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Unsteady nonlinear convective Darcy flow of a non-Newtonian fluid over a rotating vertical cone

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Abstract. A numerical model on unsteady nonlinear convective flow of a Casson fluid past a vertical rotating cone in a porous medium has been developed. The conservations laws are transformed into non-linear problem using convenient similarity transformations. The resultant equations are solved numerically using Runge-Kutta based shooting technique for the velocity, temperature and concentration distributions, highlighted by physical parameters, Casson fluid parameter, unsteady parameter, non-linear temperature and concentration effects and discussed in detailed with graphical aid. Increasing non-linear temperature and concentration parameters accelerates the tangential velocity while normal and azimuthal velocities are decreased. Temperature and concentration distributions are also decreased as well. This study finds applications in industries like pharmaceutical industries, aerospace technology and polymer production etc.

1. Introduction

The non-Newtonian fluids have wide range of applications in nature as well as industries. In particular, the Casson fluid is for expressing the yield stress property of non-Newtonian fluid flow. It has been developed due to viscous suspension of cylindrical particles in a fluid flow. A few fluids are report well due to their highly non-linearity in the fluid flow, pseudo plasticity and yielding stresses in nature. Yield stress is a special case of non-Newtonian power-law model. For example slurries, blood, chocolate, waxy crude oil, wastewater sludge, gum solutions. Casson model is a suitable one for the non-linear reaction of pseudo plastic-yield stress. Recently, Rashad et al. [1] observed free convectiveflow of anano fluid about a porous cone in a saturated porous medium. Nadeem et al. [2] analyzed MHD convective flow of a Casson fluid from an exponentially sheet. Mukhopadhyay et al. [3] examined unsteady convection of a Casson fluid flow over stretching surface. Pramanik [4] attempted and investigated effect of thermal radiation on Casson fluid over an exponentially porous stretching surface. Afikuzzaman et al. [5] used finite difference technique to analyze hall current effects on unsteady magneto hydro dynamic Casson fluid flow through a parallel plate. Imran et al. [6] analyzed Falkner-Skan flow of MHD Casson fluid from a moving wedge. Naveed et al. [7] studied MHD effects on Casson fluid of squeezing flow through parallel plates.

Recent growths of progressive technologies have stimulated in convective fluid flows through a vertical cone. This study encounter in many industrial and engineering applications, such as solar collectors, hydrology, geosciences, manufacturing of transmission missile gun, development of electronic chips,aeronautical engineering, astrophysics, homeo-therapy treatment, endoscopy scanning and threaded connections etc. Unsteady convection from a cone plays vital role in science and engineering processes such as petroleum industries, environmental controlling and pharmaceutical chemistry. Shevchuk [8] observed transport process of heat and mass transfer in two rotating devices of cone-and-plate. Chamkha et al. [9] discussed double diffusive unsteady flow from a rotating sphere with different surface conditions. Rahman [10] analyzed the effects of partial slip, radiation, cross diffusion and thermophoretic of Nanoparticles over a rotating porous disk. Nadeem&Saleem [11] studied rotating flow of Non-Newtonian Nanofluid. Malik et al. [12] investigated dissipative effects on combined convection flow from a rotating cone with variable properties. Mallikarjuna et al. [13] studied thermophoresis effects on convective non-Darcy flow over a rotating porous cone in a porous medium. Athirah, Iliyas et al. [13] discussed unsteady heat transfer effects on MHD flow of rotating Jeffrey nanofluid.

With aforesaid applications, authors envisage to investigate unsteady non-linear convective flow of a Casson fluid over a rotating cone in a porous medium.

