

Spatiotemporal stability and sensitivity analysis of a holling type-II prey predator system with Allee effect

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Abstract. In this article, an analytical investigation is carried out on two species prey predator model having Holling type II interaction with Allee effect. We analysed the local and global behaviour of the interior equilibrium point. The theoretical aspects such as the investigation of the existence and stability of the equilibriums as well as the oscillatory and sensitivity behaviours of the corresponding diffusive system have been pursued. Also, analysed the effect of white noise and diffusion analysis of the proposed system. Finally, all the analytical results are interpreted ecologically and compared with the numerical results generated by MATLAB.

1. Introduction

Both Biologists and economists have given close concentration towards the dynamical performances of various populaces and their assets. The subject mathematical model of ecosystem evolutions has a long history. The interactions between predator-prey and parasite-host and herbivore-plant are fundamentally the same. The mathematicised modelling of these interactions looks often similar. In this paper, x will denote biomass or density of prey species and y will denote the biomass or density of the predator species. Since the original work of Lotka and Volterra[1-2], the research concern in predator-prey dynamics has accomplished constant attention. It is well known that these models can directly reflect changes in the size of populaces. Considerable improvements are that the relevant theories become more and more complete in this category in recent years [3-7].

The basic model is Lotka-Volterra system, given by the system of mathematical equations of the form

$$x'(t) = M(x) - N(x, y) \quad (1.1)$$

$$y'(t) = kN(x, y) - H(y) \quad (1.2)$$

where $M(x)$ denotes the function explains the growth rate of prey, $N(x, y)$ is a function which gives the information about how prey reduces due to a predator by y , k denotes the yield factor which explains at what rate prey losses to predator gain, H is a function explains the predator removal rate in the absence of prey. The basic form which explains Lotka-Volterra system can take is

$$x'(t) = rx - nxy \quad (1.3)$$

$$y'(t) = knxy - hy \quad (1.4)$$

where f, n, k, h are constants. Equations (1.3)-(1.4) explain that in the absence of a predator, prey grows exponentially and predators starve at a constant rate, which leads to the exponential decay of the predators in the absence of prey. The rate of predation is modelled as proportional to the number of predator-prey attacks and it is proportional to the product of biomasses of prey and predator. The coexisting state of the steady state is neutrally stable. The perturbation about this steady state point



does not bring the system to the same steady. The oscillations of prey and predator population and their amplitudes are depended upon initial population from steady state point. The prey species grow exponentially to the infinity if predators are absent. This short growing in prey population is modified by introducing logistic growth of prey population. The improvement of the model explains by using the model equations.

$$x'(t) = rx \left(1 - \frac{x}{k} \right) - nxy \quad (1.5)$$

$$y'(t) = knxy - hy \quad (1.6)$$

In which r is the maximized relative growth rate and k is the carrying capacity of the ecosystem. The equilibrium point is the point of intersection of two nulclines. The y - coordinate of the equilibrium point is a function of x and the equilibrium point is stable. This improvement of the model (1.5)-(1.6) merely does not explain the predator-prey dynamics observed in nature. The main imperfection of the model (1.5)-(1.6) is that the relative growth rate of prey biomass increases. But in reality, the growth rate of predator is limited due to other diverse factors. The most easily understood reasons of which are maximal ability to cause the growth of the predator and the handling time which is the time required to predator to process the prey species. During the interval of handling time, the predator cannot attack another prey species. The improvement of mathematical models mere focuses on functions which give the information about how prey reduces due to a predator. The Holling type model is given by

$$N(x, y) = \frac{nxy}{\beta + x} \quad (1.7)$$

In which β is the saturation constant. If prey biomasses are high then the predator biomass grows at a maximum relative growth rate n , when prey biomass is low, then $N(x, y)$ approximates Lotka-

Volterra(1.7) model and it is asymptotically approaching $\frac{nxy}{\beta + x}$ as x tending to zero. Predator-prey

interactions are quite delicate to the slopes of the respective equilibrium lines. Many predators are very efficient at seeking out and capturing their prey. Predators are repressed when their populations become very dense they interfere with each other hunting activities. The equilibrium line for such predator may be very steep at first, but slope will decrease in direct relationship to the density of the predator population. In biology oriented problems, there are extensive applications of prey predator models. Many biologists studied the relationship between species and various domains.[8–12]. The idea of Allee effect was firstly elevated out with the effective resultant studies of Allee and Bowen [13]. Since then, this idea established a different kind of base for many researchers [14–17]. Aulisa and Jang [18] have recognized a new model in dynamical behaviour studies with reference to prey population. They identified that both the species will become wiped out if the prey population size falls below a certain threshold. Pan et al. [19] have deliberated a reaction-diffusion phytoplankton-zooplankton model with double Allee effects on prey population. They pointed out that the Allee effect can turn the system as complex and critical. This effect can be depicted by the following form:

$$x'(T) = rx \left(1 - (x / K) \right) \left((x / m) - 1 \right)$$

Now a day's population dynamics play a major role in all aspects. Lotka and volterra [20-21] introduced research work on theoretical ecology. Ecological replicas have been in the attention of environmental discipline as predation of interrelating species, have an emotional impact on population dynamics importantly. Review on the steadiness of mechanism and the concept of spatial pattern construction through transmission driven instability of a system of intermingling populations in which a nonlinear Structure is asymptotically steady in the non-presence of transmission, but unsteady in the presence of diffusion show an important role in bio mathematics and other areas of science [21-27]. Spatial outlines alter the chronological dynamics and stability possessions of population concentrations at a variety of spatial scales, their consequences must be assimilated in chronological

ecological replicas that do not signify space explicitly. And the spatial section of ecological interactions has been recognized as a significant feature in how environmental societies are shaped. In the spatial heterogeneity system, the response item and the diffusion element comes by reason of quest-elusion phenomenon predators hunting prey and prey absconding predators. In such a system, there is an affinity that the effects of convective and dispersive interactions on the stability of two species are studied [28-29]. The consequence of transmission on the spatiotemporal prey- predator model has been explored by many scientists to their insightful work [30-32]. Recently, the consequence of self, as well as cross-diffusion in diffusion systems, has established much attention by both environmentalists and theoreticians also. Balram Dubey et.al.[33], motivated us to do this diffusion analysis of the proposed model.

2. Mathematical model:

We assume the following prey predator system with the assumptions (i) The prey predator model is following logistic growth. (ii) The interaction between prey and predator is assumed to be Holling type-II. (iii) The parameters are which are available in the mathematical model are all greater than zero. (iv) The quantity of focal prey approaches the environmental carrying capacity the amount of alternative prey consumed by the predator will tend to be zero

$$x'(T) = rx(1 - (x/K))((x/m) - 1) - (ax/K(b+x))y + \eta_1\phi_1(t) \quad (2.1)$$

$$y'(T) = (aex/(b+x))y + sy(1 - (x/K)) - dy + \eta_2\phi_2(t) \quad (2.2)$$

where x represents the biomass density of prey, y represents biomass density of predator at time T , r represents is the intrinsic growth rate of the prey without any environment limitations, K represents carrying capacity of the prey in the absence of the predator, $[ax/(b+x)]y$ represents Holing type II functional response which is used to depict the average feeding rate of the predator when the predator spends time seeking prey, b represents the half saturation constant for the Holling type-II, a_1 represents Grazing rate of the predator population, e represents the conversional rate, d represents mortality rate of predator. $[1 - (x/K)]sy$ represents the portion of biomass of predator increments from the alternative prey, s represents the growth rate of the predator on account of the alternative prey, m represents threshold value of Alle effect, η_1, η_2 are the real constants and $\phi_i(t) = [\phi_1(t), \phi_2(t)]$ is a two dimensional Gaussian white noise process satisfying $E(\phi_i(t)) = 0$; $i = 1, 2$; $E[\phi_i(t)\phi_j(t)] = \delta_{ij}\delta(t - t^1)$; $i = j = 1, 2$ where δ_{ij} is the Kronecker delta function, δ is the Dirac -delta function.

Analysis of the model (2.1)-(2.2) without noise: For easiness, we write the model (2.1)-(2.2) in dimensionless form as follows by the scaling.

Put $x = KX$ $y = rY/a$ $T = t/r$, then the model (2.1) -(2.2) becomes as follows

$$X'(t) = X(1 - X)(m_1X - 1) - (X/K(b_1 + X))Y = G_1 \quad (2.3)$$

$$Y'(t) = (e_1X/(b_1 + X))Y + s_1Y(1 - X) - d_1Y = G_2 \quad (2.4)$$

where $e_1 = ae/r$ $b_1 = b/K$ $s_1 = s/r$ $d_1 = d/r$ $m_1 = K/m$ and apparently $m_1 > 1$.

3. Study of equilibrium points: The possible steady states of the system (2.3)-(2.4) are as $E_1(0, 0)$, $E_2(\bar{X}, 0)$, $E_3(0, Y^\phi)$ and $E_4(X^*, Y^*)$ where $\bar{X} = 1/m$, $Y^\phi = 0$

$$X^* = \frac{(s_1(1-b_1)-d_1+e_1) \pm \sqrt{(s_1-s_1b_1-d_1+e_1)^2 - 4s_1b_1(d_1-s_1)}}{2s_1}. \quad \text{This is in the form}$$

$$X^* = (\alpha \pm \sqrt{\beta}) / 2s_1, \text{ where } \alpha = (s_1(1-b_1)-d_1+e_1); \beta = (\alpha)^2 - 4s_1b_1(d_1-s_1)$$

For X^* to be positive, we must have $s_1 + e_1 > s_1b_1 + d_1$; $(\alpha)^2 > 4s_1(d_1b_1 - s_1b_1)$ and $d_1 \geq s_1$.

