarity transformations, the governing equations have been transformed into non-linear ordinary differential equations and solved by a powerful technique known as shooting method along with R-K fourth order scheme. Graphical results displaying the influence of pertinent physical parameters on the velocity, concentration profile, skin-friction coefficient and Sherwood number are given.

## 1. Introduction

The level headed discussion of boundary layer stream and also mass exchange past a broadening layer has acquired extraordinary consideration because of its assorted exploratory uses, for example, drawing of plastic movies, paper generation, counterfeit filaments, the expulsion of a polymer sheet from a bite the dust, metal turning and metal expulsion. Amid the make of these sheets, the liquefy issues from an opening and afterward reached out to accomplish the coveted consistency. The mechanical things of a definitive item entirely rely upon the extending and cooling rates all the while. For this reason, the flow of viscous fluid over a stretching sheet has produced many associated problems for example [1-4], for each incorporating an innovative effect and yet providing an exact solution.

A foresaid studies are all confined to the flow of Newtonian liquids. Nowadays, it has been broadly predictable that in business and manufacturing uses, non-Newtonian liquids are more appropriate than Newtonian liquid. Owing to this, a diversity of non-Newtonian liquid models is obtainable in the literature [5-7]. Satya Narayana et al. [8] and Harish Babu et al. [9] analyzed heat and mass transfer of non-Newtonian liquid past extending sheet with magnetic field effect. Hussain et al. [10] studied the flow of a Walter's-B fluid past a stretching cylinder. Hayat et al. [11] investigated the flow of third grad fluid due to an exponentially extending layer in the current of attractive field. Harish Babu et al. [12] analyzed the influence of magnetic and heat transfer Jeffrey liquid through extending layer. Ahmed et al. [13] considered the flow of power law fluid through an unsteady stretching sheet with magnetic effect. Amongst non-Newtonian fluids, Eyring-Powell liquid [14] has mathematically complex describes the flow behaviour at low and high shear rates. In particular, it can be utilised to express the flows of recent industrialised equipments like powdered graphite and ethylene glycol. Paanigrahi et al. [15] addressed the boundary layer stream of a Eyring-Powell liquid over a non-linear



stretching layer along with thermal diffusion and diffusion thermo. Hayat et al. [16] considered radiative flow of an Eyring-Powell liquid with magnetic nanoparticles.

The main aim of the current analysis is to study the MHD mass transfer flow over a stretching surface for the Eyring-Powell model (see Powell and Eyring, 1994). The dimensionless mathematical problems solved numerically by shooting method [17]. Shooting method is one of the highly powerful numerical techniques in solving various kinds of non-linear equations for example, fixed, homogeneous, decoupled and non-homogeneous. Liquid velocity and mass gradient discussed, explored through graphs and tables.

## 2. Mathematical formulation of the problem

Let us read the steady 2-D flow of an incompressible Eyring-Powell liquid extending layer that happens together with plane  $y=0$  and the liquid flow in the half space  $y>0$ . Additionally, the mass transfer results taken into account. The velocity  $U_w(x)$  and the concentration  $C_w(x)$  of the extending layer are related to the distance  $x$  from starting point O, Here  $C_w(x) > C_\infty$  (see Fig. 1). It is imagined that the extending velocity at the layer is  $u_w(x) = ax$  where  $a>0$  is a constant. An applied magnetic field of strength  $B_0$  is encountered perpendicular to the flow direction. Further, it is assumed that the induced magnetic field is negligible caused by small magnetic Reynolds numbers. Under the foregoing assumptions, the boundary layer flow governed by below expressions: [18, 19]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \left( \nu + \frac{1}{\rho \beta_1 c} \right) \frac{\partial^2 u}{\partial y^2} - \frac{1}{2\rho \beta_1 c^3} \left( \frac{\partial u}{\partial y} \right)^2 \left( \frac{\partial^2 u}{\partial y^2} \right) - \frac{\sigma B_0^2}{\rho} u - \frac{\nu}{k_1} u \quad (2)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} \quad (3)$$

