

Brownian motion and thermophoresis effects on Peristaltic slip flow of a MHD nanofluid in a symmetric/asymmetric channel

G Sucharitha¹, S Sreenadh¹, P Lakshminarayana² and K Sushma¹

¹Department of Mathematics, Sri Venkateswara University, Tirupati-517502, India

²Department of Mathematics, School of Advanced Sciences, VIT University, Vellore-632014, India

E-mail: lakshminarayana.p@vit.ac.in

Abstract. The slip and heat transfer effects on MHD peristaltic transport of a nanofluid in a non-uniform symmetric/asymmetric channel have studied under the assumptions of elongated wave length and negligible Reynolds number. From the simplified governing equations, the closed form solutions for velocity, streamfunction, temperature and concentrations are obtained. Also dual solutions are discussed for symmetric and asymmetric channel cases. The effects of important physical parameters are explained graphically. The slip parameter decreases the fluid velocity in middle of the channel whereas it increases the velocity at the channel walls. Temperature and concentration are decreasing and increasing functions of radiation parameter respectively. Moreover, velocity, temperature and concentrations are high in symmetric channel when compared with asymmetric channel.

Nomenclature:

x, y	Cartesian coordinates	ρ	density of the fluid
u, v	fluid velocities	t	time
p	pressure	ψ	stream function
μ	viscosity	ρ_f, ρ	densities
d	width of the channel	m	non- uniform parameter
a, b	amplitudes	δ	wave number
λ	wavelength	ν	kinematic viscosity
c	wave speed	k_0	thermal conductivity



c_p	specific heat	ϕ_0	phase difference
σ	electrical conductivity	B_0	magnetic field
T	temperature	θ	non-dimensional temperature
T_0, T_1	temperatures of the fluid at walls	C_0, C_1	concentrations of the fluid at walls
τ	ratio of the heat capacity of nanoparticle and fluid	q_r	radiative heat flux
Nb	Brownian motion parameter	Nt	Thermophoresis parameter
L	slip parameter	F	mean flow in laboratory frame
Θ	mean flow in wave frame	Rn	Radiation parameter
ϕ	non-dimensional concentration	M	magnetic parameter
Gr_t	temperature Grashofnumber	Gr_c	mass Grashof number
Pr	Prandtl number	Re	Reynolds number

1. Introduction

Peristaltic pumping is well known principle for fluid transport. It has many physiological, biomedical and industrial applications such as biofluid transport in different parts of the human body, dialysis device, heart lung instrument, blood pump machine and other pumping machinery for transporting eroding industrial fluids. Moreover, due to the requirements in biomedical engineering, several authors have been studying the peristaltic transport by considering various fluids and different assumptions [1 – 8].

The consideration of slip plays a significant role in study of some polymer flow problems. Few authors have studied [9 – 14] the effects of slip, heat transfer and magnetohydrodynamics on peristaltic pumping.

It is clear that the natural fluids such as, water flops to attain the current requirements in improving thermal conductivities. But, nanofluids are useful to enhance the thermal conductivities of traditional fluids. Recently, the authors [15 – 20] have considerable interest on peristaltic flow of nanofluids due to widespread applications of nanofluids in engineering and medicine.

Motivated by the above observations, in this paper we have studied the effects of Brownian motion and thermophoresis on peristaltic slip flow of a MHD nanofluid in a symmetric/asymmetric channel. The expressions for stream function, temperature and concentrations are obtained. The influence of various parameters on the present study have explained through the graphs.