2. Formulation of the problem

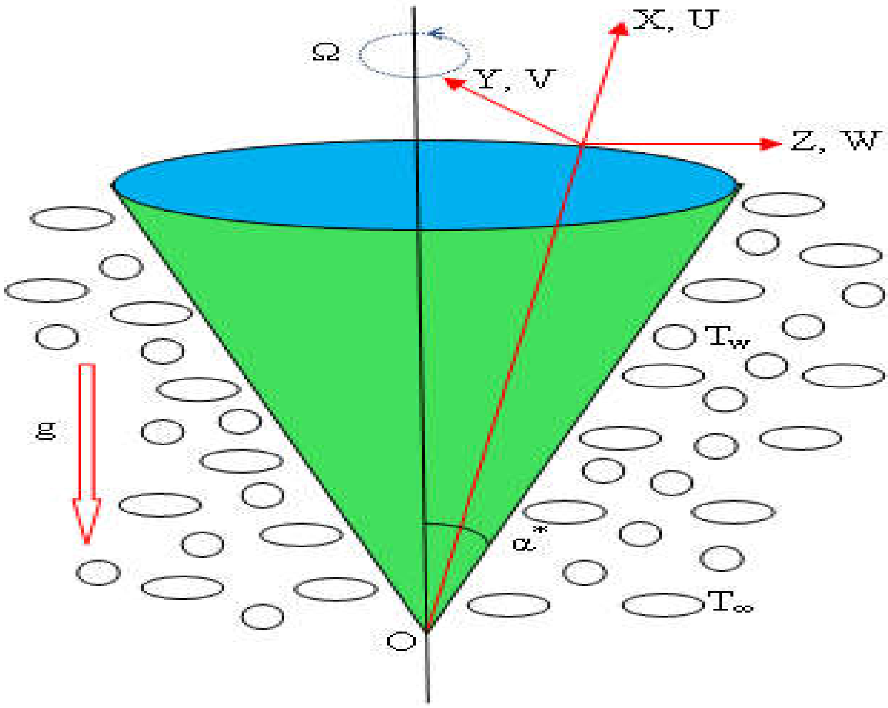

Fig. 1 Physical model of the problem

Consider unsteady viscous laminar non-Newtonian fluid flow over a rotating vertical cone in a Darcy porous medium with an angular velocity Ω. As shown in Fig. 1, a curvilinear coordinate frame is chosen in which X, Y and Z axis are respectively in tangential, azimuthal and normal directions. Governing boundary layer equations with Boussinesq approximation are given by (Roy and Anilkumar [14]):

$$X\frac{\partial U}{\partial X}+U+X\frac{\partial W}{\partial Z}=0 \qquad (1)$$

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial X}+W\frac{\partial U}{\partial Z}-\frac{V^2}{X}=\upsilon\left[1+\frac{1}{\beta}\right]\frac{\partial^2 U}{\partial Z^2}-\frac{\upsilon}{K}U$$
$$+g\cos\alpha\left[\beta_0\left(T-T_\infty\right)+\beta_2\left(C-C_\infty\right)+\beta_1\left(T-T_\infty\right)^2+\beta_3\left(C-C_\infty\right)^2\right] \tag{2}$$

$$\frac{\partial V}{\partial t}+U\frac{\partial V}{\partial X}+W\frac{\partial V}{\partial Z}+\frac{UV}{X}=\upsilon\left[1+\frac{1}{\beta}\right]\frac{\partial^2 V}{\partial Z^2}-\frac{\upsilon}{K}V \tag{3}$$

$$\frac{\partial T}{\partial t}+U\frac{\partial T}{\partial X}+W\frac{\partial T}{\partial Z}=\frac{k_e}{\rho c_p}\frac{\partial^2 T}{\partial Z^2} \tag{4}$$

$$\frac{\partial C}{\partial t}+U\frac{\partial C}{\partial X}+W\frac{\partial C}{\partial Z}=D\frac{\partial^2 C}{\partial Z^2} \tag{5}$$

and associated boundary conditions are
flow conditions:

$$U=0, V=\gamma\Omega\sin\alpha^*\left(1-st^*\right)^{-1}, W=0, \text{ at } Z=0$$
$$U=0, V=0 \text{ as } Z\to\infty \tag{6}$$

Thermal and concentration conditions are:

$$T=T_w, C=C_w \text{ at } Z=0 \text{ and}$$
$$T=T_\infty, C=C_\infty \text{ as } Z\to\infty \tag{7}$$

where U, V and W are velocities along X, Y and Z directions respectively.