$$Y^* = (X^* + b_1)(-mX^{*2} + (m+1)X - 1). \text{ For } Y^* \text{ to be positive, we must have } X^* > (1/m_1)$$

4. Study of stability:

(i) *Local stability:* To ascertain the nature of local steadiness of the interior equilibrium point $E_4(X^*, Y^*)$, we work out the variational matrix about $E_4(X^*, Y^*)$

$$M = \begin{bmatrix} -3mX^{*2} + 2X^*(1+m_1) - \frac{b_1Y^*}{(b_1+X^*)K} & \frac{-X^*}{(b_1+X^*)K} \\ \frac{Y^*e_1b_1}{b_1+X^*} - s_1Y^* & \frac{e_1X^*}{b_1+X^*} + s_1(1-X^*) - d_1 \end{bmatrix}$$

The characteristic equation of M is $\lambda^2 + \lambda(A_1) + A_2 = 0$

$$\text{Where } A_1 = -\left(\left(-3mX^{*2} + 2X^*(1+m_1) - \frac{b_1Y^*}{K(b_1+X^*)} \right) + \left(\frac{e_1X^*}{b_1+X^*} + s_1(1-X^*) - d_1 \right) \right)$$

$$A_2 = -\left(\left(-3mX^{*2} + 2X^*(1+m_1) - \frac{b_1Y^*}{K(b_1+X^*)} \right) \left(\frac{e_1X^*}{b_1+X^*} + s_1(1-X^*) - d_1 \right) \right. \\ \left. - \left(\left(\frac{Y^*e_1b_1}{b_1+X^*} - s_1Y^* \right) \left(\frac{-X^*}{K(b_1+X^*)} \right) \right) \right)$$

By Routh-Hurwitz criteria, we analyse the conditions $A_1 > 0$, $A_2 > 0$ when

$$s_1 > d_1 + X^* \left(s_1 + 2 + 2m_1 \right) \left(2 + 2m_1 + \frac{e_1}{b_1+X^*} + s_1 \right) < 0. \text{ Hence the given system is locally}$$

asymptotically stable about $E_4(X^*, Y^*)$.

(ii) *Global stability:* In this section, we present the essential results on the global stability of non-negative equilibria.

Theorem 1: The Equilibrium point $E_4(X^*, Y^*)$ is globally asymptotically stable, if $V(t) = (X - X^* - X^* \ln(X/X^*)) + l_1(Y - Y^* - Y^* \ln(Y/Y^*))$, $l_1 > 0$ is a Lyapunov function.

Proof: The time derivative of the positive definite function $V(t)$ is considered to verify the global stability behaviour of the interior equilibrium point using Lyapunov theorem.

$$V(t) = \left(X - X^* - X^* \ln\left(\frac{X}{X^*}\right) \right) + l_1 \left(Y - Y^* - Y^* \ln\left(\frac{Y}{Y^*}\right) \right) \quad l_1 > 0$$

then,

$$V'(t) = (X - X^*) \left((X-1)(m_1X-1) - \left(\frac{Y}{K(b_1+X)} \right) \right) + l_1(Y - Y^*) \left(\left(\frac{e_1X}{b_1+X} \right) + s_1(1-X) - d_1 \right)$$

$$V'(t) = (X - X^*) \left[\begin{aligned} & m_1 (X - X^*) - m_1 (-X^2 + X^{*2}) + (X - X^*) \\ & - \left(\frac{Y - Y^*}{K^2 (b_1 + X)(b_1 + X^*)} \right) \\ & + l_1 (Y - Y^*) \left(\left(\frac{e_1 (X - X^*)}{(b_1 + X)(b_1 + X^*)} \right) + s_1 (X - X^*) \right) \end{aligned} \right]$$

$$V'(t) = m_1 (X - X^*)^2 + m_1 (X - X^*)^2 (X + X^*) + (X - X^*)^2 \\ - \left[\frac{2b_1 + (X - X^*) + 2X^*}{K(b_1 + X)(b_1 + X^*)} \right] + l_1 \left(\frac{e_1 b_1 (X - X^*)(Y - Y^*)}{(b_1 + X)(b_1 + X^*)} \right)$$

Clearly, $V'(t) < 0$, hence, the non-stochastic system (2.3)-(2.4) is globally asymptotically stable,

$$\text{provided } \frac{2b_1 + (X - X^*) + 2X^*}{K(b_1 + X)(b_1 + X^*)} > 0 \text{ and } \left(\frac{l_1 e_1 b_1 (X - X^*)(Y - Y^*)}{(b_1 + X)(b_1 + X^*)} \right) < 0$$

