Peristaltic flow and heat transfer of a Herschel-Bulkley fluid in an inclined non-uniform channel with wall properties

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Abstract. In the present work, we obtained analytical solutions for peristaltic flow and heat transfer of a Herschel-Bulkley (HB) fluid in an inclined non-uniform conduit with elastic walls. The effects of the physical parameters on the axial velocity, the streamlines, the temperature and the rate of heat transfer are discussed through graphs. Further, we discussed the results for the peristaltic transport of Newtonian, Bingham and power-law fluids. The variation in the power-law index n shows that the values of the velocity and the temperature fields are grater for shear thinning ($n < 1$) HB fluids than the shear thickening ($n > 1$) HB fluids. Also for $n = 1$, the results agree very well with the available results in the literature. Further, the inclination angle has a strong influence on the velocity and the temperature fields.

1. Introduction

Peristalsis is a familiar mechanism for fluid transport. This mechanism has applications to biological, medical and engineering fields; such as taking food through esophagus, blood circulation in small blood vessels, passage of urine from kidney to bladder, transport of nuclear waste. After the initial investigations (seethe Ref. [1-4]), many works on peristaltic flow in various geometries have been analyzed [5-7]. It is a well-known fact that most of the industrial and biological fluids (blood, bile, chime, some industrial toxic fluids etc.) shows non-Newtonian behaviour and the non-Newtonian fluid flows have many applications in engineering and medicine. Hence, several investigations on non-Newtonian fluids have been presented in Refs. [8-15].

Peristaltic transport with heat transfer has numerous applications, such as bio-heat conduction in tissues, oxygenation and hemodialysis, heat exchangers and solar energy etc. Keeping these applications in mind, several authors studied the heat transfer problems with peristalsis [16, 17]. The properties of a flexible wall are very important in physiological fluid flow problems [18-22]. Moreover, most of the physiological organs like small blood vessels, ducts afferents of the male re-productive tracts, oesophagus, lymphatic vessel, intestine, cervical canal are found to be non-uniform and having some inclination. In view of these practical applications,some researchers studied the peristaltic flow problems by considering inclined or non-uniform geometry [23-24].

In this studythe influences of both slip and the heat transfer on peristaltic flow of a HB fluid in an inclined non-uniform elastic channel have been investigated. The expressions for stream functionand temperature are obtained. The effects of several physical parameters on velocity, temperature and Nusselt number are discussed. The obtained results brought out variousmotivating behaviours of the non-Newtonian fluid phenomena, particularly the shear-thinning phenomena. The present results are not only useful for industrial applications; but also provide a basic understanding of the physical model.

2. Mathematical formulation

Consider the peristaltic flow of a Herschel-Bulkley fluid in a two-dimensional inclined non-uniform channel bounded by flexible walls (see Figure A for details). The flow is generated by sinusoidal wave trains spreading on the channel walls with a continuous speed c. The wall geometry is given by

$$\bar{h}(\bar{x},\bar{t}) = a\,Sin\frac{2\pi}{\lambda}\left(\bar{x}-c\bar{t}\right)+d(x) \tag{1}$$

where $d(x)=\bar{m}x+d,\ \bar{m}<<1.$

The basic equations (continuity, momentum and energy) for the present investigation are given by

$$\frac{\partial \bar{u}}{\partial \bar{x}}+\frac{\partial \bar{v}}{\partial \bar{y}}=0, \tag{2}$$

$$\rho\left[\frac{\partial}{\partial \bar{t}}+\bar{u}\frac{\partial}{\partial \bar{x}}+\bar{v}\frac{\partial}{\partial \bar{y}}\right]\bar{u}=-\frac{\partial \bar{p}}{\partial \bar{x}}+\mu\frac{\partial^2 \bar{u}}{\partial \bar{x}^2}-\frac{\partial}{\partial \bar{y}}\left(\bar{\tau}_0+\mu\left(-\frac{\partial \bar{u}}{\partial \bar{y}}\right)^n\right)+\rho g\sin\alpha, \tag{3}$$

