

# Effects of variable viscosity and porosity of fluid, Soret and Dufour mixed double diffusive convective flow over an accelerating surface

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**Abstract:** Numerical computation has been carried out for the study of non-linear coupled partial differential equations for combined effects of Soret, Dufour and variable fluid properties like variable viscosity and variable permeability on thermal and diffusion mixed convection flow fluid through porous media over an accelerating surface. The study has been carried out for other additional terms like viscous dissipation and ohmic effect due to the presence of viscous and porous media. By applying the shooting method, we have drawn the numerical results for the flow characteristics like velocity, temperature and concentration for the Soret and Dufour effects on mixed( free and forced) convective flow for various non-dimensional parameters of the problem. Our computational results are compared with the earlier works in the absence of the additional terms and effects on the physical system and found to be good agreement.

## Nomenclature:

Pr	Prandtl number ( $Pr = \frac{\rho v_{\infty} C_p}{k}$ )
E	Eckert number ( $E = \frac{b^2 x^2}{C_p (T_w - T_{\infty})}$ )
Sc	Schmidt number ( $Sc = \frac{v_{\infty}}{D_m}$ )
Df	Dufour number ( $Df = \frac{D_m K_T (C_w - C_{\infty})}{C_s C_p v_{\infty} (T_w - T_{\infty})}$ )
Sr	Soret number ( $Sr = \frac{K_T D_m (T_w - T_{\infty})}{T_m v_{\infty} (C_w - C_{\infty})}$ )
$G_s$	Temperature buoyancy parameter ( $G_s = \frac{Gr_t}{Re^2}$ )



$G_c$	Mass buoyancy parameter ( $G_c = \frac{Gr_m}{Re^2}$ )
$Gr_t$	Local temperature Grashof number ( $Gr_t = \frac{g\beta (T_w - T_\infty)x^3}{v_\infty^2}$ )
$Gr_m$	Local mass Grashof number ( $Gr_m = \frac{g\beta^*(C_w - C_\infty)x^3}{v_\infty^2}$ )
Re	Reynolds number ( $Re = \frac{u_0 x}{v}$ )
$\sigma$	Local porous parameter ( $\sigma = \frac{v}{bk_0}$ )
$\alpha^*$	Ratios of viscosities ( $\alpha^* = \frac{\mu}{\mu}$ )
$\theta_r$	Viscosity parameter ( $\theta_r = \frac{T_r - T_\infty}{T_w - T_\infty} = -\frac{1}{\gamma(T_w - T_\infty)}$ )
$v_w$	$v_w = m(bv)^{1/2}$ , m being constant.
$\epsilon_0$	Porosity at the edge of the boundary layer
$k_0$	Permeability at the edge of the boundary layer
$\eta$	Dimensionless similarity variable
d	Constant having value 3.0
$d^*$	Constant having value 1.5
$\theta$	Dimensionless temperature
$f$	Non dimensionless stream function
$r$	Temperature parameter
$s$	Heat flux parameter
$H$	Dimensionless concentration
$g$	Gravitational acceleration

## 1. Introduction

In the process of estimating the characteristic behavior of the movement of the flow, temperature variations of the fluid and changes in the concentration or species of the fluid depends on different forces (external or internal) and boundary conditions which attracted many scholars for the importance of understanding natural phenomena occurs in nature and industrial applications. In the recent and past, the study of fluid flow plays an important role for the development or analysis of a physical system. At present some of the applications arises in industrial and science and technology like drying of solids, soil physics, petroleum industry, food processing, paper production, coating and polymer processing, geothermal reservoirs, drying process (food and wood products), air conditioning, filtration of solids from liquids, hydro-geology, chemical engineering, casting and welding, enhanced oil recovery, cloth industry, thermal insulation and many other.

In these applications we come across constant fluid properties i.e. without varying viscosity, porosity, permeability and diffusivity etc. i.e. which are not changing with respect to the physical quantities like temperature or concentration or position or time. Many authors have studied mathematically the above application problems by assuming constant fluid properties over non-accelerating surface and which may satisfy approximately with the real values of the experimental or industrial results. Later, the researchers improved these models by incorporating the accelerating surface with constant fluid properties. Different aspects of the above practical application problems for a gradual increase in investigating the problem of flow induced by a solid surface and continuous flat surface have been investigated to name a few [1-2] and their references. The effect of the temperature heat sources or sinks in stagnant flow carried out by Sparrow and Cess [3]. Several earlier research works are carried out on non-accelerating surface with constant surface temperature and velocity but for many industrial and practical applications the surface undergoes stretching and heating or cooling which is due to that the surface velocity and temperature variations and which is investigated by Tsou[4]. Later the authors [5-7] have examined about the stretching surface with constant surface temperature.

