Magnetic field dependent intensity variation in the hyperfine split Rb D_1 and D_2 lines

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Abstract. We present the variation of intensity in Doppler-free hyperfine lines for rubidium atoms with the application of magnetic field. Different polarization configurations have been studied systematically with varying magnetic fields. There is a significant increase in the intensity variation with applied magnetic field related to different polarization configurations. These variations are explained with the theoretical calculations. The calculations are performed by adopting Nakayama's four level model with the varying magnetic field induced transition probability.

1. Introduction

Atom-laser interaction in the presence of a magnetic field (magneto-optical effects) has been a subject of study for several decades [1]. The availability of narrow line width and tunable diode lasers made these experiments possible in many laboratories [2-3]. Conventional absorption spectroscopy techniques [4-5] results in Doppler broadened absorption spectrum. Doppler-free saturation absorption spectroscopy can be used to resolve the Zeeman lines in the case of weak or intermediate magnetic fields. Alkali metal atoms provide the simplest prototype for high-resolution spectroscopic studies of atomic energy level structure and transition probabilities under the influence of applied electric and magnetic fields. Even though the spectra of [87]Rb have already been studied extensively [6-10], there have been few systematic investigations of Zeeman spectral intensity as a function of an applied magnetic field [11]. In the present work, optical pumping effects, polarization effects and magnetic field induced transition probability have been taken into account in order to account for the complete study of the Zeeman profile.

In this paper the magnetic field dependent intensity variation of rubidium hyperfine split D_1 (5 $^2S_{1/2}$ → 5 $^2P_{1/2}$) and D_2 (5 $^2S_{1/2}$ → 5 $^2P_{3/2}$) lines have been discussed. Tremblay et al. [12] has estimated the magnetic field dependent transition probability and Nakayama [13] has developed four level model to calculate the relative intensity of hyperfine transitions. We have adopted these two models and found a unique way of calculating relative intensity in the presence of magnetic field, which is described in detail in the next section. We have also calculated the magnetic field dependent intensity variation with different polarization configurations of pump and probe beams.

2. Theoretical Formalism

The Hamiltonian which demonstrates the hyperfine structure with the magnetic field is:

$$H = A_J(\mathbf{I}.\mathbf{J}) + B_J \frac{3(\mathbf{I}\mathbf{J})^2 + \frac{3}{2}(\mathbf{I}\mathbf{J}) - I(I+1)J(J+1)}{2I(2I-1)J(2J-1)} + H_B \qquad (1)$$

where A_J is the magnetic hyperfine structure constant and B_J is the electric quadrupole constant. H_B is the interaction Hamiltonian due to external magnetic field. The interaction Hamiltonian;

$$H_B = \left(\frac{-\mu_B}{\hbar}\right) \mathbf{B}.(\mathbf{L} + g_S \mathbf{S} + g_I \mathbf{I}) \qquad (2)$$

where μ_B is the Bohr magneton, **B** is the static magnetic field, g_S is the electron spin Landé factor and g_I is the nuclear spin Landé factor. **L, S** and **I** are orbital, spin and nuclear spin quantum numbers.

The Hamiltonian can be represented in the matrix form as

$$\langle F, m_F|H|F, m_F\rangle = H_{HFS} - \mu_B g_F m_F B_z \qquad (3)$$

This gives the diagonal matrix elements of the Hamiltonian with the unperturbed state vectors.

Here g_F is the Landé factor and m_F is the magnetic quantum number.

The off- diagonal matrix elements can appear only when the selection rules satisfy this condition as $\Delta F = \pm 1$ and $\Delta m_F = 0$

$$\langle F - 1, m_F|H|F, m_F\rangle = \langle F, m_F|H|F - 1, m_F\rangle =$$

$$\frac{-\mu_B}{2}(g_J - g_I)B_z \left(\frac{[(J+I+1)^2 - F^2][F^2 - (J-I)^2]}{F}\right)^{\frac{1}{2}} \left(\frac{F^2 - m_F^2}{F(2F+1)(2F-1)}\right)^{1/2} \qquad (4)$$

The energy eigen values for different magnetic fields are obtained by diagonalizing the hyperfine structure Hamiltonian. Tremblay et al. [12] has introduced a way to calculate the transition probability with the application of magnetic field. The hyperfine total angular momentum quantum number (F) is no longer considered as a good quantum number to describe the system completely with the application of magnetic field. Hence we have to assume that the magnetic sublevels of the hyperfine levels are mixed due to the influence of magnetic field.