$$\rho\left[\frac{\partial}{\partial \bar{t}}+\bar{u}\frac{\partial}{\partial \bar{x}}+\bar{v}\frac{\partial}{\partial \bar{y}}\right]\bar{v}=-\frac{\partial \bar{p}}{\partial \bar{y}}+\mu\left(\frac{\partial^2 \bar{v}}{\partial \bar{x}^2}+\frac{\partial^2 \bar{v}}{\partial \bar{y}^2}\right)+\rho g\cos\alpha, \tag{4}$$

$$\begin{aligned}\xi\left[\frac{\partial}{\partial \bar{t}}+\bar{u}\frac{\partial}{\partial \bar{x}}+\bar{v}\frac{\partial}{\partial \bar{y}}\right]T=\frac{k}{\rho}\left(\frac{\partial^2 T}{\partial \bar{x}^2}+\frac{\partial^2 T}{\partial \bar{y}^2}\right)+\\ \upsilon\left\{2\left[\left(\frac{\partial \bar{u}}{\partial \bar{x}}\right)^2+\left(\frac{\partial \bar{v}}{\partial \bar{y}}\right)^2\right]+\left(\frac{\partial \bar{v}}{\partial \bar{x}}+\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2\right\}\end{aligned} \tag{5}$$

where $\bar{u}$ and $\bar{v}$ are the axial and transverse velocities, ρ is the fluid density, μ is the viscosity, p is the pressure, d is the width of the channel, a is the amplitude, λ is the wavelength, c is the wave speed, $\bar{m}$ is the dimensional non-uniform parameter of the channel, ξ is the specific heat at fixed volume, υ is the kinematic viscosity, k is the conductivity of the fluid, T is the thermal temperature, τ_0 is the yield stress, α is the inclination angle, and g is the acceleration due to gravity.

Using the momentum equation in the *x*- direction, yield (for details see Ref. [22])

$$\frac{\partial}{\partial x}\bar{L}^*(\bar{h})=\frac{\partial \bar{p}}{\partial x}=\mu\frac{\partial^2 \bar{u}}{\partial \bar{x}^2}-\frac{\partial}{\partial y}\left[\bar{\tau}_0+\mu\left(-\frac{\partial \bar{u}}{\partial y}\right)^n\right]-\rho\left[\frac{\partial}{\partial \bar{t}}+\bar{u}\frac{\partial}{\partial x}+\bar{v}\frac{\partial}{\partial y}\right]\bar{u}+\rho g\sin\alpha, \tag{6}$$

$$\bar{u}=-\eta\frac{\partial \bar{u}}{\partial y}\ \text{at}\ \bar{y}=\bar{h}=[d+\bar{m}\bar{x}+a\sin\frac{2\pi}{\lambda}(\bar{x}-c\bar{t})], \tag{7}$$

$$\frac{\partial T}{\partial \bar{y}}=0\ \text{on}\ \bar{y}=\bar{y}_0,\ T=T_1\ \text{on}\ \bar{y}=\bar{h}. \tag{8}$$

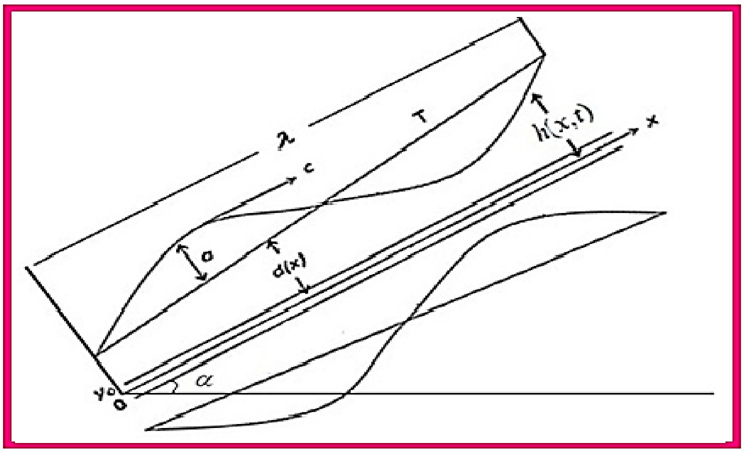

Figure A. Flow configuration and geometry of the problem

Let us introduce ψ (such that $\bar{u} = \frac{\partial \bar{\psi}}{\partial \bar{y}}$ and $\bar{v} = -\frac{\partial \bar{\psi}}{\partial \bar{x}}$)and the non-dimensional parameters