During the past several decades, the study of convective fluid flow in a or through flow permeable medium called porous medium taken a lead role for the researchers due to wide diverse range of engineering applications as mentioned below. The industrial applications for the study of porous media are seen in heat exchangers in high heat flux applications such as electronic equipment, insulation of the heated body, thermal energy storage and sensible heat storage beds, drying process (wood and food products), air conditioning and filtration process. During the last decades, several researchers studied free convection heat and mass transfer in a porous medium [8-10]. Mass transfer effects on flow past an accelerated vertical plate has already been well studied [11-12]. Furthermore, the problem of mass and heat transfer of non-Newtonian fluids through porous media has been a subject of interest in many research projects [13-15]. The authors [16-17] are analyzed the problem of combined heat and mass transfer fluid flow over an accelerating sheet which is referred as stretching sheet across the boundary layer through porous media has been studied due to the application in metallurgy and chemical engineering fields. Crane [18] has carried out his investigation first on the fluid flow due to stretching sheet surface with a constant surface temperature.

Eckert and Drake [19] investigated that the Dufour and Soret effect helps in understanding the study between the mixtures of gases with modeling weight can be ignored in electronic technology field. Kafoussias and Williams [20] incorporated a similar effect on mixed free forced convective and mass transfer with variable viscosity, which is a function of temperature. Later same effect of dufour and soret over a vertical surface embedded in a porous media is carried out by Anghel et al. [21]. The effect of surface heat flux power law and surface temperature power law in the heat transfer characteristics of a continuous linear stretching surface was analyzed by Chen and Char [22]. The non-Darcy free and forced convection along a vertical wall in a saturated porous medium has studied by Lai and Kulacki [23]. Alabraba et al. [24] examined on binary mixture of the laminar convection and hydromagnetic flow fluid along with radiative heat transfer.

Further Seddeek [25] carried out his research on effect of variation of variable viscosity on hydro magnetic fluid flow and heat transfer of radiation and oscillatory boundary layer through porous media. The free convection and mass transfer flow of a viscous incompressible and electrically conducting fluid through a highly porous medium bounded by vertical infinite surface with constant suction velocity and constant heat flux is analyzed by Acharya et al [26]. Seddeek [27] have studied on soret and dufour effects on mixed convective flow and mass transfer in the presence of suction and blowing over an accelerating surface under along with a heat source and variable viscosity. Rudraiah [28] studied convection by incorporating inertial effect at the boundary layer. Chandrasekhar and Namboodiri [29] have studied the behavior of velocity distribution and heat transfer with variable permeability of the porous medium. Importance of inertia effects of variable fluid properties through

sparsely packed porous medium continued with Mohammadein and El-Shaer [30]. Very recently Dinesh et al. [31-34] studied the mixed convection on a non-accelerating vertical plate by varying fluid properties in terms of permeability, porosity and diffusivity for a more practical situation problem.

In all the above applications we come across the variable fluid properties which changes due to the physical quantities or parameters like temperature or concentration or position or time. As per our knowledge the researchers mentioned above are mainly concentrated to analyze the fluid flow behavior like velocity, concentration and temperature with varying only viscosity or non-accelerating surface. Also the variation of viscosity of the fluid changes with temperature or permeability of the porous medium varies with position or time which is observed in practical applications which are mentioned above and these changes affects the characteristics of the fluid flow. Hence the main novelty in our paper is to study mathematically the physical behavior of varying fluid properties (viscosity, porosity and permeability) along with accelerating boundary surface. To estimate the variations will affect the flow behavior compared to the earlier works in the absence of variation of fluid properties and the work satisfy exactly with the earlier works and found an excellent agreement of the numerical results.