The related boundary conditions are

$$\left. \begin{aligned} u = u_w(x) = ax, v = 0, C = C_w \text{ at } y = 0 \\ u \rightarrow 0, C \rightarrow C_\infty \text{ as } y \rightarrow \infty \end{aligned} \right\} \quad (4)$$

To introduce the followings transformations

$$\eta = \sqrt{\frac{a}{\nu}} y, u = ax f'(\eta), v = -\sqrt{a\nu} f(\eta), \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \quad (5)$$

The continuity equation is identically satisfied eqs. (1) -(4), and eqs. (2) -(5) become:

$$(1 + \gamma) f''' - f'^2 + ff'' - \gamma \beta f''' f''^2 - \left( M^2 + \frac{1}{K} \right) f' = 0 \quad (6)$$

$$\phi'' + Sc f \phi' = 0 \quad (7)$$

Boundary conditions (4) can be converted into

$$\left. \begin{aligned} f(\eta) &= 0, f'(\eta) = 1, \phi(\eta) = 1 \text{ at } \eta = 0 \\ f'(\eta) &\rightarrow 0, \phi(\eta) \rightarrow 0 \text{ as } \eta \rightarrow \infty \end{aligned} \right\} \quad (8)$$

Here,  $\gamma = \frac{1}{\mu\beta_1 c}$  and  $\beta = \frac{(ax)^3}{2x\nu c^2}$  are material constants of Eyring-Powell fluid,  $M^2 = \frac{\sigma B_0^2}{a\rho}$  is

Hartmann number and the permeability parameter  $K = \frac{\nu}{ck_1}$ ,  $Sc = \frac{\nu}{D}$  is the Schmidt number.

The physical quantities of skin-friction coefficient  $C_f$  and Sherwood number  $Sh$  which is well-defined as

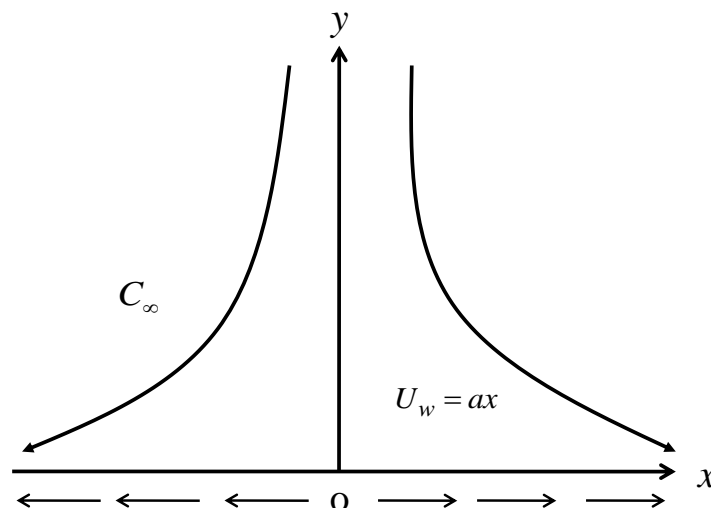
$$\text{Re}^{1/2} C_f = \left[ (1 + \gamma) f'' - \frac{\beta\gamma}{3} f''' \right]_{\eta=0} \quad (9)$$

$$Sh \text{Re}_x^{-1/2} = -\phi'(0) \quad (10)$$

Where the local Reynolds Number  $\text{Re}_x = \frac{u_w(x) \cdot x}{\nu}$ .