3. Solution and Procedure

Introducing the following non dimensional variables

$$\eta=\left(\frac{\Omega\sin\alpha^*}{\upsilon}\right)^{\frac{1}{2}}\left(1-st^*\right)^{-\frac{1}{2}}Z, t^*=(\Omega\sin\alpha^*)t, P_r=\frac{\rho c_p\upsilon}{k_e}, S_c=\frac{\upsilon}{D}$$

$$\left[U,V\right](t,X,Z)=\left[-2^{-1}f'(\eta),\ g(\eta)\right]\Omega X\sin\alpha^*\left(1-st^*\right)^{-1}$$

$$W(t,X,Z)=\left(\upsilon\Omega\sin\alpha^*\right)^{\frac{1}{2}}\left(1-st^*\right)^{-\frac{1}{2}}f(\eta),$$

$$\left[T,C\right](t,X,Z)-T_\infty=\left[\left(T_w-T_\infty\right)\Theta(\eta),\left(C_w-C_\infty\right)\Phi(\eta),\right],$$

$$\text{where}\left[T_w-T_\infty, C_w-C_\infty\right]=\left[\left(T_0-T_\infty\right),\left(C_0-C_\infty\right)\right]\left(\frac{X}{L}\right)\left(1-st^*\right)^{-2}$$

Using above equations, equations (2)-(6) becomes

$$\left(1+\frac{1}{\beta}\right)f'''+\frac{1}{2}\left(f'\right)^2-ff''-2g^2-Da^{-1}f'-s\left(f'+\frac{\eta}{2}f''\right)-$$
$$2\Delta\left[\Theta+\alpha_1\Theta^2+N\left(\Phi+\alpha_2\Phi^2\right)\right]=0 \tag{8}$$

$$\left(1+\frac{1}{\beta}\right)g''-s\left(g+\frac{\eta}{2}g'\right)-fg'+gf'-Da^{-1}g=0 \tag{9}$$

$$\frac{1}{P_r}\Theta''-s\left(2\Theta+\frac{\eta}{2}\Theta'\right)+\frac{1}{2}f'\Theta-f\Theta'=0 \tag{10}$$

$$\frac{1}{S_c}\Phi'' - s\left(2\Phi + \frac{\eta}{2}\Phi'\right) - f\Phi' + \frac{1}{2}f'\Phi = 0 \tag{11}$$

And the associated boundary conditions are

$$\begin{aligned} &f = 0, f' = 0, g = 1, \Theta = 1, \Phi = 1 \text{ at } \eta = 0 \text{ and} \\ &f' = 0, g = 0, \Theta = 0, \Phi = 0 \text{ as } \eta \to \infty \end{aligned} \tag{12}$$

Where β is Casson fluid parameter

$Da^{-1} = \dfrac{\upsilon}{K\Omega \sin\alpha^* \left(1 - st^*\right)^{-1}}$ is the inverse Darcy number

$\Delta = \dfrac{Gr}{\mathrm{Re}^2}$ is mixed convection parameter

$Gr = \dfrac{g\beta_0 \left(T_0 - T_\infty\right) L^3 \cos\alpha}{\upsilon^2}$ is the Grashof number

$\mathrm{Re} = \dfrac{\Omega \sin\alpha^* L^2}{\upsilon}$ is the Reynold's number

$\alpha_1 = \dfrac{\beta_1}{\beta_0}\left(T_w - T_\infty\right)$ is the non-linear thermal convection parameter

$\alpha_2 = \dfrac{\beta_3}{\beta_2}\left(C_w - C_\infty\right)$ is the non-linear concentration convection parameter

And $N = \dfrac{\beta_2}{\beta_0}\left(\dfrac{C_0 - C_\infty}{T_0 - T_\infty}\right)$ is the buoyancy ratio

Flow characteristics of the fluid, which are the local surface tangential skin friction coefficient and the azimuthal skin friction coefficientalong X and Y directions are $C_{fx} = \dfrac{2\mu\left(\dfrac{\partial U}{\partial Z}\right)_{Z=0}}{\rho\left(\Omega_0 X \sin\alpha\right)^2}$ and $C_{fy} = \dfrac{-2\mu\left(\dfrac{\partial V}{\partial Z}\right)_{Z=0}}{\rho\left(\Omega_0 X \sin\alpha\right)^2}$, the local Nusseltnumber is $Nu_x = \dfrac{-X\left(\dfrac{\partial T}{\partial Z}\right)_{Z=0}}{\left(T_w - T_\infty\right)}$ and the local Sherwood number is $Sh_x = \dfrac{-X\left(\dfrac{\partial C}{\partial Z}\right)_{Z=0}}{\left(C_w - C_\infty\right)}$.