## 2. Mathematical formulation

Consider a two-dimensional ( $x$  &  $y$ ), steady (independent of time), combined effects of concentration/species and temperature for a laminar, viscous, incompressible fluid flow in the presence of uniform porous medium over an accelerating flat plate. The following assumptions are made:

- (a) The Boussinesque approximation is considered,
- (b) Porosity and permeability are the expressions in terms of the vertical coordinate  $y$ ,
- (c) Ohmic, Soret and Dofour effects are taken into account,
- (d) Assume the molecular transport properties are constant,
- (e) Due to variation of the viscosity we assumed that which is inversely proportional to function of temperature,

which is in the form of  $\frac{1}{\mu} = \frac{1}{\mu_\infty} [1 + \gamma(T - T_\infty)]$  or  $\frac{1}{\mu} = a(T - T_r)$ , (see Lai and Kulacki [23]) where

$$a = \frac{\gamma}{\mu_\infty}, \quad T_r = T_\infty - \frac{1}{\gamma}.$$

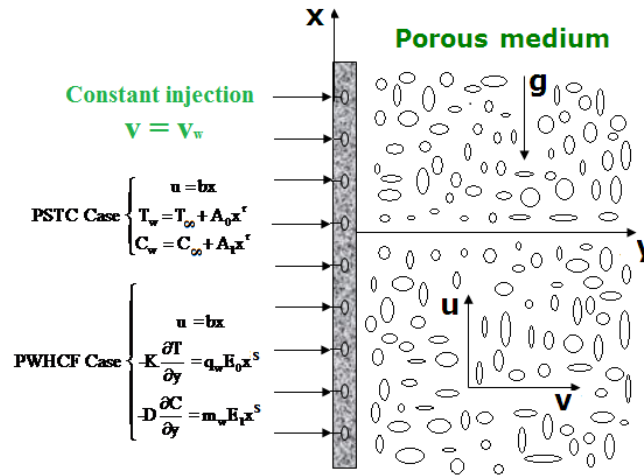
Both 'a' and  $T_r$  are constants, which depends on the thermal property of the fluid  $\gamma$ . In general  $\begin{cases} a > 0 & \text{for liquids} \\ a < 0 & \text{for gases} \end{cases}$ . The physical model of the fluid flow is shown in Fig.1. Based on all the above assumptions the governing equations of motion are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{1}{\rho} \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right) + g\beta (T - T_\infty) - g\beta^* (C - C_\infty) - \frac{\epsilon(y) \bar{\mu} u}{\rho k(y)}, \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2} + \frac{D_m K_T}{C_s C_p} \frac{\partial^2 C}{\partial y^2} + \frac{\mu}{\rho C_p} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{\bar{\mu} \epsilon^2(y) u^2}{\rho C_p k(y)}, \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} + \frac{D_m K_T}{T_m} \frac{\partial^2 T}{\partial y^2}, \quad (4)$$



**Figure1.** Physical Configuration

where the velocity components  $u$  and  $v$  are in the direction of  $x$  and  $y$  respectively. The accelerating surface moves along the  $x$ -axis and both  $x$ -axis and  $y$ -axis are mutually perpendicular to each other,  $k(y)$  is the variation of permeability of the porous medium,  $\varepsilon(y)$  is the variation of porosity of the porous medium,  $\theta_r$  is a viscosity parameter, which is defined by (Seddeek [25]) and all other physical quantities have their usual meanings.

$$\theta_r = \frac{T_r - T_\infty}{T_w - T_\infty} = - \frac{1}{\gamma(T_w - T_\infty)}. \quad (5)$$

The governing equations of the physical model are to be analyzed for various boundary conditions which is observed across the boundaries, the boundary of the system under consideration is seen one at the accelerating surface and other one at the far away from the oscillatory surface in the porous medium. These special types of boundary conditions have been discussed in the following cases: (i) Prescribed Surface Temperature and Concentration (PSTC) and (ii) Prescribed Wall Heat and Concentration Flux (PWHCF).