5. Analysis of the model (2.1)-(2.2)

Now, this section is meant for the extension of the deterministic model (2.3)-(2.4), which is obtained by adding noise term. There are several ways in which environmental noise may be incorporated in the model system (2.3)-(2.4). External noise may arise from random fluctuations of finite number of parameters around some known mean values of the populace densities around some fixed values. The populace intensities of oscillations are calculated near the inner steady states due to environmental attributes by applying the method of [17] and [18]. Since the aquatic ecosystem which always has unsystematic fluctuations of the environment, it is difficult to define the usual phenomenon as a deterministic ideal. The stochastic investigation benefits us to get an extra intuition about the continuous changing aspects of any ecological unit. The deterministic model (2.3)-(2.4) with the effect of random noise of the environmental results in a stochastic system ((5.1)-(5.2)) given in the following discussion.

$$X'(t) = X(X-1)(m_1 X - 1) - (X/K(b_1 + X))Y + \eta_1 \varphi(t) \quad (5.1)$$

$$Y'(t) = [e_1 X / (b_1 + X)]Y + s_1 Y(1 - X) - d_1 Y + \eta_2 \varphi_2(t) \quad (5.2)$$

where, η_1, η_2 are the real constants and $\varphi_i(t) = [\varphi_1(t), \varphi_2(t)]$ is a two-dimensional Gaussian white noise process satisfying $E(\varphi_i(t)) = 0$; $i = 1, 2$; $E[\varphi_i(t)\varphi_j(t')] = \delta_{ij}\delta(t-t')$; $i = j = 1, 2$ where δ_{ij} is the Kronecker delta function; δ is the Dirac -delta function.

The linear part of the system (5.1)-(5.2) is

$$u_1'(t) = K(1 + m_1)b_1 u_1 S^* - u_2 S^* + \eta_1 \varphi_1(t) \quad (5.3)$$

$$u_2'(t) = (s_1 - d_1)u_1 P^* + \eta_2 \varphi_2(t) \quad (5.4)$$

by putting $X(t) = u_1(t) + S^*$; $Y(t) = u_2(t) + P^*$

Now applying Fourier transform on both sides of (5.3)-(5.4), we get,

$$i\omega \tilde{u}_1(\omega) = K(1 + m_1)b_1 S^* \tilde{u}_1(\omega) - S^* \tilde{u}_2(\omega) + \eta_1 \tilde{\varphi}_1(\omega) \quad (5.5)$$

$$i\omega \tilde{u}_2(\omega) = (s_1 - d_1)P^* \tilde{u}_2(\omega) + \eta_2 \tilde{\varphi}_2(\omega) \quad (5.6)$$

The matrix form of (5.5)-(5.6) is in the form of $\tilde{\phi}(\omega) = M(\omega)\tilde{u}(\omega)$ where

$$M(\omega) = \begin{bmatrix} i\omega - K(1 + m_1)b_1 S^* & -S^* \\ -(s_1 - d_1)P^* & i\omega \end{bmatrix}; \tilde{u}(\omega) = \begin{bmatrix} u_1(\omega) \\ u_2(\omega) \end{bmatrix}; \tilde{\phi}(\omega) = \begin{bmatrix} \eta_1 \varphi_1(\omega) \\ \eta_2 \varphi_2(\omega) \end{bmatrix}$$

The above equation can also be written as $\tilde{u}(\omega) = [A(\omega)]^{-1} \tilde{\varphi}(\omega) = B(\omega) \tilde{\varphi}(\omega)$, where $B(\omega) = [M(\omega)]^{-1}$ is the inverse matrix of $M(\omega)$. The components of the solution (5.5) are given by

$\tilde{u}(\omega) = \sum_{j=1}^2 B_{ij}(\omega) \eta_j \tilde{\phi}_j(\omega); i=1, 2$. Where $B_{ij}(\omega)$ are the elements of the matrix $B(\omega)$ and $\tilde{u}_j(\omega)$ are the

mean values of populations. The intensities of fluctuations of u_i ; $i=1, 2$ are given by

$$\sigma_{u_i}^2 = \frac{1}{2\pi} \sum_{j=1}^2 \int_{-\infty}^{\infty} \eta_j |B_{ij}(\omega)|^2 d\omega; i=1, 2 \text{ where } B_{ij} = \frac{G_{ij}(\omega)}{\det M(\omega)}; i=1, 2$$

We have the population variances of the model (5.1)-(5.2) as follows

$$\sigma_{u_i}^2 = \frac{1}{2\pi} \left\{ \eta_1 \int_{-\infty}^{\infty} \frac{|G_{ij}(\omega)|^2}{|\det M(\omega)|^2} d\omega + \eta_2 \int_{-\infty}^{\infty} \frac{|G_{ij}(\omega)|^2}{|\det M(\omega)|^2} d\omega \right\};$$

where $G_{mn} = X_{mn} + iY_{mn}$; $m, n=1, 2$; $X_{11}=0$; $Y_{11}=\omega$; $X_{12}=(s_1-d_1)P^*$; $Y_{12}=0$; $X_{21}=-S^*$; $Y_{21}=0$;