$$\left.\begin{aligned}
&x = \frac{\bar{x}}{\lambda},\ y = \frac{\bar{y}}{d},\ \psi = \frac{\bar{\psi}}{cd},\ p = \frac{d^{n+1}\bar{p}}{\mu c^n \lambda},\ t = \frac{c\bar{t}}{\lambda}, K = \frac{k}{d^2},\ m = \frac{\lambda \bar{m}}{d}, \delta = \frac{d}{\lambda},\\
&\varepsilon = \frac{a}{d}, y_0 = \frac{\bar{y}_0}{d}, h = \frac{\bar{h}}{d} = 1 + mx + \varepsilon \sin 2\pi(x-t), R = \frac{\rho c d}{\mu}, \theta = \frac{(T - T_0)}{(T_1 - T_0)},\\
&\tau_0 = \frac{\bar{\tau}_0}{\mu\left(c/d\right)^n}, \tau_{xy} = \frac{d\overline{\tau_{xy}}}{\mu c}, \eta_0 = \frac{\rho g d^2}{\mu c}, \Pr = \frac{\rho \upsilon \xi}{k}, Ec = \frac{c^2}{\xi(T_1 - T_0)},\\
&E_1 = \frac{-\tau d^3}{\lambda^3 \mu c}, E_2 = \frac{m_1 c d^3}{\lambda^3 \mu}, E_3 = \frac{c d^3}{\lambda^2 \mu}, \beta = \frac{h}{d}
\end{aligned}\right\} \tag{9}$$

where R, Pr and Ec are the Reynolds,Prandtland Eckert numbers, $\delta, \varepsilon, E_1, E_2, E_3, m$ and β are the dimensionless geometric, rigidity, stiffness parameter, viscous damping force, non-uniform parameter andslip parameters .

3. Solution of the problem

Using the non-dimensional quantities and the assumptions of lengthy wavelength and small Reynolds number, the equations (3) - (8) moderate as

$$0 = -\frac{\partial p}{\partial x} - \frac{\partial}{\partial y}\left[\tau_0 + \left(-\frac{\partial^2 \psi}{\partial y^2}\right)^n\right] + \eta_0 \sin\alpha, \tag{10}$$

$$0 = \frac{\partial p}{\partial y}, \tag{11}$$

$$0 = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} + Ec\left(\frac{\partial^2 \psi}{\partial y^2}\right)^2, \tag{12}$$

$$-\frac{\partial}{\partial y}\left[\tau_0 + \left(-\frac{\partial^2 \psi}{\partial y^2}\right)^n\right] + \eta_0 \sin\alpha = \left[E_1 \frac{\partial^3 h}{\partial x^3} + E_2 \frac{\partial^3 h}{\partial x \partial t^2} + E_3 \frac{\partial^2 h}{\partial x \partial t}\right], \tag{13}$$

$$\frac{\partial \psi}{\partial y} = -\beta \frac{\partial^2 \psi}{\partial y^2} \text{ at } y = h = [1 + m x + \varepsilon \sin 2\pi (x - t)]. \tag{14}$$

We assumed that the streamline takes zero value at the line $y = 0$. That is,

$$\psi_p(0) = 0,\ \psi_{yy}(0) = \tau_0 \text{ at } y = 0,$$

$$\psi = \psi_p \text{ at } y = y_0, \tag{15}$$

$$\frac{\partial \theta}{\partial y} = 0 \text{ at } y = y_0,\ \theta = 1 \text{ at } y = h. \tag{16}$$