*(i) Prescribed Surface Temperature and Concentration (PSTC)*

A polynomial degree of  $r$  has been maintained at the boundary on temperature of the vertical plate i.e. the accelerating temperature of the vertical plate  $T_w$  is assumed in the form of  $T_w = T_\infty + A_0 x^r$ , similarly the concentration of the accelerating wall  $C_w$  is considered in the form of  $C_w = C_\infty + A_1 x^r$ . The following are the boundary conditions on velocity, temperature and concentration fields:

$$u = bx, \quad v = v_w, \quad T_w = T_\infty + A_0 x^r, \quad C_w = C_\infty + A_1 x^r \quad \text{at } y = 0, \quad (6)$$

$$u = 0, \quad T = T_\infty, \quad C = C_\infty \quad \text{as } y \rightarrow \infty, \quad (7)$$

where  $A_0, A_1$  are arbitrary constants and  $b$  is the constant stretching rate,  $r$  is the temperature parameter and if  $r = 0$  then the thermal boundary conditions become isothermal. Since the Eqs. (1)-(4) are highly non-linear coupled partial differential equations, so it is difficult to solve those equations using analytical methods or procedures. But in order to solve them numerically Acharya et al [26] introduced the following similarity variable  $\eta$  and dimensionless parameters  $f, \theta, H$ .

$$\psi = (vb)^{\frac{1}{2}}xf(\eta), \quad \eta = \left(\frac{b}{v}\right)^{\frac{1}{2}}y, \quad \theta(\eta) = \frac{T-T_{\infty}}{T_w-T_{\infty}}, \quad H(\eta) = \frac{C-C_{\infty}}{C_w-C_{\infty}}, \quad (8)$$

the velocity equation (1) will satisfy automatically when the stream function  $\psi(x, y)$  is defined in such way that  $u = \frac{\partial\psi}{\partial y}$  and  $v = -\frac{\partial\psi}{\partial x}$  and the corresponding velocity components are given by

$$u = bxf'(\eta), \quad v = -(bv)^{\frac{1}{2}}f(\eta). \quad (9)$$

Following Chandrasekhara and Namboodiri [29], the variable permeability  $k(\eta)$  and the variable porosity  $\varepsilon(\eta)$  are given by

$$k(\eta) = k_0(1 + de^{-\eta}), \quad (10)$$

$$\varepsilon(\eta) = \varepsilon_0(1 + d^*e^{-\eta}), \quad (11)$$

where  $k_0$  is the permeability and  $\varepsilon_0$  is the porosity of the porous medium at the edge of the boundary layer,  $d = 3.0$  and  $d^* = 1.5$  are considered as constants for variable permeability and porosity respectively. Substituting Eqs. (8) & (9) in Eqs. (2)-(4) using Eqs. (10) & (11), we get the following transformed non-linear couple ordinary differential equations.

$$f''' - \left(\frac{1}{\theta - \theta_r}\right)\theta'f'' - \left(\frac{\theta}{\theta_r} - 1\right)(ff'' - f'^2) + \alpha^*\varepsilon_0\sigma\left(\frac{1+d^*e^{-\eta}}{1+de^{-\eta}}\right)\left(\frac{\theta}{\theta_r} - 1\right)f' = \left(\frac{\theta}{\theta_r} - 1\right)(G_s\theta - G_cH), \quad (12)$$

$$\theta'' + P_r f\theta' - r P_r f'\theta = -P_r D_f H'' - E P_r f'^2 - E\sigma P_r \varepsilon_0^2 \left[\frac{(1+d^*e^{-\eta})^2}{1+de^{-\eta}}\right]f'^2, \quad (13)$$

$$H'' + S_c fH' - r S_c f'H = -S_c S_r \theta'', \quad (14)$$

where all the dimensionless parameters are defined in nomenclature section and the corresponding transformed boundary conditions in non-dimensional form are:

$$f(0) = -\frac{v_w}{(bv)^{\frac{1}{2}}} = -m, \quad f'(0) = 1, \quad \theta = 1, \quad H = 1 \quad \text{at } y = 0, \quad (15)$$

$$f' = 0, \quad \theta = 0, \quad H = 0 \quad \text{as } y \rightarrow \infty. \quad (16)$$

(ii) *Prescribed Wall Heat and Concentration Flux (PWHCF)*

A dirichelet conditions on temperature is maintained at the wall which is assumed in the form  $-k\frac{\partial T}{\partial y} = q_w E_0 x^s$  and  $-D\frac{\partial C}{\partial y} = m_w E_1 x^s$ . The following are the boundary conditions on velocity, temperature and concentration fields:

$$u = bx, \quad v = v_w, \quad -k\frac{\partial T}{\partial y} = q_w E_0 x^s, \quad -D\frac{\partial C}{\partial y} = m_w E_1 x^s \quad \text{at } y = 0, \quad (17)$$

$$u = 0, \quad T = T_{\infty}, \quad C = C_{\infty} \quad \text{as } y \rightarrow \infty, \quad (18)$$

where  $E_0$ ,  $E_1$  are arbitrary constants,  $s$  is the heat flux parameter and the accelerating sheet is subject to uniform heat flux when  $s = 0$ . Since the Eqs. (1)-(4) are highly non-linear coupled partial differential equations, so it is difficult to solve those equations using analytical methods or procedures. But in order to solve them numerically Acharya et al [26] introduced the following similarity variable  $\eta$  and dimensionless parameters  $f, g, h$ .