In non-dimensional form, tangential skin friction coefficient is $\mathrm{Re}^{1/2} C_{fx} = -f''(0)$,azimuthal skin friction coefficient is $2^{-1}\mathrm{Re}^{1/2} C_{fy} = -g'(0)$, Nusselt number is $\mathrm{Re}^{-1/2} Nu_x = -\Theta'(0)$ and Sherwood number is $\mathrm{Re}^{-1/2} Sh_x = -\Phi'(0)$.

4. Results and Discussions

The Analytical solutions of a set of equations (8) – (11) with conditions (12) are difficult due to coupled non-linearity. Therefore, numerical method, namely the shooting technique (Rashad et al.[16]) is used and presented the results graphically on fluid flow velocities, (tangential velocity f', circumferential velocity g and normal velocity f, temperature Θ and concentration Φ distributions for different physical parameters. The physical parameters are fixed with the values $Da^{-1} = 0.5$, $\beta = 0.5$, $s = 2$, $\Delta = 10$, $\Pr = 0.71$, $Sc = 0.22$, $N = 1$, $\alpha_1 = 0.5, \alpha_2 = 0.5$ unless specified. The obtained results are compared and correlated with existing results Hering and Grosh [17],Himasekhar and Sarma [18] and Mallikarjuna et.al [19] for linear convection of a Newtonian fluid without porous media as shown in Table-1.

Table-1. Comparison results in the absence of the concentration equation for linear convection of Newtonian fluid

	$-\Theta'(0)$			
Δ	Hering and Grosh [22]	Himasekhar and Sarma [23]	Mallikarjuna et.al [24]	Present results
0	0.42852	0.4316	0.42842	0.42849
0.1	0.46156			0.46114
1.0	0.61202		0.61213	0.61206
10	1.0173		1.07018	1.01728

Table 2 shows that skin friction coefficients, Nusselt(Nu) and Sherwood number (Sh)) results for different values of β, s, α_1 and α_2. It is observed that as increase in α_1 and α_2, skin friction coefficients, rate of thermal and concentration transportations are accelerated near the cone surface. Skin friction coefficients along tangential and azimuthal direction and local Nusselt and Sherwood numbers are enhanced with increasing values of Casson fluid parameter and unsteadiness parameter.

Table 2. Tangential and circumferential (azimuthal) Skin friction coefficients, local Nusselt number and Sherwood number values for $\alpha_1, \alpha_2, \beta$ and s.

α_1	α_2	β	s	$-f''(0)$	$-g'(0)$	$-\Theta'(0)$	$-\Phi'(0)$
0.1	0.1	0.5	2	6.2618	1.5820	1.7810	1.0185
5.0				12.3526	1.8	1.8828	1.0750
10				17.9585	1.9631	1.9642	1.1194
	5.0			14.8405	1.9416	1.9554	1.1235
	10			22.4556	2.1745	2.0812	1.1977
		1		8.3550	1.6797	1.8256	1.0442
		2		10.2059	1.7527	1.8601	1.0630
			3	5.5996	1.7328	2.0687	1.1720
			4	5.0640	1.8920	2.3311	1.3131

Figs. 2-6 depicts unsteady and Casson fluid parameters on velocities, temperature and concentration profiles respectively. The tangential velocity profile gets peak near the cone and tends to zero as moving far from the surface. As unsteady parameter increases f' results are reduced as shown in fig. 3. The circumferential and the normal velocity profiles are get enhanced with increase in s. But the temperature and the concentration distributions are reduced for increasing the values of s. Increase in β influences to increase the tangential profile from cone surface to certain point and then reduced as η tends to large value, while circumferential velocity g and normal velocity f are decreased. As β increasing from 0.5 through 1 and 2, the temperature and the concentration profiles are decelerated enormously as shown in figs. 5 and 6.