From equation (10) we obtain

$$-\frac{\partial^2}{\partial y^2}\left[\tau_0 + \left(-\frac{\partial^2 \psi}{\partial y^2}\right)^n\right] = 0. \tag{17}$$

By simplifying equations (13), (14),(15) and(17) we get the stream functions and corresponding velocities in the plug $(0 \le y \le y_0)$ and non-plug $(y_0 \le y \le h)$ flow regions as

$$\psi_p = \left[\frac{(-A y_0 + B - \tau_0)^{\frac{1}{n}+1}}{A\left(\frac{1}{n}+1\right)} + C\right] y \tag{18}$$

$$u_p = \frac{(-A y_0 + B - \tau_0)^{\frac{1}{n}+1}}{A\left(\frac{1}{n}+1\right)} + C \tag{19}$$

$$\psi = \frac{-(-A y + B - \tau_0)^{\frac{1}{n}+2}}{A^2\left(\frac{1}{n}+1\right)\left(\frac{1}{n}+2\right)} + C y + E \tag{20}$$

$$u = \frac{(-A y + B - \tau_0)^{\frac{1}{n}+1}}{A\left(\frac{1}{n}+1\right)} + C. \tag{21}$$

Where $y_0 = \frac{B - \tau_0}{A}$, $A = -8\varepsilon \pi^3\left[(E_1 + E_2)\cos 2\pi(x-t) - \frac{E_3}{2\pi}\sin 2\pi(x-t)\right] - \eta_0 \sin\alpha$,

$$C = \frac{-(-A\eta + B - \tau_0)^{\frac{1}{n}+1}}{A\left(\frac{1}{n}+1\right)} + \beta(-A\eta + B - \tau_0)^{\frac{1}{n}},\quad B = (-\tau_0)^n + \tau_0.$$

$$E = \left[\frac{(-A y_0 + B - \tau_0)^{\frac{1}{n}+1}}{A\left(\frac{1}{n}+1\right)} + C\right] y_0 + \frac{(-A y_0 + B - \tau_0)^{\frac{1}{n}+2}}{A^2\left(\frac{1}{n}+1\right)\left(\frac{1}{n}+2\right)} - C y_0.$$

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By solving (12) with the help of equation (20) and (16) the temperature field is obtained as

$$\theta = -Br\left[\frac{\left(-Ay+B-\tau_0\right)^{\frac{2}{n}+2}}{A^2\left(\frac{2}{n}+1\right)\left(\frac{2}{n}+2\right)}\right]+C_1 y+C_2 \tag{22}$$

where $C_1=-Br\left[\frac{\left(-Ay_0+B-\tau_0\right)^{\frac{2}{n}+1}}{A\left(\frac{2}{n}+1\right)}\right]$, $C_2=1-C_1 h+Br\left[\frac{\left(-Ah+B-\tau_0\right)^{\frac{2}{n}+2}}{A^2\left(\frac{2}{n}+1\right)\left(\frac{2}{n}+2\right)}\right]$,

$Br = Ec\,\mathrm{Pr}$ is the Brinkman number.

The Nusselt number is given by

$$Nu=-\left(\theta_y\right)_{at\ y=h}, \tag{23}$$

4. Results and Discussions

Figs.1–7 are plotted to analyze the effects of various parameters on the velocity distribution. For this purpose we used the fixed values as $x=0.2, t=0.1,\ \beta=0.2, \varepsilon=0.2,\ \tau_0=0.5,\ m=0.1,\ n=1.1, \eta_0=1,\ \alpha=\pi/4$, $E_1=0.5, E_2=0.3, E_3=0.2$.