The change of wave function for the ground state can be written as;

$$|\psi(F_g, m_g)\rangle = \sum_{F_g'} C_{F_g F_g'} |F_g', m_g\rangle ;$$

and for the excited state;

$$|\psi(F_e, m_e)\rangle = \sum_{F_e'} C_{F_e F_e'} |F_e', m_e\rangle \qquad (5)$$

where $|F_g', m_g\rangle, |F_e', m_e\rangle$ are the state vectors for the ground and excited states. $C_{F_g F_g'}$ and $C_{F_e F_e'}$ are mixing coefficients depending on the field strength and magnetic quantum number m_F. There are four hyperfine levels in the $5\,^2P_{3/2}$ excited state and two hyperfine levels in the $5\,^2S_{1/2}$ ground state. The linear combination of different hyperfine levels represents the new atomic states $|\psi(F_g, m_g)\rangle$, $|\psi(F_e, m_e)\rangle$, which is due to the nonlinear nature of Zeeman sublevels. The hyperfine sublevels with the same magnetic quantum numbers are likely to be mixed with the application of magnetic field. The mixing coefficients of the hyperfine states are the eigenvectors and the energies of these corresponding levels are eigenvalues. This can be obtained using the above mentioned equations.

The dipole matrix element is proportional to the square of the transfer coefficients due to the application of a magnetic field.

$$|\langle e|D_q|g\rangle|^2 \approx a^2[\psi(F_e, m_e); \psi(F_g, m_g); q] \qquad (6)$$

The transfer coefficients with the influence of magnetic field can be written as

$$a[\psi(F_e, m_e); \psi(F_g, m_g); q] = \sum_{F_g' F_e'} C_{F_e F_e'} a(F_e', m_e; F_g', m_g; q) C_{F_g F_g'}, \text{ where}$$

$$a(F_e', m_e; F_g', m_g; q) =$$

$$(-1)^{1+I+J_e+F_e+F_g-m_e}\sqrt{2J_e+1}\sqrt{2F_e+1}\sqrt{2F_g+1}\begin{pmatrix} F_e & 1 & F_g \\ -m_e & q & m_g \end{pmatrix}\begin{Bmatrix} F_e & 1 & F_g \\ U_g & I & J_e \end{Bmatrix} \qquad (7)$$

In equation (7), the parentheses and the curly brackets are the representation of 3*j* and 6*j* symbols.

Nakayama's four level model can be used to calculate the relative line intensities of the Doppler-free spectra [13]. Optical pumping effects are taken into consideration in this model.

The electric susceptibility can be written as

$$\zeta_{P_r P_r}^{(P_u)} \approx \sum_{u,r} I_{P_r P_r}^{(P_u)} \exp\left[-\left(\frac{\Delta\omega_{ur}}{2kv}\right)^2\right]/(\omega - \omega_{ur} - i\Gamma) \qquad (8)$$

The indices *u, r* represent the pump and probe beam transitions. p_u, p_r correspond to the polarization states. ω_u, ω_r are the angular frequencies, $\Delta\omega_{ur} = (\omega_u - \omega_r)$, $\omega_{ur} = (\omega_u + \omega_r)/2$. *k* is the wave number and Γ is the line width of the absorption signal. The exponential term is related to the Doppler factor. The most probable velocity $v = \sqrt{\frac{2K_B T}{M}}$. In this approximation, saturation effects are not taken into account because we are considering only the lower laser intensity approximation. In this case the optical coherences in the atom-laser interaction can be neglected.

The relative intensity for the four-level system is

$$I_{P_r P_r}^{(P_u)} = |\mu_u|^2 |\mu_r|^2 \left(-\delta_{u,sp} + |\mu_{sp}|^2/\eta\right) \qquad (9)$$

where $|\mu_u|^2$, $|\mu_r|^2$ and $|\mu_{sp}|^2$ are the transition probabilities for the pump, probe and the spontaneous emission transitions. η is the total transition probability. The value of Kronecker's delta ($\delta_{u, sp}$) is unity for I (two-level) and V- type (three-level) resonances. It is zero for Λ (three-level) and N- type (four-level) resonances. The absorption coefficient is related to the electric susceptibility, which is represented as

$$\chi_{P_r P_r}^{(P_u)} \approx Im\left(\zeta_{P_r P_r}^{(P_u)}\right) \qquad (10)$$

Nakayama used the four level model to compute the intensity variation of hyperfine lines without the magnetic field. We have extended this work by applying the magnetic field induced transition probability to calculate the intensity variation of different hyperfine lines in the presence of magnetic field.

