

# Gain scheduling controller for pitch control of a TRMS system

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**Abstract:** Gain scheduling is a control technique which is applied for the control of non-linear systems by using a family of linear controllers at different operating points so as to increase the range of operation of the process. The dynamics of any nonlinear system changes with respect to operating points. These operating points are characterized by one or more variables known as scheduling variables. In such cases, we linearize the system at different equilibrium points. Due to the high amount of non-linearities and complexity in the aerodynamic design, modelling of other unmanned aerial vehicles have been replaced here with twin rotor system. The linearized system is then controlled by using PID controllers which are designed with respect to the obtained operating points. Linearizing of the non-linear system, designing and tuning of PID are being implemented using different MATLAB functions.

## 1. Introduction

Recently, Unmanned Aerial Vehicle has become a strong platform to provide innovations in the field of flight systems. Control methodologies applied to flight systems have gained significant importance due to complexity in its implementation. This has put forward experimentations regarding control of quad rotor as well as twin rotor models. This paper deals with controlling a twin rotor system using PID controller so as to achieve robustness in position and speed using adaptive control techniques [1].

Adaptive control is a methodology utilized by the controllers to adapt to a controlled system environment with an initially uncertain or varying condition. Adaptive control techniques are applied to systems which undergoes nonlinearities and other uncertainties and the control schemes mainly involves gain scheduling, model reference adaptive control, self-tuning regulators etc[2]. Gain scheduling involves breaking down a system with respect to many operating points by using a series of controllers so as to achieve robustness in the system operation. This project handles gain scheduling for a Twin Rotor system using PID controllers. The twin rotor system is a non-linear system which mainly consist of rotors and a beam which is pivoted to its base to allow easy horizontal and vertical rotation of the system [1]. The upward and downward movement of the TRMS system is obtained by



controlling the pitch by considering the voltages applied at the main and tail rotors. In order to maintain stability and constancy in the performance of system during take-off and landing, so as to achieve the desired position, gain scheduling strategy is being implemented. The nonlinearities associated with the TRMS model is suppressed by linearizing the system using different MATLAB functions. The mathematical modelling of TRMS along with its physical parameters are also discussed here [2].

## 2. System Description

### 2.1 Model Design:



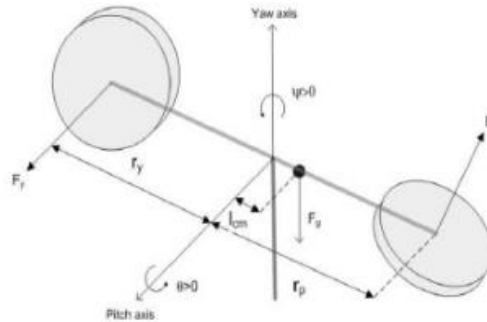
**Figure 1.** Twin Rotor System

Figure 1 shows the model of a Twin rotor system. A TRMS consists of rotors and a beam pivoted to its base to allow easy horizontal and vertical rotation of the system. The mechanical unit of the system consist of two rotors connected on opposite side of a beam. The rotors have their axis of rotation orthogonal to each other to counter balance them. The connected beam is placed on shaft with two co-axial bearings. This set-up is mounted on a base for provide mechanical stability for the system. The electrical unit of the system consists of motors, microcontroller modules programmed to the system move in horizontal and vertical planes [4].

The movement of the system being in horizontal and vertical planes corresponds to its yaw and pitch axis movement. The axis movements are produced based on the propeller thrust. The pitch axis rotation is for up/down movement while the yaw axis is for sideways movement of the twin rotor. Based on the voltage supplied to the main and tail rotor, the movement of the TRMS is achieved. The system being non-linear is linearized and Gain scheduled using PID controller to achieve the desired position [4].

### 2.2 Mathematical Modelling:

The pitch and yaw axis control represent a 2-DOF system. Pitch axis is positive when system moves up while yaw is positive for clockwise movement. Figure 2 shows the free body diagram of the system.