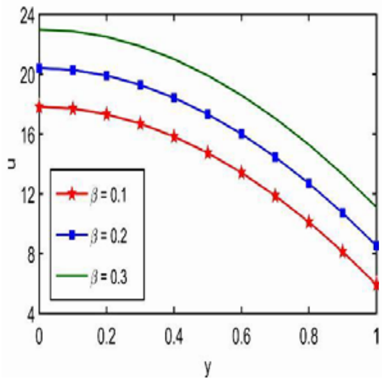

Figure1. Velocity profiles for β

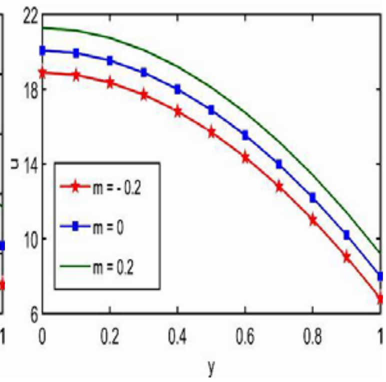

Figure 2. Velocity profiles for m

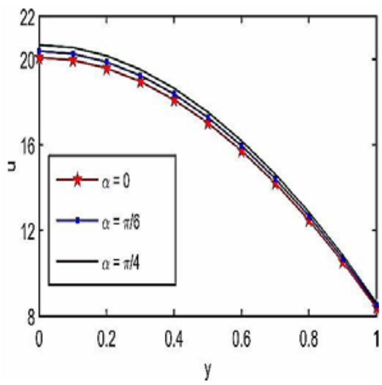

Figure3. Velocity profiles for α

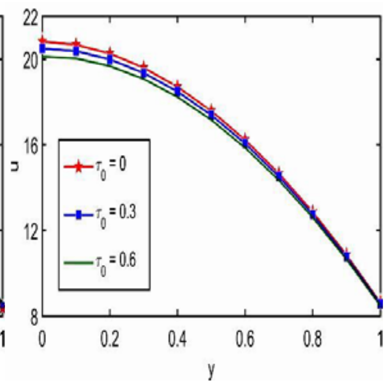

Figure 4. Velocity profiles for τ_0

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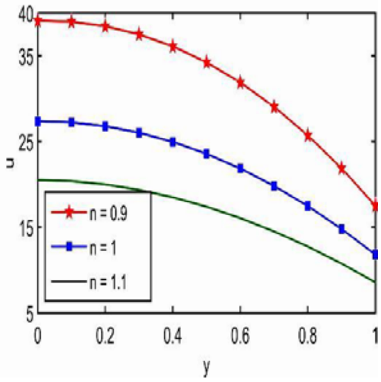
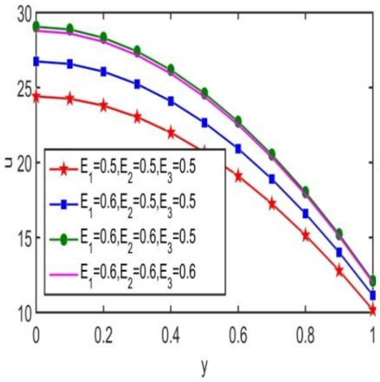


Figure 5. Velocity profiles for n



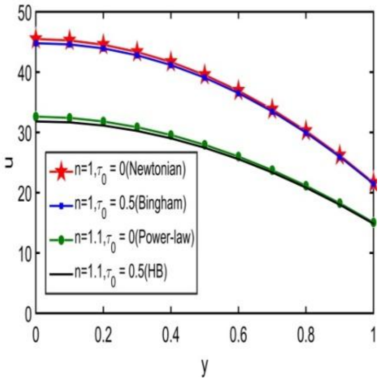


Figure. 6 Velocity profiles for different values of elasticity parameters

Figure 7. Velocity profiles for different values of 'τ_0' and 'n'

From Figs. 1–3, we observe that the increment in slip parameter β leads to improvement in velocity of the fluid which is due to the reduction of friction between the channel wall and the fluid. Also we noticed an increase in the non-uniform parameter m and the inclination parameter α enhance the velocity. Fig. 4 and Fig. 5 show that the velocity decreases with increasing yield stress τ_0 and n. Further, Fig. 6 depicts that increase in E_1 and E_2 enhances the velocity while it is quite the opposite with the parameter E_3. In general, an increment in E_3 leads to a damping force which reduces the fluid velocity. Fig. 7 depicts that the comparison of the velocity profiles in peristaltic pumping of Newtonian (N), Bingham (B), power-law (P) and Herschel-Bulkley (HB) fluids. It is noticed that the velocity in HB fluid case < the velocity in P fluid case < the velocity in B fluid case < the velocity in the N fluid case.