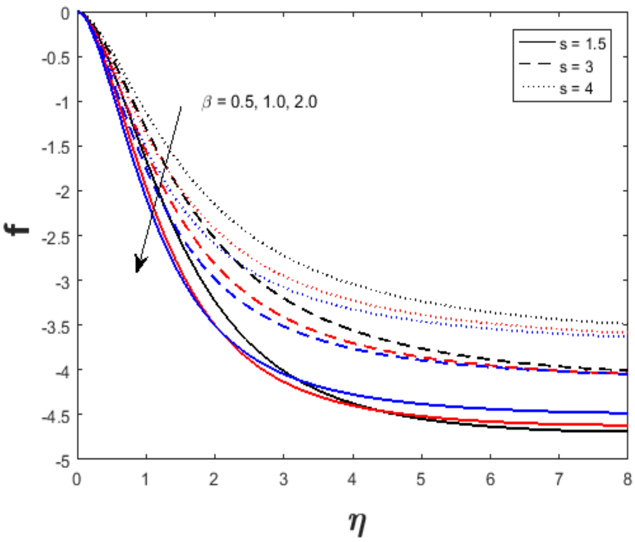

Figure 2. Variation of β and s on f

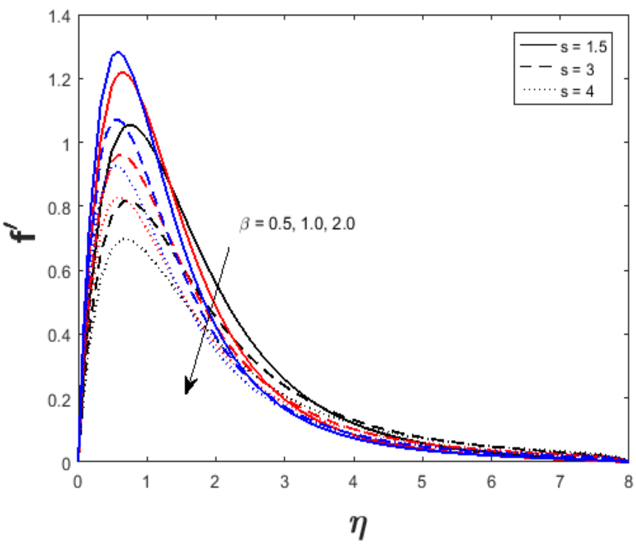

Figure 3. Variation of β and s on f'