2. Modelling of the problem

We consider the peristaltic slip flow of a conducting nanofluid in a non-uniform vertical asymmetric channel. The flow is formed by a sinusoidal wave propagation along the walls of the channel with a constant speed c (Fig.1). The flow geometry of the present problem is defined as

$$\bar{h}_1(\bar{x}, \bar{t}) = -a_1 \sin \left[\frac{2\pi}{\lambda} (\bar{x} - c\bar{t}) + \phi_0 \right] - \bar{m}\bar{x} - d \quad (1)$$

$$\bar{h}_2(\bar{x}, \bar{t}) = \bar{m}\bar{x} + a_2 \sin \left[\frac{2\pi}{\lambda} (\bar{x} - c\bar{t}) + \phi_0 \right] + d \quad (2)$$

As per the above assumptions and long wave length and small Reynolds number approximations the simplified non-dimensional governing equations for the present study are given by (Kothandapani and Prakash [15])

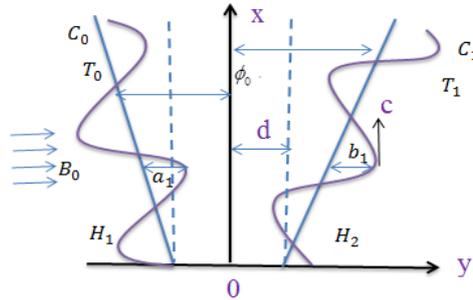


Figure 1. Flow configuration

$$\frac{\partial p}{\partial x} = \frac{\partial^3 \psi}{\partial y^3} - M^2 \frac{\partial \psi}{\partial y} + Gr_t \theta + Gr_c \phi, \tag{3}$$

$$\frac{\partial p}{\partial y} = 0, \tag{4}$$

$$\frac{\partial^2 \theta}{\partial y^2} + \frac{Pr Nb}{(1 + Pr Rn)} \left(\frac{\partial \theta}{\partial y} \right) \left(\frac{\partial \phi}{\partial y} \right) + \frac{Pr Nt}{(1 + Pr Rn)} \left(\frac{\partial \theta}{\partial y} \right)^2 = 0, \tag{5}$$

$$\frac{\partial^2 \phi}{\partial y^2} + \frac{Nt}{Nb} \frac{\partial^2 \theta}{\partial y^2} = 0, \tag{6}$$

Corresponding boundary conditions are

$$\psi = \frac{F}{2}, \frac{\partial \psi}{\partial y} + L \frac{\partial^2 \psi}{\partial y^2} = -1, \theta = \phi = 1, \text{ at } y = h_2 = b \sin[2\pi(x-t)] + 1 + mx, \tag{7}$$

$$\psi = -\frac{F}{2}, \frac{\partial \psi}{\partial y} - L \frac{\partial^2 \psi}{\partial y^2} = -1, \theta = \phi = 0, \text{ at } y = h_1 = -a \sin[2\pi(x-t) + \phi_0] - 1 - mx, \tag{8}$$

The non-dimensional quantities in the above simplified equations are defined as :

$$\left. \begin{aligned} x\lambda = \bar{x}, yd = \bar{y}, \psi cd = \bar{\psi}, uc = \bar{u}, vc = \bar{v}, p = \frac{d^2 \bar{p}}{\mu c \lambda}, t = \frac{ct}{\lambda}, \delta = \frac{d}{\lambda}, a = \frac{a_1}{d}, \\ b = \frac{b_1}{d}, m = \frac{\lambda \bar{m}}{d}, M = \sqrt{\frac{\sigma}{\mu}} B_0, Re = \frac{\rho_f cd}{\mu}, \theta = \frac{(T - T_0)}{(T_1 - T_0)}, \phi = \frac{(C - C_0)}{(C_1 - C_0)}, \\ Gr_t = \frac{(1 - C_0) \rho_f g \alpha d^2 (T_1 - T_0)}{c \mu}, Gr_c = \frac{(\rho_p - \rho_f) \rho_f g \beta d^2 (C_1 - C_0)}{c \mu}, \\ Pr = \frac{\rho v c_f}{k_0}, h_1 = \frac{\bar{h}_1}{d}, h_2 = \frac{\bar{h}_2}{d}, N_t = \frac{\tau D_T (T_1 - T_0)}{T_m v}, N_b = \frac{\tau D_B (C_1 - C_0)}{v}, \\ Rn = \frac{16 \sigma^* T_0^3}{3 K^* \mu c_f}, \frac{\partial \psi}{\partial y} = u, \frac{\partial \psi}{\partial x} = -v, F = \frac{Q}{cd}, \Theta = \frac{q}{cd} \end{aligned} \right\} \tag{9}$$