3. Results and Discussion

3.1. Relative intensity of Doppler- free hyperfine D₂ lines in the presence of magnetic field
The magnetic field dependent transition probabilities for the $\Delta m_F = 0$ transitions in the case of F=2 to F'=1, 2, 3 for ^{87}Rb (D₂ line) is shown in figure 1. The dotted lines in these figures correspond to the non- dipole allowed transitions which have non zero transition probability in the presence of magnetic field. In order to explain the relative magnitude of the observed Doppler- broadened hyperfine lines in the presence of moderate magnetic field (up to 50G), Tremblay's field induced transition probability and Nakyama's four level model has been followed.
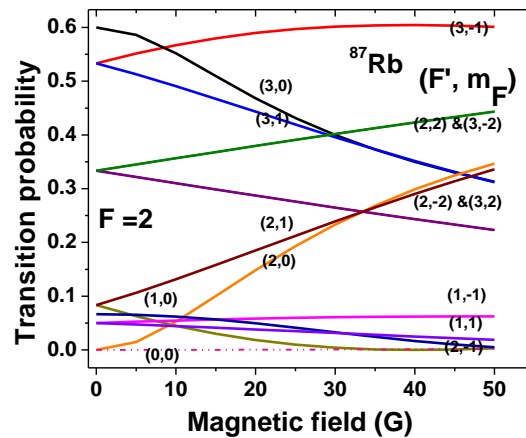

Figure 1. Transition probabilities for the F = 2 to F' = 0, 1, 2, 3 transitions in π polarization of ^{87}Rb D₂ line as a function of the magnetic field

The non-dipole forbidden transitions ($\Delta m_F = 0$, $\Delta F = 0$; $\Delta F = \pm 2$) will become allowed in the presence of a magnetic field. In the case of high magnetic field region, these forbidden transitions play a significant role in the absorption spectrum. In this work we focus on the weak/intermediate magnetic field region, hence it is no longer important to consider the forbidden transitions. Here we only consider the dipole allowed transitions for calculating the magnetic field dependent relative intensities.

Relative magnitude of Doppler- free hyperfine lines have been measured using saturation spectroscopy. All possible polarization configurations of pump and probe beams such as (π, σ_-); (σ_+, σ_-); (σ_+, σ_+); and (π, π) have been considered. In the four level model, number of separate resonances for (π, $\sigma_\pm$) polarization configuration is 214 for ^{87}Rb atoms. Likewise (σ_+, σ_-), (π, π) and (σ_+, σ_+) have 107, 106 and 95 resonances. We have calculated the magnetic field dependent transition probabilities for each of these resonances and added them together for particular pump-probe polarization scheme.
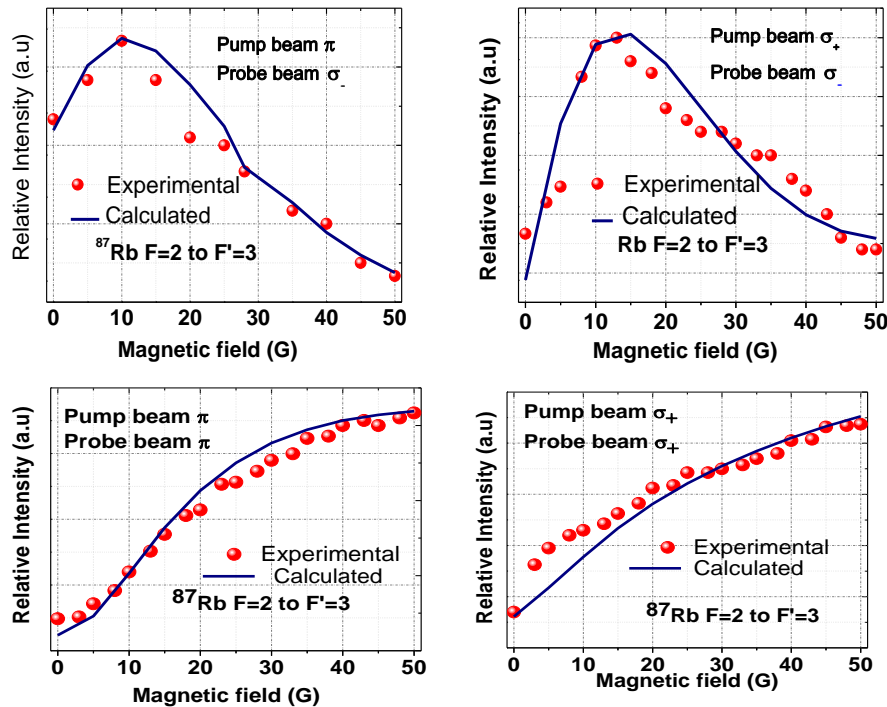

Figure 2. Relative intensities as a function of magnetic field for the transition F=2 to F'=3 in ^{87}Rb for different pump-probe polarization schemes.