$$X_{22}=(1+m_1)b_1S^*; Y_{22}=\omega$$

$$\sigma_{u_1}^2 = \frac{1}{2\pi} \left\{ \eta_1 \int_{-\infty}^{\infty} \frac{|G_{11}(\omega)|^2}{|\det M(\omega)|^2} d\omega + \eta_2 \int_{-\infty}^{\infty} \frac{|G_{12}(\omega)|^2}{|\det M(\omega)|^2} d\omega \right\};$$

$$\sigma_{u_2}^2 = \frac{1}{2\pi} \left\{ \eta_1 \int_{-\infty}^{\infty} \frac{|G_{21}(\omega)|^2}{|\det M(\omega)|^2} d\omega + \eta_2 \int_{-\infty}^{\infty} \frac{|G_{22}(\omega)|^2}{|\det M(\omega)|^2} d\omega \right\}$$

Substituting the above values we get

$$\sigma_{u_1}^2 = \frac{1}{2\pi} \left\{ \int_{-\infty}^{\infty} \frac{\eta_1(\omega^2) + \eta_2((s_1-d_1)P^*)^2}{\left((S^*P^*(s_1d_1-\omega^2))^2 + (K(1+m_1)b_1S^*\omega)^2\right)} d\omega \right\};$$

$$\sigma_{u_2}^2 = \frac{1}{2\pi} \left\{ \int_{-\infty}^{\infty} \frac{\eta_1(S^{*2}) + \eta_2((1+m_1)^2 b_1^2 S^{*2} + \omega^2)}{\left((S^*P^*(s_1d_1-\omega^2))^2 + (K(1+m_1)b_1KS^*\omega)^2\right)} d\omega \right\};$$

Case(i): when $\eta_1=0$, then

$$\sigma_{u_1}^2 = \frac{\eta_2}{2\pi} \int_{-\infty}^{\infty} \frac{((s_1-d_1)P^*)^2}{\left((S^*P^*(s_1d_1-\omega^2))^2 + (K(1+m_1)b_1S^*\omega)^2\right)} d\omega$$

$$\sigma_{u_2}^2 = \frac{\eta_2}{2\pi} \int_{-\infty}^{\infty} \frac{((1+m_1)^2 K^2 b_1^2 S^{*2} + \omega^2)}{\left((S^*P^*(s_1d_1-\omega^2))^2 + ((1+m_1)b_1KS^*\omega)^2\right)} d\omega$$

Case(ii): when $\eta_2=0$ then

$$\sigma_{u_1}^2 = \frac{\eta_1}{2\pi} \int_{-\infty}^{\infty} \frac{\omega^2}{\left((S^*P^*(s_1d_1-\omega^2))^2 + ((1+m_1)b_1KS^*\omega)^2\right)} d\omega$$

$$\sigma_{u_2}^2 = \frac{\eta_1}{2\pi} \int_{-\infty}^{\infty} \frac{(S^{*2})}{\left((S^*P^*(s_1d_1-\omega^2))^2 + ((1+m_1)b_1KS^*\omega)^2\right)} d\omega$$

The population variances point out the stability of population for smaller values of mean square fluctuations, while the larger values of population variances indicate the populations.

6. Spatiotemporal analysis

The current article deals with a class of extended tri trophic prey predator systems in environmental science, modeled by diffusion equations. Although the dispersal system is a relatively simple model for the raid of prey species by predators in a spatial domain, the solutions exhibit an extensive spectrum of ecologically pertinent behaviour. Spatiotemporal dynamics includes chaos, target patterns [19-23]. By constructing a structure consists of prey, predator and top predator system with constant harvesting rates. The populace of the system is prey, predator and top predator. The populations are subject to dispersal. The spread of the population is observed by the pattern. These are two kinds of spread (i) The propagation of continuous traveling population fronts of high species density. (ii) The formation & movement of paths of high density separated by areas with density near to zero. The actual dynamics of the species spread is a result of the inter play between diffusion and deterministic factors. We shall study the effect of diffusion of ecological population on the model system. Let us consider the diffusive equation system as

$$X'(t) = X(X-1)(m_1X-1) - [X/K(b_1+X)]Y + D_1 \frac{\partial^2 X}{\partial u^2} \quad (6.1)$$

$$Y'(t) = [e_1X/(b_1+X)]Y + s_1Y(1-X) - d_1Y + D_2 \frac{\partial^2 Y}{\partial u^2} \quad (6.2)$$

In this D_1, D_2 represents the constant diffusion coefficients of the prey, predator. The model system (6.1)-(6.2) are inhomogeneous as the reaction diffusion system. For such introduction of the diffusion term of the populations, it has become a spatiotemporal dynamical system. We consider the following conditions of the population $X(u, t), Y(u, t)$ in $0 \leq u \leq L, L > 0$ as follows

$$\frac{\partial X(0, t)}{\partial t} = \frac{\partial X(L, t)}{\partial t} = \frac{\partial Y(0, t)}{\partial t} = \frac{\partial Y(L, t)}{\partial t} = 0$$