**Figure 2.** Free body diagram

The system being non-linear, let us define the ODE equations for the system. Consider moment of inertia about roll ( $J_\phi$ ), pitch ( $J_\theta$ ), and yaw ( $J_e$ ). The angular momentum about the axis is  $n_\epsilon, n_\theta$  &  $n_\phi$ . Taking the mass and length of the beam, rotors and whole system the ODE is defined. The parameter values are defined in Table. 1. The inputs to the system are Forward voltage ( $V_f$ ) and Backward Voltage ( $V_b$ ). Equation (1) represent the ODE for TRMS.

$$\dot{x}(t) = Fx(t) + Gu(t) \quad (1)$$

Where  $\dot{x}(t) = [\ddot{\epsilon}; \dot{\epsilon}; \ddot{\theta}; \dot{\theta}; \ddot{\phi}; \dot{\phi}]$

$$F_x = [\ddot{\epsilon}; -1.378\cos(\epsilon) - 0.1494\sin(\epsilon) - 0.0012\dot{\epsilon}; \ddot{\theta}; -6.1535\sin(\theta) - 0.0227\dot{\theta}; \ddot{\phi}; -0.0061\dot{\phi}]$$

$$G = [0, 0; 0.3605 \cos(x(3)), 0.3605 \cos(x(3)); 0, 0; 20.114, -2.0114; 0, 0; -0.3780 \sin(x(3)), -0.3780 \sin(x(3))]$$

$$U = [V_f; V_b]$$

**Table 1.** Parameter values

| Parameter  | Value                 | Parameter    | Value                      |
|------------|-----------------------|--------------|----------------------------|
| $J_e$      | 0.86 kgm <sup>2</sup> | $M_f$        | 0.69 kg                    |
| $J_\phi$   | 0.44 kgm <sup>2</sup> | $M_b$        | 0.69 kg                    |
| $J_\theta$ | 0.82 kgm <sup>2</sup> | $M_c$        | 1.67 kg                    |
| $L_a$      | 0.62 m                | Km           | 0.5 N/V                    |
| $L_c$      | 0.44 m                | g            | 9.8 m/s <sup>2</sup>       |
| $L_d$      | 0.05 m                | $n_\epsilon$ | 0.001 kg m <sup>2</sup> /s |
| $L_h$      | 0.177 m               | $n_\theta$   | 0.001 kg m <sup>2</sup> /s |
| $L_e$      | 0.02 m                | $n_\phi$     | 0.005 kg m <sup>2</sup> /s |

### 3. Controller Design

TRMS system is a non-linear system. This paper concentrates on controlling the pitch angle/theta to achieve desired position. The approach followed to achieve this is Linearization based on operating point, Gain scheduling using PID controller [5].

#### 3.1 Linearization based on zones:

The system is operated at different inputs of  $V_f$  and  $V_b$  and the corresponding response for roll, pitch and yaw are obtained. From the response recorded, the system is analysed for different operating points and an operating point is selected around which the system is linearized and further analysis is carried out [6].

The stability of theta depends on the input voltage  $V_f$  and  $V_b$ . In order for theta to settle, a range of input within which the system should operate is selected. The input range is given in Table 2. The system should not be operated beyond this input as it makes theta unstable.

**Table 2.** Input Range

| Input | Range      |
|-------|------------|
| $V_f$ | [1.1 -1.1] |
| $V_b$ | [-1.1 1.1] |

The linearized model obtained is given as in Equation (2) and (3)

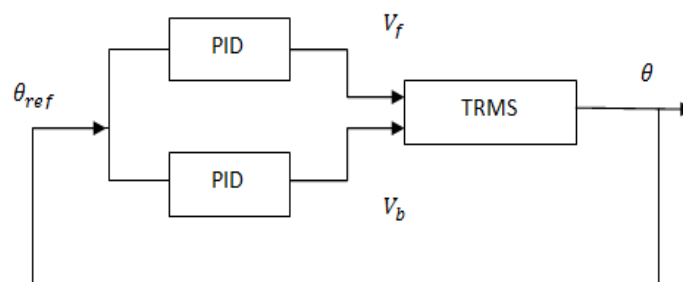
Operating Point:  $V_f = 0.3V$  and  $V_b = -0.08V$

$$\text{Transfer function from input1: } \frac{\theta(s)}{V_f + V_b} = \frac{2.011}{s^2 + 0.0277s + 6.153} \quad (2)$$

$$\text{Transfer function from input2: } \frac{\theta(s)}{V_f - V_b} = \frac{-2.011}{s^2 + 0.0277s + 6.153} \quad (3)$$

### 3.2 Controller design:

The system after linearizing is subjected to Gain scheduling using PID controller. Gain scheduling is an adaptive control process applied for non-linear system. It uses linear controllers to provide required control for different operating points. Figure. 3 shows the block diagram of system with PID controller [7].