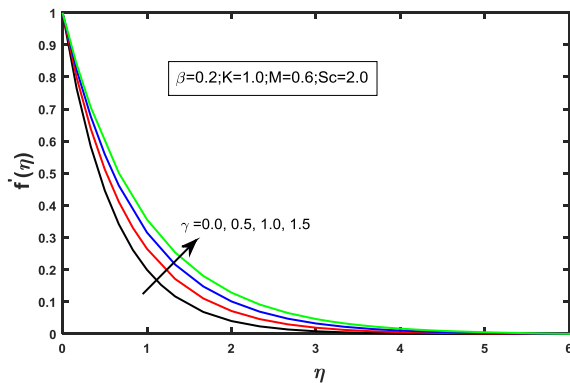
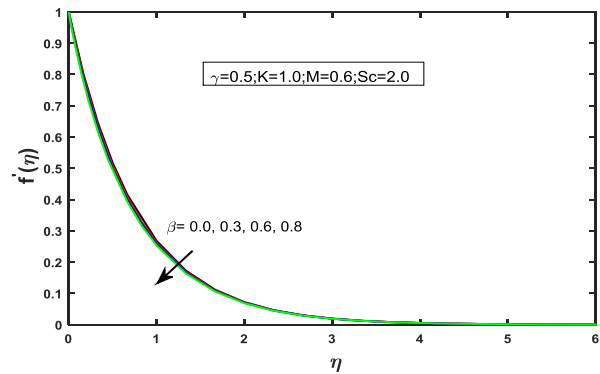
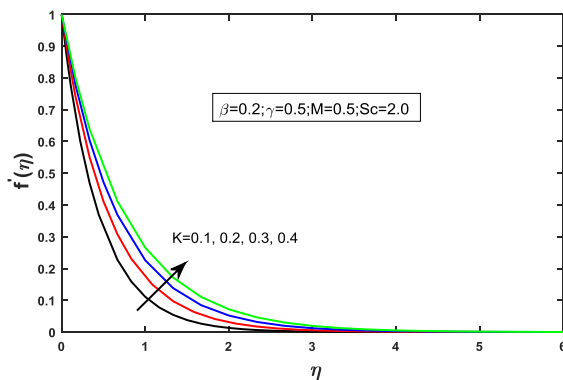
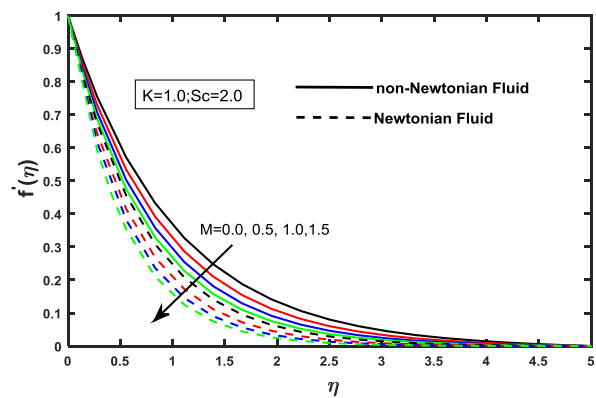
### 3. Result and Discussions

The non-linear OD Eqs. (6) and (7) with the boundary conditions Eq. (8) have been solved numerically using the shooting design with 4<sup>th</sup> order R-K scheme. The influence of Hartmann number,  $K$  and  $\beta$  on the  $f(\eta)$ ,  $\phi(\eta)$  fields, skin friction coefficient and Sherwood number are analyzed and discussed through graphs, which are plotted in Figs. 2-9.

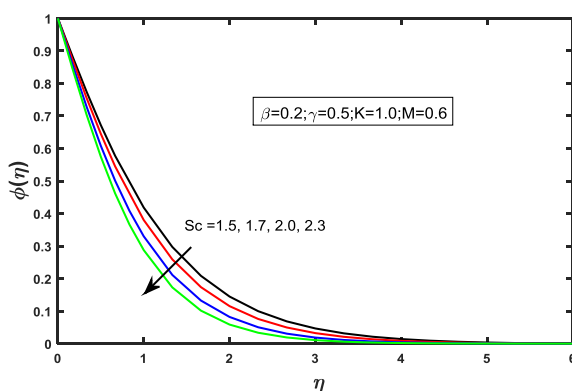
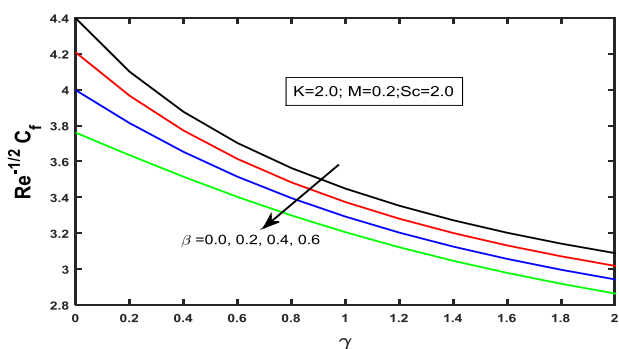


**Figure 1.** The physical model of the problem

We compared existing results gained by shooting design with the results gained by Khan et al. [20]. The validated result comparison of our present skin-friction co-efficient specified in Table 1. Hence, the solutions obtained are in good coherence.