The effect of heat transfer on peristalsis is illustrated in Figs. 8-14 for fixed $x=0.2, t=0.1$, $\tau_0=0.5, \varepsilon=0.3, n=2, Br=1, m=0.1, E_1=0.4, E_2=0.3, E_3=0.2$, $\eta_0=1$, $\alpha=\pi/3$. From Figs. 8-12, we observe that increment in Br and τ_0 enhance the temperature field. Further it is noticed that for large values of m and α temperature increase whereas the increment in n reduces the temperature. Also from Fig. 13 we see that the temperature increases with increasing E_1 and E_2; whereas it decreases with increasing E_3. Further, Fig. 14 presents that the temperature in HB fluid case < the temperature in P fluid case < the temperature in B fluid case < the temperature in the N fluid case. The results in Fig. 15 and Fig. 16 are in conformity with the results of Lakshminarayana et al. [22].

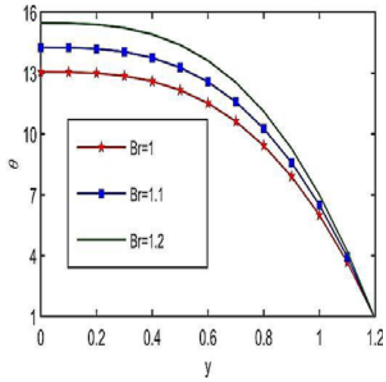
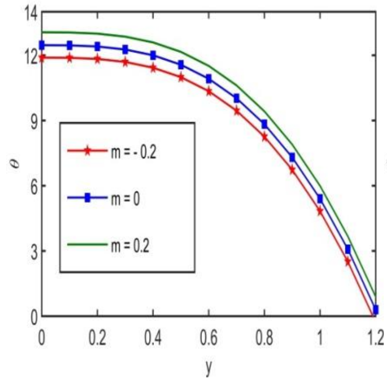


Figure 8. Temperature profiles for different values of 'Br'



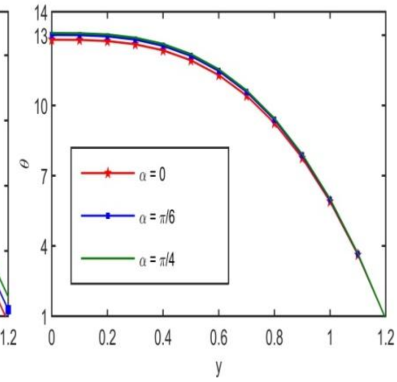
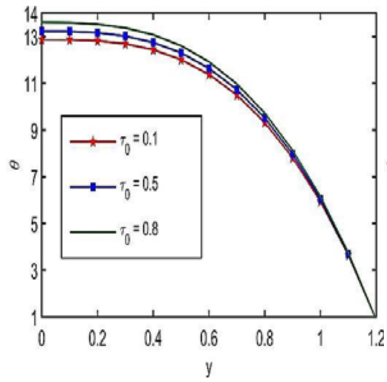


Figure 9. Temperature profiles for different values of 'm'

Figure 10. Temperature profiles for different values of 'α'



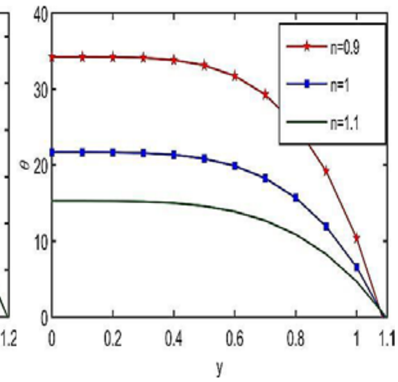
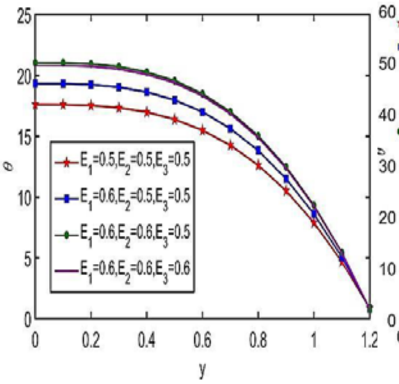