$$\psi = (vb)^{\frac{1}{2}}xf(\eta), \quad \eta = \left(\frac{b}{v}\right)^{\frac{1}{2}}y, \quad T - T_{\infty} = \frac{E_0x^s}{k} \left(\frac{v}{a}\right)^{\frac{1}{2}}g(\eta), \quad C - C_{\infty} = \frac{E_1x^s}{D} \left(\frac{v}{a}\right)^{\frac{1}{2}}h(\eta), \quad (19)$$

the velocity equation (1) will satisfy automatically when the stream function  $\psi(x, y)$  is defined in such way that  $u = \frac{\partial\psi}{\partial y}$  and  $v = -\frac{\partial\psi}{\partial x}$  and the corresponding velocity components are given by Eq. (9). Substituting Eqs. (9) & (19) in Eqs. (2)-(4) and using Eqs. (10) & (11), we get the following transformed non-linear couple ordinary differential equations

$$f''' - \left(\frac{1}{g-g_r}\right)g'f'' - \left(\frac{g}{g_r} - 1\right)(ff'' - f'^2) + \alpha^*\varepsilon_0\sigma \left(\frac{1+d^*e^{-\eta}}{1+de^{-\eta}}\right)\left(\frac{g}{g_r} - 1\right)f' = \left(\frac{g}{g_r} - 1\right)(G_s g - G_c h), \quad (20)$$

$$g'' + P_r f g' - s P_r f' g = -P_r D_f h'' - E P_r f'^2 - E \sigma P_r \varepsilon_0^2 \left[\frac{(1+d^*e^{-\eta})^2}{1+de^{-\eta}}\right] f'^2, \quad (21)$$

$$h'' + S_c f h' - s S_c f' h = -S_c S_r g'', \quad (22)$$

where all the dimensionless parameters are defined in nomenclature section. The corresponding transformed boundary conditions in non-dimensional form are:

$$f(0) = -\frac{v_w}{(bv)^{\frac{1}{2}}} = -m, \quad f'(0) = 1, \quad g = -1, \quad h = -1 \quad \text{at } y = 0, \quad (23)$$

$$f' = 0, \quad \theta = 0, \quad H = 0 \quad \text{as } y \rightarrow \infty. \quad (24)$$

### 3. Numerical solution

The Eqs. (12) - (14) along with the boundary conditions (15) & (16) are the functions of  $f(\eta), \theta(\eta)$  &  $H(\eta)$  in the case of PSTC and the equations (20) - (22) along with the boundary conditions (23) & (24) are the functions of  $f(\eta), g(\eta)$  &  $h(\eta)$  in case of PWHCF and which are to be determined; we observe that the ODE's are not only non-linear in nature but involved variable coefficient like transcendental terms as well as the corresponding boundary conditions. The known methods pertaining to analytical nature of the solution cannot be applied here because of its highly coupled non-linear nature involved between the equations of the fluid flow model. To overcome the difficulty raised in the analytical method here we adopt the numerical method to find the solution by transforming the higher order coupled non-linear ODE's into system of first order ODE's and these first order ODE's can be solved by Runge-Kutta-Fehlberg explicit method and second order accuracy of Newton-Raphson method. The combination of these two numerical techniques to estimate an approximate solution of the problem is referred as shooting technique. By this we stop the iterative procedure for evaluating the numerical results for the above system of equations for the given boundary conditions. We developed these computations using the software like Mat lab in which the computational work is carried out for an accuracy of  $10^{-6}$  i.e. the difference between the two iterative successive values is less than the prescribed accuracy of  $10^{-6}$ .