The relation between non-dimensional mean flows F and Θ is given by

$$F = \Theta + \sin[2\pi(x-t)]a + \sin[2\pi(x-t) + \phi]b \tag{10}$$

Where
$$F = \int_{h_1}^{h_2} u dy = \int_{h_1}^{h_2} \frac{\partial \psi}{\partial y} dy \tag{11}$$

3. Solution of the problem

By solving equations (5) and (6) by utilizing the boundary conditions (7) and (8) we obtain the temperature and nanoparticle concentration as

$$\theta = C_2 + C_3 e^{-C_1 y} \tag{12}$$

$$\phi = -\frac{Nt}{Nb} C_3 e^{-C_1 y} + C_1 y + C_4 \tag{13}$$

Differentiation of equation (3) with respect to y yields as

$$\frac{\partial^4 \psi}{\partial y^4} - M^2 \frac{\partial^2 \psi}{\partial y^2} + Gr_t \frac{\partial \theta}{\partial y} + Gr_c \frac{\partial \phi}{\partial y} = 0 \tag{14}$$

From equation (12), (13) and (14) with the help of boundary conditions (7) and (8) we obtain the stream function and the velocity as

$$\psi = C_5 + C_6 y + C_7 \cosh My + C_8 \sinh My + l_3 e^{-C_1 y} + l_4 y^2 \tag{15}$$

$$u = C_6 + C_7 M \sinh My + C_8 M \cosh My - C_1 l_3 e^{-C_1 y} + 2l_4 y \tag{16}$$

Where $C_1 = \frac{N_b + N_t}{N_b(h_2 - h_1)}$, $C_2 = -\frac{e^{-C_1 h_1}}{e^{-C_1 h_2} - e^{-C_1 h_1}}$, $C_3 = \frac{1}{e^{-C_1 h_2} - e^{-C_1 h_1}}$, $C_4 = \frac{N_t}{N_b} C_3 e^{-C_1 h_1} - C_1 h_1$,

$$C_5 = \frac{F}{2} - C_6 h_2 - C_7 \cosh mh_2 - C_8 \sinh mh_2 - l_3 e^{-C_1 h_2} - l_4 h_2^2, C_6 = -1 - C_7 l_8 - C_8 l_9 - l_{10}, C_7 = \frac{l_{16} - C_8 l_{15}}{l_{14}}, C_8 = \frac{l_{16} l_{17} - l_{14} l_{19}}{l_{15} l_{17} - l_{14} l_{18}}$$