Figure 11. Temperature profiles for different values of 'τ_0'

Figure 12. Temperature profiles for different 'n'

The Nusselt number Nu variations (for fixed : $x = 0.2, t = 0.1, m=0.1, n = 3$, $Br=1, \varepsilon = 0.4, \alpha = \pi/4, E_1 = 0.5, E_2 = 0.3, E_3 = 0.2$) shown in Figs. 17 - 19. The absolute value of Nu increases with aggregate values of Br and τ_0 while it decreases with growing n .



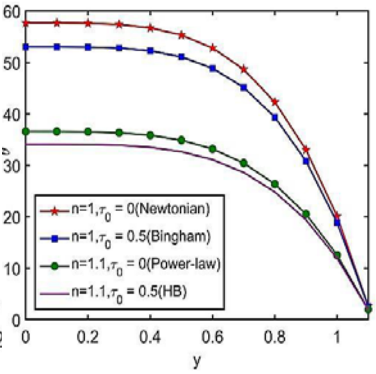
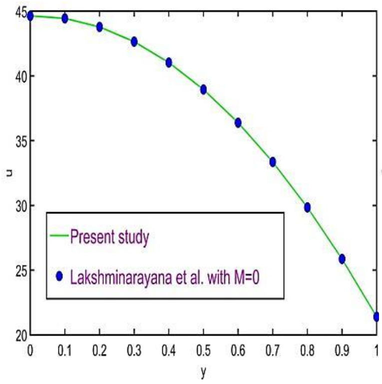


Fig.ure13. Temperature profiles for different values of elasticity parameters

Figure 14. Temperature profiles for different values of 'τ_0'



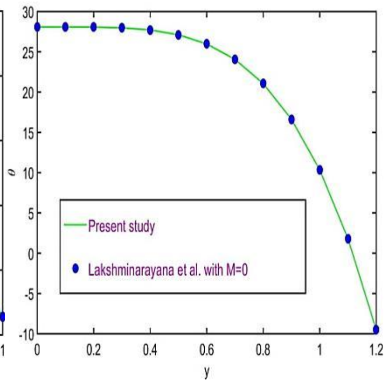


Figure 15. Validation of velocity profiles for fixed values: $x = 0.2, t = 0.1, \beta = 0.2,$
$\tau_0 = 0.4, m{=}0.1, \varepsilon = 0.3, n = 1.1, \eta_0 = 1, E_1{=}0.5, E_2{=}0.3, E_3{=}0.2, \alpha{=}\pi/4$

Figure 16. Validation of temperature profiles for fixed values: $x = 0.2, t = 0.1,$
$Br = 1, \tau_0 = 0.5, n = 1, m{=}0.1, \ \varepsilon = 0.2, E_1{=}0.4, E_2{=}0.3, E_3{=}0.2, \eta_0 = 1, \alpha = \pi/3$

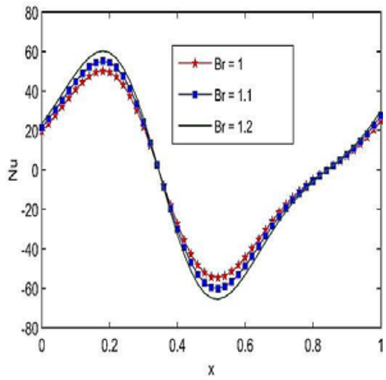
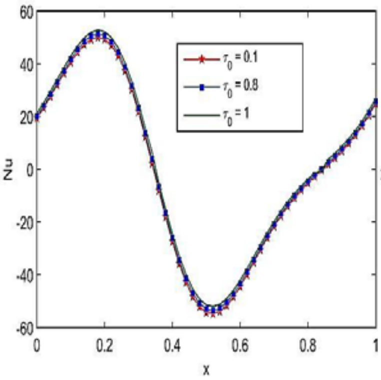