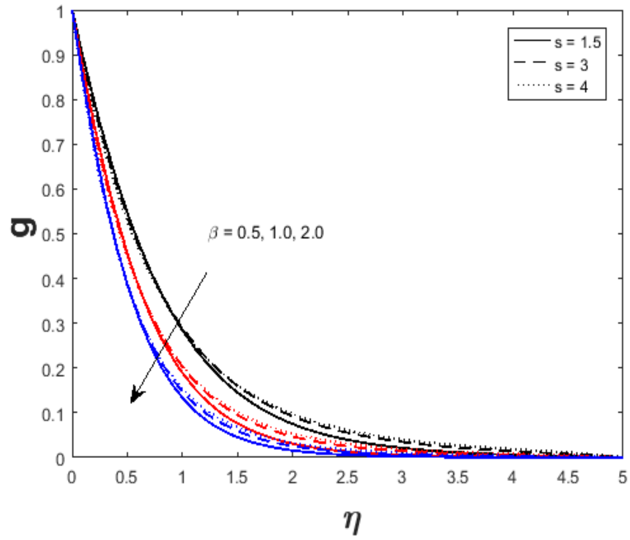

Figure 4. Variation of β and s on *g*

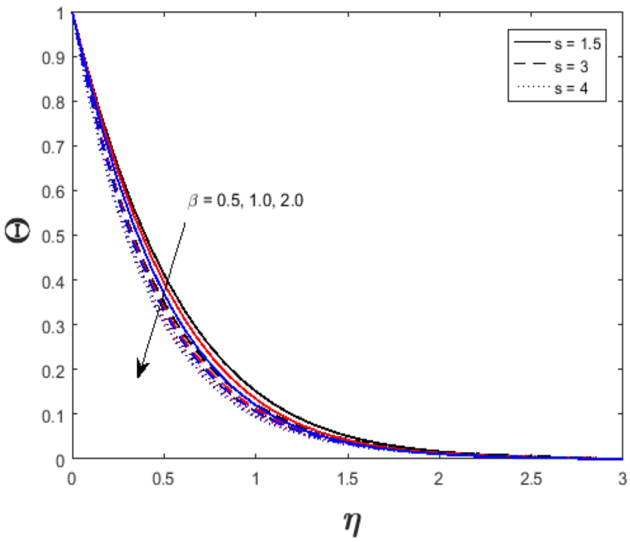

Figure 5. Variation of β and s on Θ

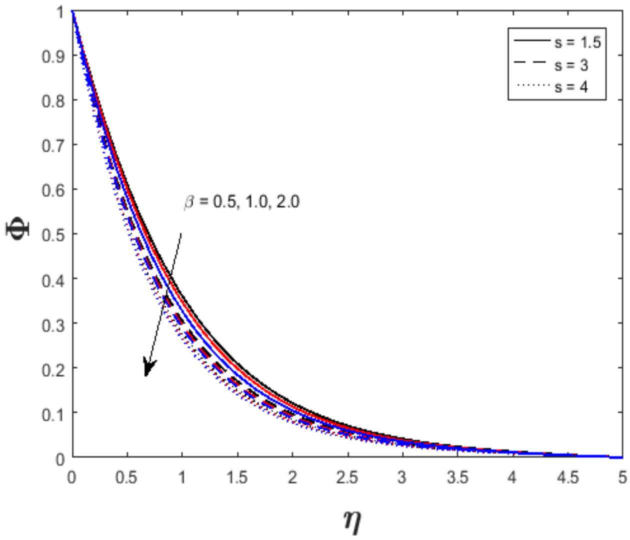

Figure 6. Variation of β and s on Φ

Fig. 7-11 represents the nonlinear thermal convection α_1 and the concentration convection α_2 parameters on velocity, temperature and concentration distributions respectively. Increasing α_1 and α_2 tends to enhance the tangential velocity near the cone surface to certain point, and decelerates afterwards as shown in fig. 8, while azimuthal and normal profiles and temperature and concentration distributions are reduced with increase in α_1 and α_2 .

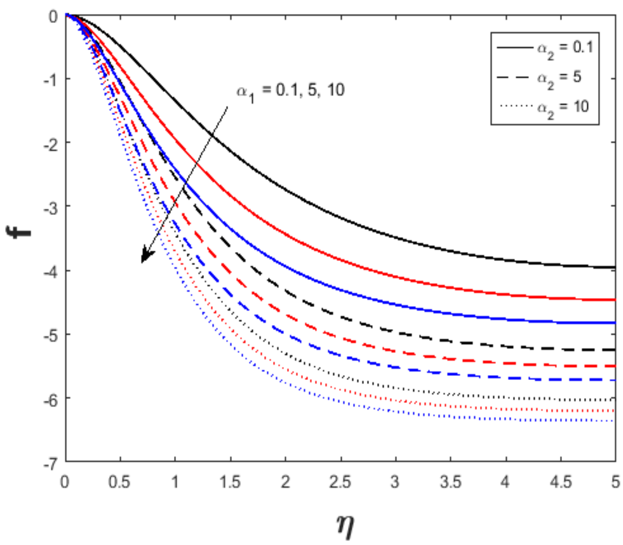

Figure 7. Variation of α_1 and α_2 on f

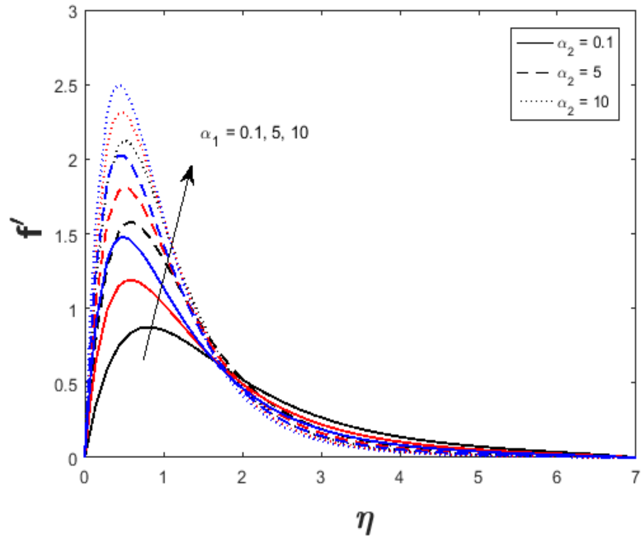

Figure 8. Variation of α_1 and α_2 on f'