The variation of relative intensities for the closed transition (F=2 to F'=3) for ^{87}Rb is shown in figure 2. In the case of (π, σ_-) and (σ_+, σ_-) pump- probe polarization configurations, the relative intensity increases and reaches a maximum around 10 G and then decreases. This has been predicted in the theoretical calculation as well. The mixing of different hyperfine F levels with the same m_F value and the influence of optical pumping is responsible for this intensity variation. This has been observed only for the closed transition of ^{87}Rb. The other hyperfine transitions of the D_2 line (F=2 to F'=1 and F=2 to F'=2) do not have such characteristics with the magnetic field. Hence it is an interesting feature which can be taken into consideration while designing rubidium atomic clock and vapor cell magnetometer.

3.2.   Relative intensity of Doppler-free hyperfine D_1 lines in the presence of magnetic field
The magnetic field induced transition probability for the rubidium D_1 line is calculated and is shown in figure 3. The transition probability varies linearly with respect to magnetic field. There are two different F levels and the number of Zeeman sublevels in $5\ ^2P_{1/2}$ state is 8 for ^{87}Rb. The hyperfine energy levels are widely spaced (~812 MHz for ^{87}Rb) in comparison with $5\ ^2P_{3/2}$ state. This makes the study of $5\ ^2P_{1/2}$ state less of an issue with respect to magnetic field.
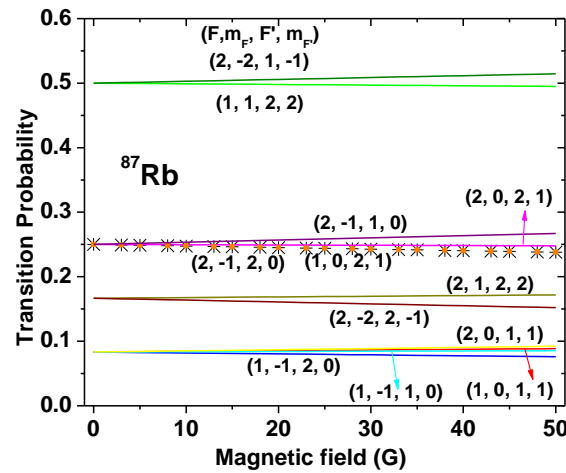

Figure 3. Transition probabilities for the F = 1, 2 to F$^{'}$ = 1, 2 transitions in $\sigma_{\pm}$ polarization of ^{87}Rb D_1 line as a function of the magnetic field
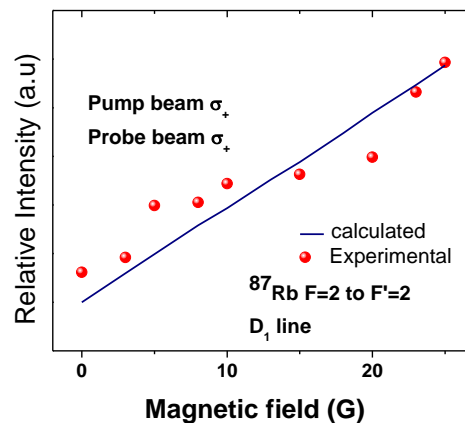

Figure 4. Relative intensities as a function of magnetic field for the transition F=2 to F'=2 in ^{87}Rb for (σ_+,σ_+) polarization configuration

In the case of D_1 line ^{87}Rb spectrum, the field induced transition probability varies linearly with respect to magnetic field. All the four hyperfine transitions for the D_1 line are open transitions and there is an equal distribution of population among these appropriate hyperfine ground levels. According to four level model the intensity varies linearly with respect to magnetic field, which is depicted in figure 4.

4. Summary and conclusions
The relative intensity is computed using theoretical models of field induced transition probability and the four level model. The study of magnetic field dependent intensity variation shows the role of non-

linearity of Zeeman levels, optical pumping and polarization effects. The relative magnitude of intensity is maximum at a field of 10 G in the ^{87}Rb D_2 line transition F=2 to F'=3 for the polarization configurations (σ_+, π) and (σ_+, σ_-). This is purely a non-linear effect which needs to be taken into account while designing magnetic devices with rubidium atoms.

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