The zero isoclines of model equations (6.1)-(6.2) also give the steady state which is same as we have obtained for homogeneous system. The linear part of the system (6.1)-(6.2) is (in view of inner steady state)

$$x'(t) = (Kb_1(1+m_1)xX^*) - yX^* + D_1 \frac{\partial^2 X}{\partial u^2} \quad (6.3)$$

$$y'(t) = (e_1 + s_1 + b_1s_1 - d_1)xY^* + D_2 \frac{\partial^2 Y}{\partial u^2} \quad (6.4)$$

by putting, $X = X^* + x, Y = Y^* + y$

The solution of (6.3) - (6.4) is as $X(u, t) = \alpha_1 e^{\lambda t} e^{iku}$; $Y(u, t) = \alpha_2 e^{\lambda t} e^{iku}$, then the model becomes

$$x'(t) = (b_1K(1+m_1)xX^*) - yX^* + D_1(-k^2x) \quad (6.5)$$

$$y'(t) = (e_1 + s_1 + b_1s_1 - d_1)xY^* + D_2(-k^2y) \quad (6.6)$$

The characteristic equation of (6.5)-(6.6) is $\lambda^2 + A\lambda + B = 0$ (6.7)

where $A = b_1K(1+m_1)X^* - k^2(D_1 + D_2)$

$B = -b_1K(1+m_1)X^*k^2D_2 + k^4D_1D_2 + X^*Y^*(e_1 + s_1(1+b_1) - d_1)$;

By applying Routh-Hurwitz criterion, to satisfy and make it possible if and only if $A > 0, B > 0$ (which is definitely possible).

Theorem: The system in the absence of spatiotemporal attributes at the inner steady state (X^*, Y^*) attains steadiness, then the corresponding uniform steady state of the model (6.1)-(6.2) in the presence of spatiotemporal attributes also attains steadiness.

Proof:- Consider a function $V_1(t)$ as $V_1(t) = \int_0^R V(X, Y) du$

$$V(X, Y) = \left[(X - X^*) - X^* \ln(X / X^*) \right] + I_1 \left[(Y - Y^*) - Y^* \ln(Y / Y^*) \right]$$

$$V_1'(t) = \int_0^R \left(\frac{\partial v}{\partial X} \cdot \frac{\partial X}{\partial t} + \frac{\partial v}{\partial Y} \cdot \frac{\partial Y}{\partial t} \right) du = I_1 + I_2$$

where $I_1 = \int_0^R \frac{dv}{dt} du$ and $I_2 = \int_0^R \left(D_1 \frac{\partial v}{\partial X} \frac{\partial^2 X}{\partial u^2} + D_2 \frac{\partial v}{\partial Y} \frac{\partial^2 Y}{\partial u^2} \right) du$

$$I_2 = -D_1 \int_0^R \frac{\partial^2 v}{\partial X} \left(\frac{\partial X}{\partial u} \right)^2 du - D_2 \int_0^R \frac{\partial^2 v}{\partial Y} \left(\frac{\partial Y}{\partial u} \right)^2 du$$

$$I_2 = -D_1 \int_0^R \frac{X^*}{X} \left(\frac{\partial X}{\partial u} \right)^2 du - D_2 \int_0^R \frac{Y^*}{Y} \left(\frac{\partial Y}{\partial u} \right)^2 du$$

It is observed that, if $I_1 < 0$ then $V_1'(t) < 0$. Hence the theorem holds.

7. Numerical simulations

In this division, we established the analytical findings through numerical simulations using MATLAB.

Example 1: For the set of parameters $m_1 = 1.5$; $b_1 = 5$; $d_1 = 2.0$; $s_1 = 0.1$; $e_1 = 0.395$; $K = 0.5$;

$X = 15$; $Y = 10$, the following figures 1(a), 1(b) are the various numerical simulations.

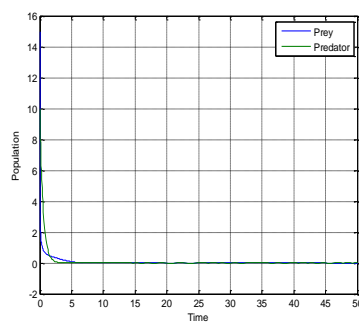


Figure 1(a)

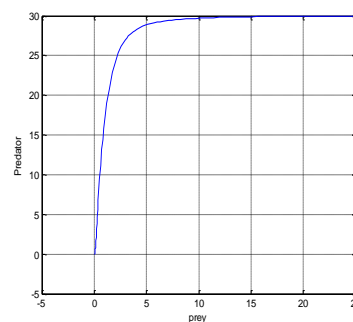


Figure 1(b)

Example 2: For the set of parameters $m_1 = 1.5$; $b_1 = 3$; $d_1 = 1$; $s_1 = 0.1$; $e_1 = 0.395$; $K = 0.5$;