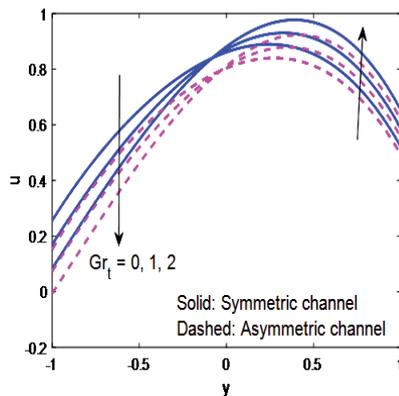


Figure 2. Velocity profiles for Gr_t

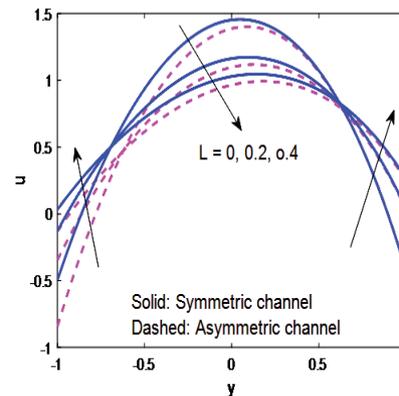


Figure 3. Velocity profiles for L

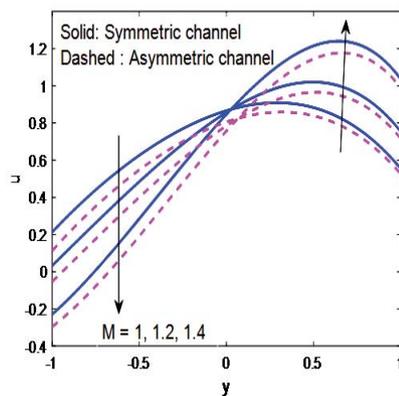


Figure 4. Velocity profiles for M

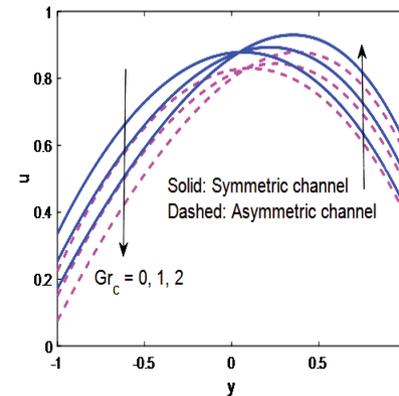


Figure 5. Velocity profiles for Gr_c

4. Results of the problem

The effects of the physical parameters on the flow phenomenon are discussed through graphs by using the fixed values of the parameters as $x = 0.6, t = 0.1, m = 0.2, a = 0.12, b = 0.2, M = 1, Nt = 2, Nb = 3, Rn = 1, Pr = 1, Gr_c = 1.5, Gr_t = 0.5, L = 1, \Theta = 1.5, \phi_0 = \pi/4$ (asymmetric channel) and $\phi_0 = 0$ (symmetric channel). It is clear that the velocity, temperature and concentration are high in symmetric channel when compared with asymmetric channel.