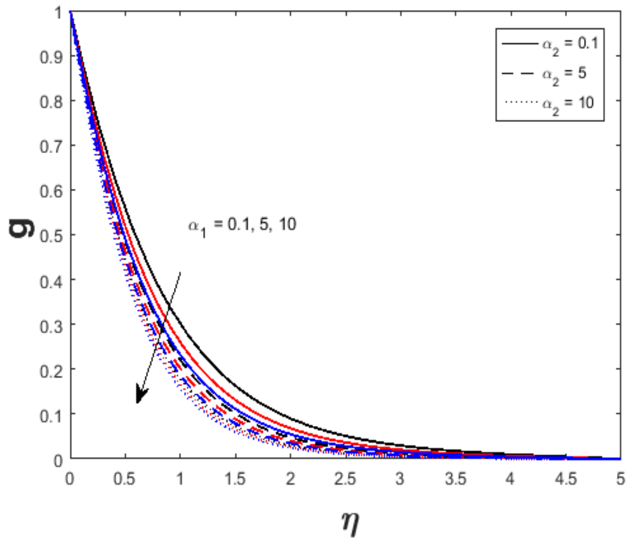

Figure 9. Variation of α_1 and α_2 on g

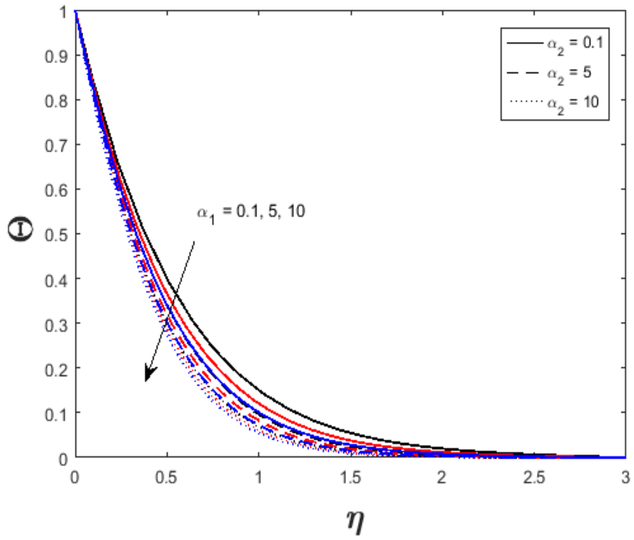

Figure 10. Variation of α_1 and α_2 on Θ

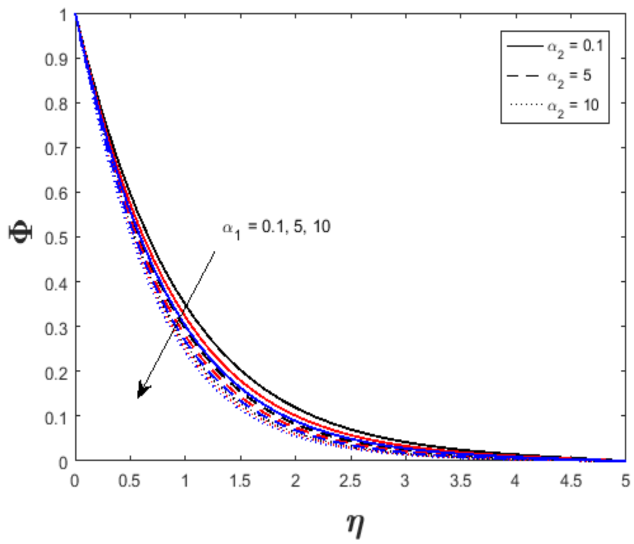

Figure11. Variation of α_1 and α_2 on Φ

5. Conclusion

In this paper, a model on nonlinear convective Darcy flow of a Casson fluid from a rotating cone has been developed and analyzed. The basic equations for the model are non-dimensionalzed and reduced to ordinary nonlinear differential equations using specified transformations. A numerical technique has been considered to produce the results of the resultant equations graphically. A comparison has been done with earlier results to check the accuracy of the numerical technique. As non-linear thermal and concentration convection parameters increase, flow characteristic profiles are reduced as well as skin friction coefficients, Nusselt and Sherwood number results.

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