$X = 35$; $Y = 40$, the following figures 2(a), 2(b) are the various numerical simulations

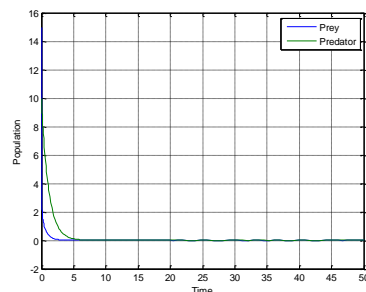


Figure 2(a)

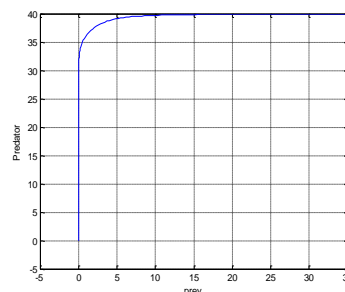
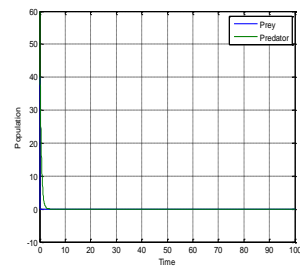
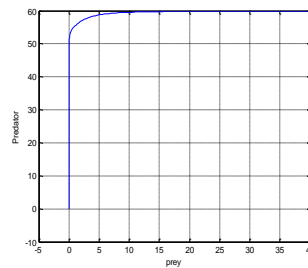


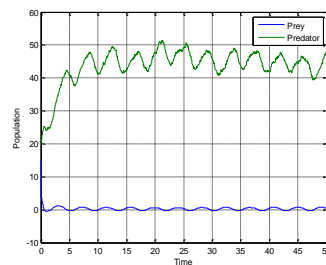
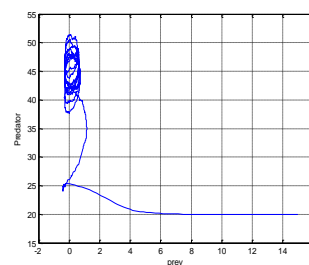
Figure 2(b)

Example 3: For the set of parameters $m_1 = 1.5$; $b_1 = 3$; $d_1 = 1$; $s_1 = 0.1$; $e_1 = 0.395$; $K = 0.5$;

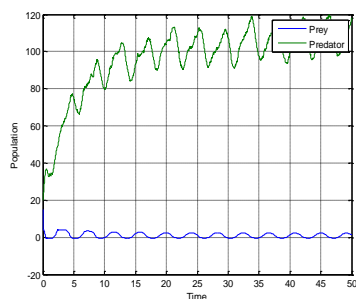
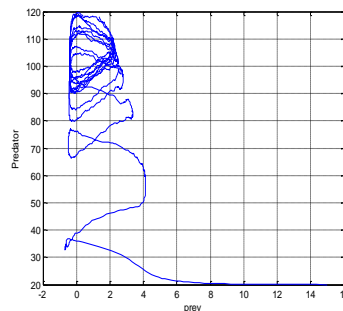
$X = 40$; $Y = 60$, the following figures 3(a), 3(b) are the various numerical simulations

**Figure 3(a)****Figure 3(b)**

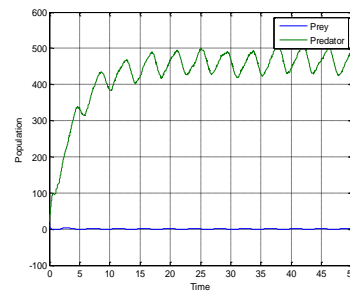
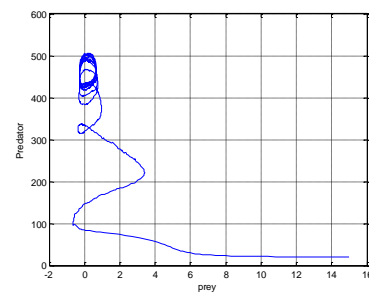
Example 4: For the set of parameters $m_1 = 0.5$; $b_1 = 1$; $d_1 = 0.4$; $s_1 = 0.1$; $e_1 = 0.395$; $K = 2$; $X = 40$; $Y = 60$, $\eta_1 = 10$, $\eta_1 = 20$; the following figures 4(a), 4(b) are the various numerical simulations with noise

**Figure 4(a)****Figure 4(b)**

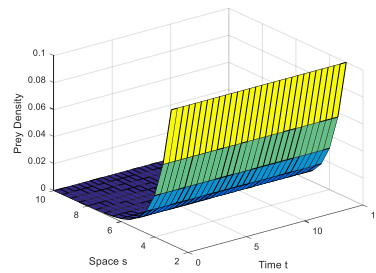
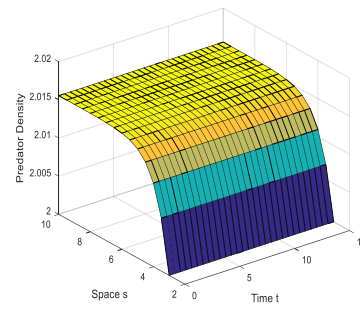
Example 5: For the set of parameters $m_1 = 0.5$; $b_1 = 1$; $d_1 = 0.4$; $s_1 = 0.1$; $e_1 = 0.395$; $K = 2$; $\eta_1 = 40$, $\eta_1 = 50$; the following figures 5(a), 5(b) are the various numerical simulations with noise.