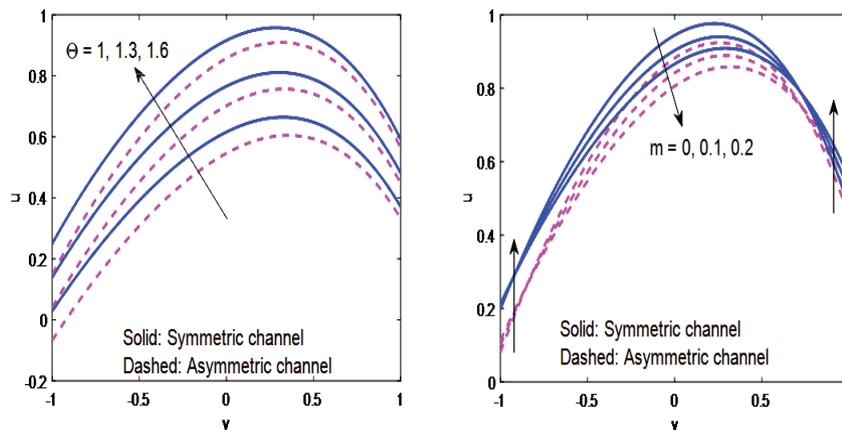


Figure 6. Velocity profiles for θ Figure 7. Velocity profiles for m

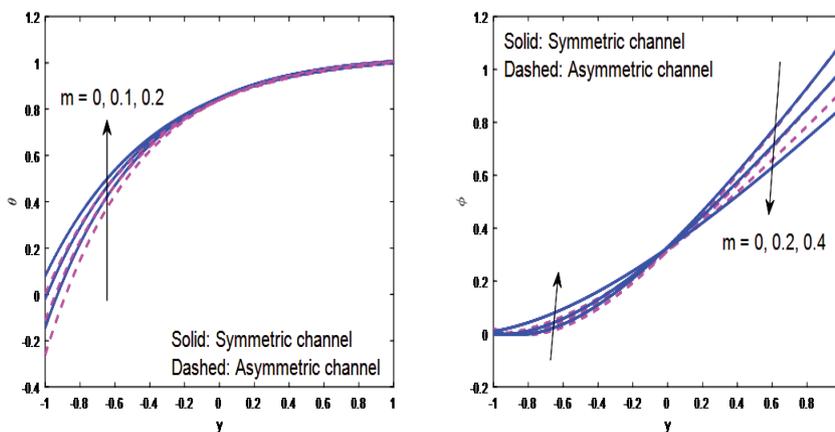


Figure 8. Temperature profiles for m Figure 9. Concentration profiles for m

From Fig.2 and Fig.3, we observed that the increase in Grashof number declines the velocity in the left half of the channel and opposite behaviour is observed in right half of the channel. Fig.4 depicts that the large values of magnetic parameter reduce the fluid velocity in the left half of the channel and enhances in right half of the channel. Further the velocity profiles are intersected at mid point of the channel. The effect of slip is shown in Fig.5. We noticed that the higher values of the slip parameter decrease the velocity in the middle of the channel but the situation is reversed at the channel walls. Fig.6 displays that the increase in flow rate improves the fluid velocity.

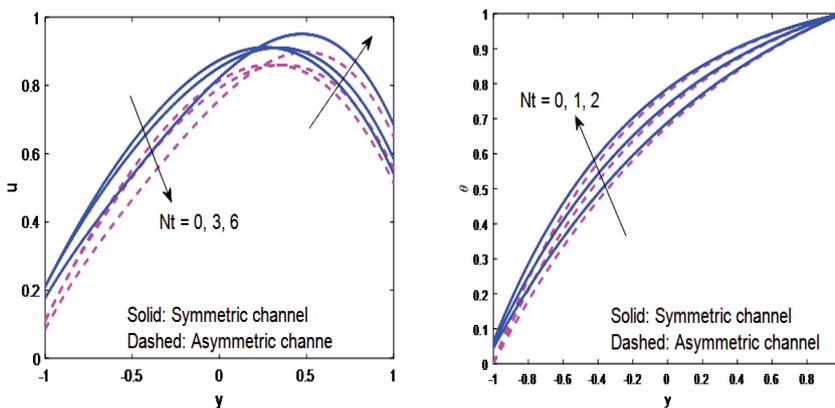


Figure 10. Velocity profiles for Nt Figure 11. Temperature profiles for Nt

Nt

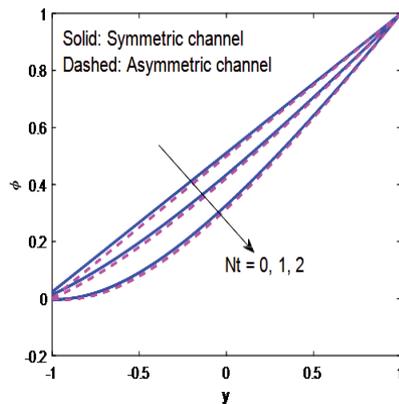


Figure 12. Concentration profiles for Nt

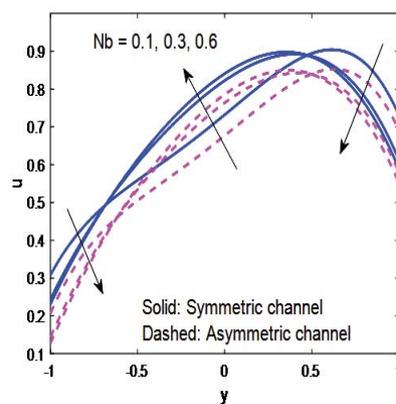


Figure 13. Velocity profiles for Nb

The impact of nonuniform parameter m shown in Figures 7 – 9. We identified that the influence of m on the velocity is same as in the case of slip parameter. Also observed that the increase in m increases the temperature field at the left wall and the profiles are coincide at the right wall of the channel while it enhances the concentration in left half of the channel and reduces in the right half of the channel. From Fig.10 – Fig.12, we noted that the higher values of thermophoresis parameter decrease the fluid velocity in the left half of the channel and the contrary behaviour is noticed in the right half of the channel. Moreover, we observed the increase in temperature field and decrease in concentration.