Figure17. Variation of Nu for different values of 'Br'



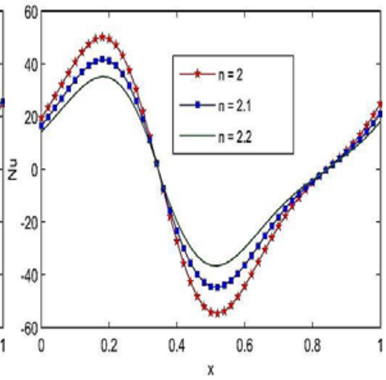


Figure 18. Variation of Nu for different values of 'τ_0'

Figure 19. Variation of Nu for different values of 'n'

5. Trapping phenomenon

Trapping is an important phenomenon in peristaltic transport. Streamlines are drawn for fixed values $m = 0.1, t = 0.2, \varepsilon = 0.3, E_1 = 0.6, E_2 = 0.4, E_3 = 0.2, \tau_0 = 0.4, n = 1, \eta_0 = 1, \alpha = \pi/4$ to study the effect of slip parameter β and yield stress τ_0 in Figs. 20-21. We observe that an increase in the slip parameter increases the size of trapped bolus while the higher value of yield stress reduces the size of the trapped bolus.

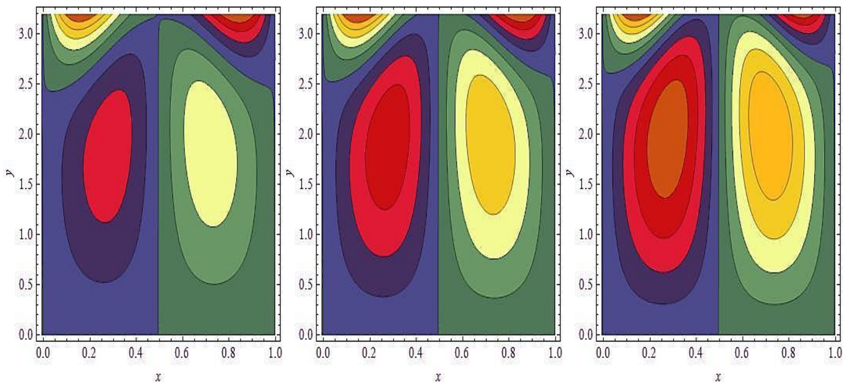


Figure 20. Streamlines for (a) $\beta = 0.1$, (b) $\beta = 0.6$, (c) $\beta = 1$

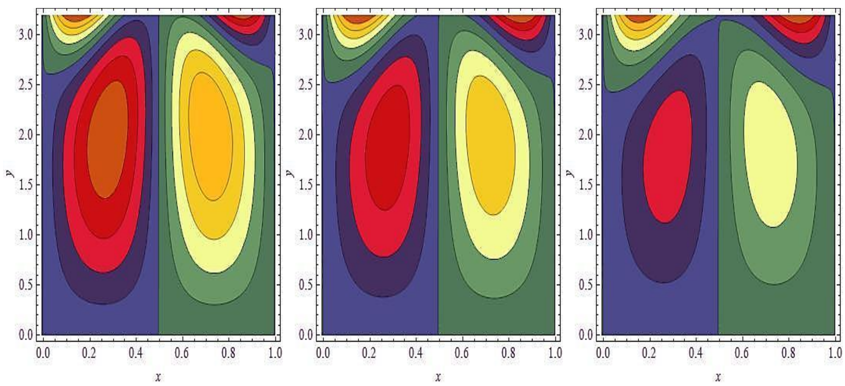


Fig.ure 21. Streamlines for (a) $\tau_0 = 0.2$, (b) $\tau_0 = 0.4$, (c) $\tau_0 = 0.6$

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