**Figure 3.** System Block with PID controller

PID controller calculates the error between the set point and the output variable and based on tuning methods, the error is reduced considerably to obtain the output as desired set point. The Equation (4) gives a general form of PID controller which has proportional, integral, derivative terms.

$$u(t) = K_p + \frac{K_i}{s} + K_d * s \quad (4)$$

Where  $K_p$ : Proportional gain

$K_i$ : Integral gain

$K_d$ : Derivative gain

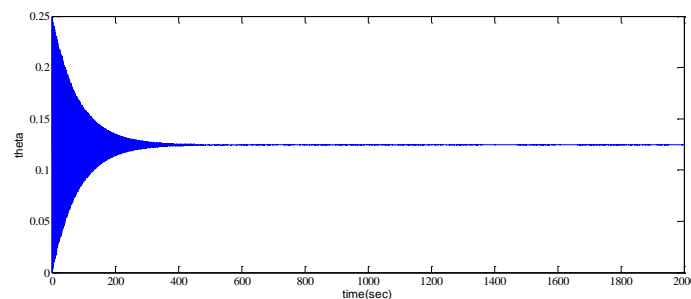
The individual terms calculates the error between set point and output, tunes the parameters to achieve stable system. The robustness and stability is obtained based on BIBO concept. The algorithm for tuning the parameters chooses a crossover frequency based on system dynamics and tunes for a target phase margin of  $60^\circ$  [8].

#### 4. Results

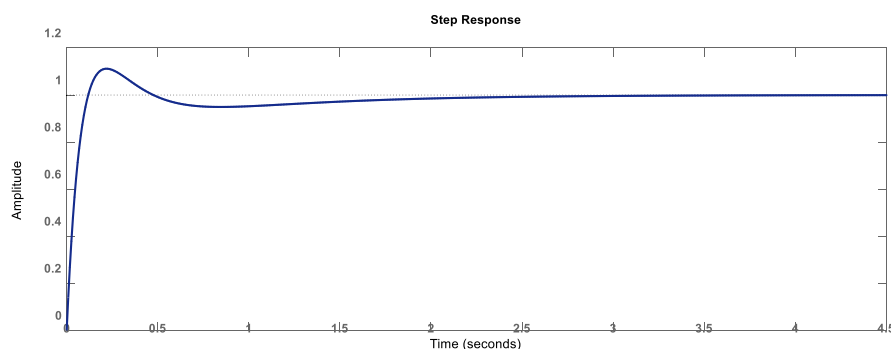
TRMS is operated with various  $V_f$  and  $V_b$  and the operating point (0.3,-0.08) is selected around which the stability of theta is measured and the model is linearized. Figure 4 shows the stability graph for theta. The settling value of theta is 0.12rad. The linearized model is subjected to PID tuning to obtain the tuned parameters. Table.3 gives the transfer function and controller parameter for TRMS model. Figure.5 shows the system response tuned with PID from input1:  $V_f + V_b$  and Figure.5 with input2:  $V_f - V_b$

**Table 3.** PID controller parameters

| Operating Point            | Transfer Function  | Controller Parameters                           |
|----------------------------|--|---|
| $V_f = 0.3V, V_b = -0.08V$ |  |   |
| Input 1<br>$V_f + V_b$     | $\frac{\theta(s)}{V_f + V_b} = \frac{2.011}{s^2 + 0.0277s + 6.153}$  | $K_p = 36.2$<br>$K_i = 37.8$<br>$K_d = 8.66$    |
| Input 2<br>$V_f - V_b$     | $\frac{\theta(s)}{V_f - V_b} = \frac{-2.011}{s^2 + 0.0277s + 6.153}$ | $K_p = -36.2$<br>$K_i = -37.8$<br>$K_d = -8.66$ |



**Figure 4.** Theta parameter



**Figure 5.** System response with PID tuning

## 5. Conclusion and Future Scope

The paper discusses modelling a robust controller which uses adaptive control scheme of gain scheduling for controlling of a twin rotor system. The mathematical modelling for the system using ODE equations has been implemented and the system is then linearized with respect to the obtained set points using MATLAB functions. The linearized model is utilised for tuning PID controllers which gives corresponding transfer functions. The response of the system with the tuned PID controllers are also obtained.

The project can be extended so as to control yaw and roll instead of pitch. Here, the control of pitch gives upward and downward movement whereas controlling of roll or yaw gives sidewise movement. The 2 DOF twin rotor system can be replaced with a 6 DOF quad rotor system with respect to the same parameters of roll, yaw and pitch.

## References

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