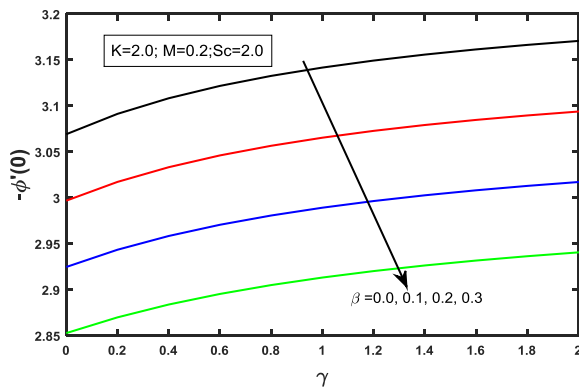
Figure 2. Result of  $\gamma$  on  $f'(\eta)$ Figure 3. Result of  $\beta$  on  $f'(\eta)$ Figure 4. Result of  $K$  on  $f'(\eta)$ Figure 5. Result of  $M$  on  $f'(\eta)$ 

Figures 2-3 describes the results of Eyring-Powell fluid parameter  $\gamma$  and  $\beta$  on  $f(\eta)$  profile. It is found that the  $f(\eta)$  profile increases with ascending values of  $\gamma$ , but opposite trend is seen in the case  $\beta$ . Also, notice that the boundary layer thickness ascending for increasing both  $\gamma$  and  $\beta$  values. Figure 4 depicts the influence of  $K$  on the  $f(\eta)$  distribution. It seen that fluid  $f(\eta)$  increases with an ascending in the values of  $K$ . In addition, it is predictable to enhance in the permeability of the permeable medium, take the lead to the rise in the liquid stream. As a result, the holes of the porous medium enhance larger and then the resistivity of the medium may be neglected.

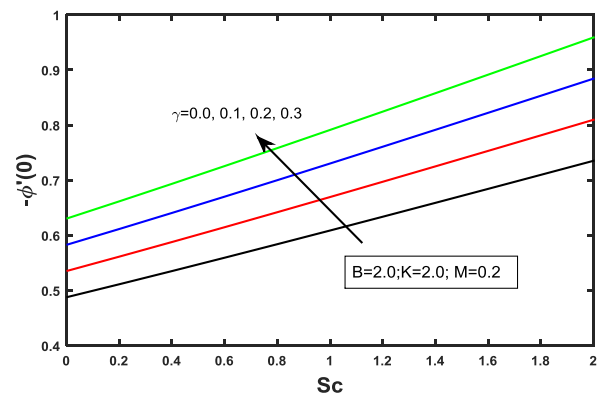
Figure 6. Result of  $Sc$  on  $\phi(\eta)$ Figure 7. Result of  $Re_x^{-1/2} c_f$  versus  $\gamma$  for different values of  $\beta$

The result of  $M$  on  $f(\eta)$  profile displayed in Figure 5 for both Newtonian and non-Newtonian cases. It is seen that,  $f(\eta)$  profiles and boundary layer thickness decrease with a rise in the  $M$ .

Figure 6 displays the result of  $Sc$  on  $\phi(\eta)$  portrait. The  $\phi(\eta)$  field decreases when  $Sc$  increases. This causes the  $\phi(\eta)$  buoyancy impacts to decrease, and there is a decline in the liquid  $f(\eta)$ .