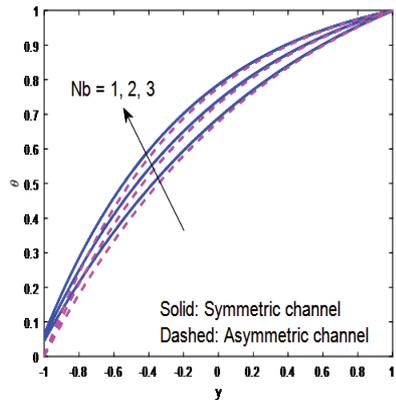


Figure 14. Temperature profiles for Nb

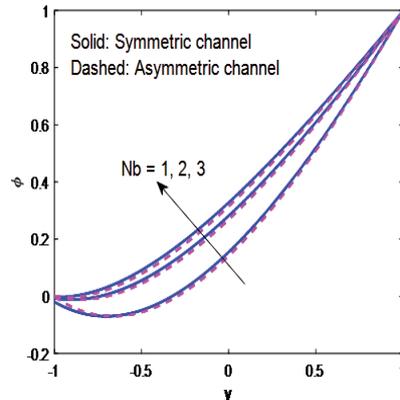


Figure 15. Concentration profiles for Nb

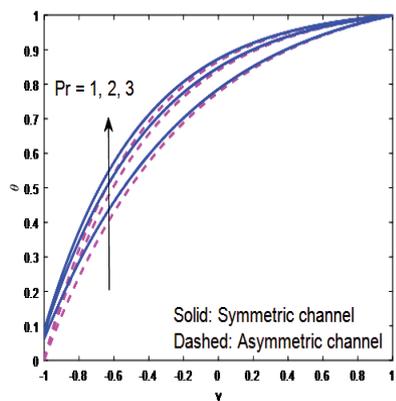


Figure 16. Temperature profiles for Pr

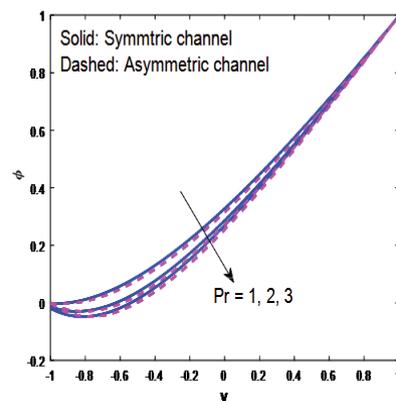


Figure 17. Concentration profiles for Pr

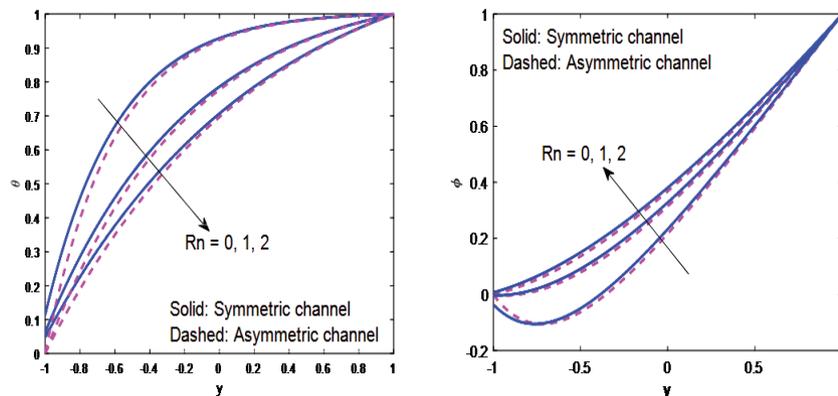


Figure 18. Temperature profiles for R_n **Figure 19.** Concentration profiles for R_n

To study the effect of Brownian motion parameter Nb we plotted the figures 13 – 15. The increase in Nb improves the velocity in the mid way of the channel but reduces at the channel walls. Further it enhances the temperature and concentration fields. From figures 16 – 19, we seen that the increment in Prandtl number boosts the temperature whereas it diminishes the concentration. Also we noticed the opposite behaviour for increasing radiation parameter. Fig.20 explains the validation of the present work.