**Figure 5(a)****Figure 5(b)**

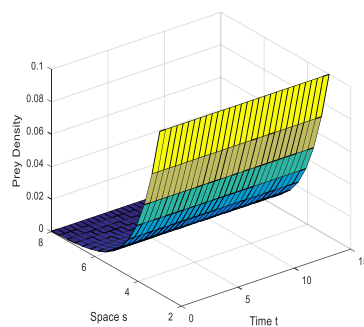
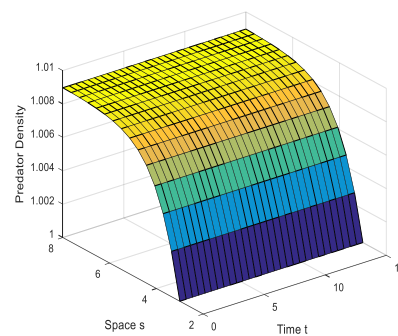
Example 6: For the set of parameters $m_1 = 0.5$; $b_1 = 1$; $d_1 = 0.4$; $s_1 = 0.1$; $e_1 = 0.395$; $K = 2$; $\eta_1 = 100$, $\eta_1 = 200$; the following figures 6(a), 6(b) are the various numerical simulations with noise.

**Figure 6(a)****Figure 6(b)**

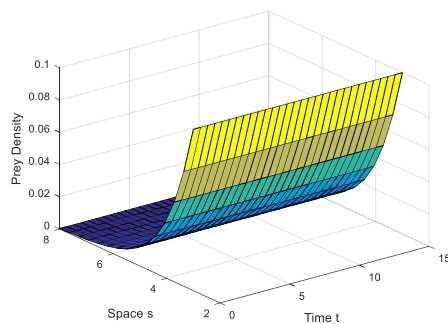
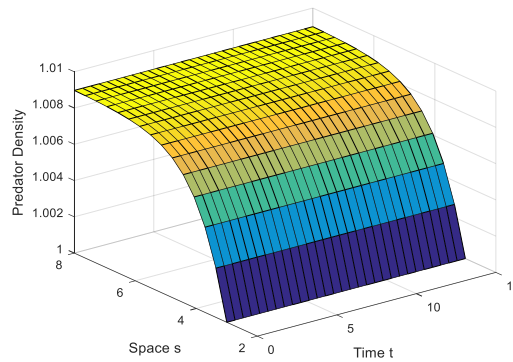
Example 7: For the set of parameters $m_1 = 0.5$; $b_1 = 1$; $d_1 = 0.4$; $s_1 = 0.1$; $e_1 = 0.395$; $K = 2$, $D_1 = 0.00001$, $D_2 = 0.00003$ the following figures 7(a), 7(b) are the various numerical simulations with diffusion.

**Figure 7(a)****Figure 7(b)**

Example 8: For the set of parameters $m_1 = 0.5$; $b_1 = 1$; $d_1 = 0.4$; $s_1 = 0.1$; $e_1 = 0.395$; $K = 2$; $D_1 = 0.1$, $D_2 = 0.3$; the following figures 8(a), 8(b) are the various numerical simulations with diffusion.

**Figure 8(a)****Figure 8(b)**

Example 9: For the set of parameters $m_1 = 0.5$; $b_1 = 1$; $d_1 = 0.4$; $s_1 = 0.1$; $e_1 = 0.395$; $K = 2$; $D_1 = 100$, $D_2 = 200$, the following figures 9(a), 9(b) are the various numerical simulations with diffusion.

**Figure 9(a)****Figure 9(b)**

7. Concluding remarks

In this, it is premeditated about a prey–predatorial with Allee influence and noise and diffusion for both prey and predator which plays a major role in turning the changing aspects of the model. We obtained all possible equilibrium points and inspected for stability, using various mathematical and scientific tools. It is shown that the dynamics of deterministic system in the figures 1(a),1(b),2(a),2(b),3(a),3(b) and also local and global stabilities are analyzed using Routh-Hurwitz criteria and Lyapunov function respectively. We also identified that the noise gives its impact on the system due to change of a suitable parameters, causes chaotic dynamics with low, medium and high variances of oscillations from figures (4(a), 4(b), 5(a), 5(b), 6(a), 6(b)). It also verified the stable oscillations of the prey and predator populations against time and space in Figures 7(a), 7(b), 8(a), 8(b), 9(a), 9(b).

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