**Figure 8.** Result of  $-\phi'(0)$  versus  $\gamma$ , for distinct values of  $\beta$



**Figure 9.** Result of  $-\phi'(0)$  versus  $Sc$ , for distinct values of  $\gamma$

Figures 7-8 demonstrate the variation of  $Re_x^{1/2} c_f$ , and  $-\phi'(0)$  versus Eyring-Powell fluid parameter such as  $\gamma$  and  $\beta$ . It is found that both  $Re_x^{1/2} c_f$  and  $-\phi'(0)$  dwindle with an ascending in the values of  $\beta$ , but reverse action is watched in the case  $\gamma$ .

Figure 9 illustrates the distinction of  $-\phi'(0)$  with  $Sc$  for diverse values of  $\gamma$ . It is viewed that the  $-\phi'(0)$  increases with increase of  $\gamma$ .

**Table 1.** Comparison of skin friction co-efficient for distant values of  $\gamma$  and  $\beta$ .

$\beta / \gamma$	$Re_x^{1/2} c_f$	
	Khan et al. [20]	Present
0.0	1.0952	1.09527
0.1	1.0939	1.09402
0.2	1.0922	1.09235
0.3	1.0909	1.09076
0.4	1.0894	1.08934
0.5	1.0878	1.08785

#### 4. Conclusions

In this work, the effect of MHD stream of an Eyring-Powell liquid past a extending layer in the presence of porous medium is investigated. Numerical results are obtained using MALAB software. Following conclusions can be illustrated from the present explore:

1. The velocity rises with an increasing value of  $\beta$  whereas opposite effect is seen in the case of  $\gamma$  and  $K$ .
2. The fluid velocity is increased for the non-Newtonian fluid as compared with the corresponding Newtonian liquid for the effect of  $M$ .
3. The skin friction co-efficient decrease with an increase in values of  $\gamma$  and  $\beta$ .
4. The Sherwood number increases with increasing values of  $\gamma$  as well as  $Sc$ .

## References

- [1] Crane L J 1970 *Z Angew Math Phys*, **21** 645–7
- [2] Gupta P S, Gupta S 1977 *J Chem Eng*, **55** 744–6
- [3] Chen, CK, Char, MI 1988 *J Maths Anal Appl*, **135** 568–80
- [4] Mukhopadhyay S 2015 *Int. J. Heat Mass Transf.* **52** 3261–3265
- [5] Harris J 1977 Longman, London
- [6] Bird RB, Curtiss CF, Armstrong RC, Hassager O 1987 *Fluid mechanics*, Wiley, New York, 1, 170–171, 227–228, 351, 234, 174, 139.
- [7] Harish Babu, D, Satya Narayana PV 2013 *ISRN Thermodynamics* **2013** 1-17
- [8] Satya Narayana, PV, Harish Babu, D 2016 *J.Taiwan Inst. of Chemical Eng.* **59** 8–25
- [9] Harish babu D, Venkateswarlu B, Satya Narayana P V 2017 *Front. in heat mass transf.* **8** 1-5
- [10] Hussain A. Ullah, A 2016 *Alexandria Eng. J.* **55** 3073–3080
- [11] Hayat T, Khan MI, Waqas M, Alsaedi A, Yasmeen T 2016 *Chinese J. Chem. Eng.* doi: 10.1016/j.cjche. 2016.06.008
- [12] Harish Babu D, Satya Narayana PV 2016 *J.Magn. Magne. Mater.* **412** 185–193
- [13] Ahmed J, Begum A, Shahzad A, Ali R 2016 *Results in Phys.* **6** 73–98
- [14] Powell RE, Eyring H 1944 *Nature*, London 18
- [15] Panigrahi S, Reza M, Kumar Mishra A 2015 *Procedia Eng.* **127** 645-651
- [16] Hayat T, Khan MI, Waqas M, Alsaedi A 2017 *J. Mole. Liq.*, doi:10.1016/j.molliq.2017.01.076
- [17] Na, TY 1979 *Academic Press*, New York
- [18] Javed T, Ali N, Abbas Z, Sajid M 2013 *Chem. Eng. Comm.*, **200**(3) 327-336
- [19] Andersson HI, Hansen OR, Holmedal B 1994 *Int. J. Heat Mass Transf.*, **37**(4) 659-664
- [20] Khan I, Qasim M, Shafie S 2016 *Thermal Sci.* **20**(6) 1903-1912