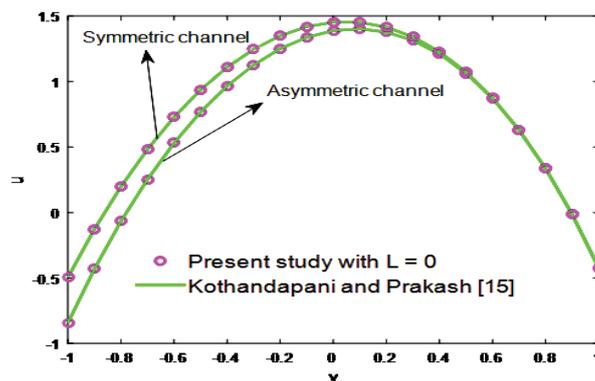


Figure 20. Validation with the existing work

References

- [1] Elshehawey E F and Mekhemier Kh S 1994 *J. Phys. D: Appl. Phys.* **27** 1163
- [2] Usha S and Ramachandra Rao A 1997 *J. Biomech. Engrg.* **119** 483-488
- [3] Vajravelu K, Sreenadh S and Ramesh Babu V 2005 *Int. J. Non-linear Mech.* **40** 83-90.
- [4] Radhakrishnamacharya G and Srinivasulu Ch 2007 *Mecanique* **335** 369-373.
- [5] Akbar N S, Hayat T, Nadeem S and Hendi A A, 2011 *Int. J. Heat Mass Transf.* **54** 1654-1664
- [6] Sucharitha G, Sreenadh S and Lakshminarayana P 2012 *Int. J. Eng. Research and Technology* **1(10)** 1-10
- [7] Chakradhar K, Nandagopal K, Sreenadh S and Lakshminarayana P 2015 *Int. J. of Math. Archive* **6 (3)** 173- 183
- [8] Vajravelu K, Sreenadh S, Lakshminarayana P, Sucharitha G and Rashidi M M 2016 *J. of Appl. Fluid Mech.* **9 (4)** 1615-1625
- [9] Akbar N S, Hayat T, Nadeem S and Hendi A A 2011 *Int. J. Heat Mass Transf.* **54** 1654-1664
- [10] Hayat T, Abbasi F M, Maryem A, Monaquel S 2014 *J. Mol. Liq.* **194** 93-99
- [11] Lakshminarayana P, Sreenadh S and Sucharitha G 2015 *Procedia Engineering* **127** 087-1094
- [12] Vajravelu K, Sreenadh S, Lakshminarayana P and Sucharitha G 2016 *Int. J. of Biomath.* **9 (2)** 1650023

- [13] Satyanarayana K V V, Sreenadh S, Sucharitha G and Lakshminarayana P 2016 *Ind. J. of Sci. and Techn.* **9 (42)** 1-9
- [14] Bhatti M M, Zeeshan A and Ijaz N 2016 *J. Mol. Liq.* **218** 240–245
- [15] Kothandapani M and Prakash J 2015 *J. of Mech. In Medi. and Biol.* **15(03)** 1550030
- [16] Tripathi D and Anwar Bég O 2015 *J. of Mech. in Medi. and Bio.* **15 (03)** 1550021
- [17] Kothandapani M and Prakash J 2015 *J. Magn. Magn. Mater.* **378** 152–163
- [18] Hayat T, Rijal Iqbal, Tanveer A and Alsaedi A 2016 *J. Mol. Liq.* **223** 1100–1113
- [19] Tanveer A, Hayat T, Alsadi F and Alsadi A 2017 *Compu. in Bio. and Med.* **82** 71-79
- [20] Sucharitha G, Lakshminarayana P and Sandeep N 2017 *Int. J. of Mech. Sci.* **131-132** 52-62