### 4. Results and Discussion

The importance of this paper is lies in understanding the variation of viscosity parameter in terms of  $\theta_r$  and the variation of permeability and porosity is seen in Darcy term in momentum equation and ohmic effect in energy equation respectively. We adopted numerical computation drawn from Mat Lab by solving the system of ODE's under two different conditions. They are (i) Prescribed Surface Temperature and Concentration (PSTC) (ii) Prescribed Wall Heat and Concentration Flux (PWHCF). The coupled non-linear ODE's with variable co-efficients are coming from the momentum, energy and species equations and which involves important non-dimensional parameters like



viscosity parameter, porous parameter, ratios of viscosities, prandtl number, Eckert number, solet number, dufour number, Temperature buoyancy parameter, Mass buoyancy parameter and all other parameters involved in the physical problem are assumed to be fixed value throughout the computation. The numerical results are carried out to discuss the variation of velocity or momentum of the fluid flow, temperature variation and solutal changes for the above non-dimensional parameters and are depicted from the following Figures [2-26].

**Effect of viscosity parameter  $\theta_r$ :** Figs.2-4 show the variations of viscosity parameter  $\theta_r$  in terms of velocity, temperature and concentration respectively. With the variation of variable viscosity parameter  $\theta_r$  there is a drastic change in velocity diminishes exponentially and decreases more in the case of PSTC compare to that of PWHCF case. This is due to the fact that higher the viscosity diminishes the flow rate or in the other words the heat transfer between wall to far away from accelerating plate is decrease and which is depicted in the Fig.2. There is an opposite behavior of heat transfer can be observed with the variation of viscosity parameter  $\theta_r$  on temperature as shown in Fig.3. Here heat transfer is more in the case of PWHCF compare to that of PSTC case. Similarly a slight variation is seen in the case of specious with respect to the viscosity parameter  $\theta_r$ , which is shown in the Fig.4. Since the effect of viscosity parameter  $\theta_r$  directly not play on the concentration but indirectly affects the concentration through temperature and momentum equations.

**Effect of porous parameter  $\sigma$ :** Figs.5-7 indicates the effects of porous parameter  $\sigma$  over the velocity, temperature and concentration respectively. The increase in the porous parameter  $\sigma$  of the fluid is due to enhancement of the viscosity of the fluid or decrease in the permeability at the edge or decrease in the stretching rate of the accelerating surface, this will result a gradual reduction in the flow of velocity of the fluid, which is illustrated in the Fig.5. An opposite behavior is observed with the case of temperature profile for various values of the porous parameter  $\sigma$ . The non-dimensional temperature enhanced both in the case of PSTC and PWHCF because the increment of fluid viscosity or decrement of the permeability of porous medium at the edge of the boundary layer which is depicted in Fig.6. Similarly for the same enhancement of the porous parameter  $\sigma$  there is a gradual increase of concentration is observed in the Fig.7, which is due to the fact that the enhancement of viscosity will reduce the movement of fluid hence the acceleration of the concentration will gradually increase. In the absence of porous parameter  $\sigma$  i.e.  $\sigma = 0$  our results have good agreement with earlier works.

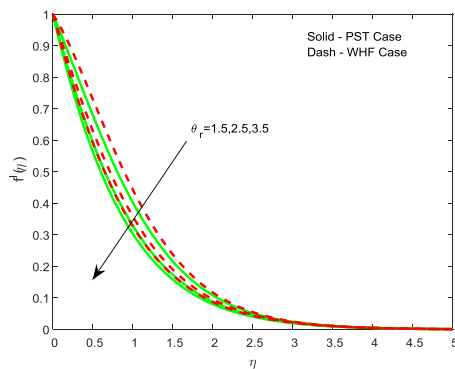
**Effect of ratios of viscosities  $\alpha^*$ :** Figs. 8-10 illustrate the variations of ratios of viscosities  $\alpha^*$  in terms of velocity, temperature and concentration respectively. For various values of ratios of viscosities  $\alpha^*$  there is decrement in non-dimensional velocity of the fluid, which is due to the decrement of viscous of the porous medium or increment in the fluid viscosity. Since the enhancement of the fluid viscosity will decrease the velocity of the fluid, which can be seen the Fig. 8. An opposite behavior can be seen in the Fig.9 of the temperature profile which is due to the fact that the enhancement of non-dimensional temperature will decrease of the fluid viscosity and which implies the increment of ratios of viscosity  $\alpha^*$ . Similarly the same behavior can be noticed in the Fig.10 of the concentration profile. In the absence of the viscosity of the porous medium our results are coincide with the previous results.

**Effect of Prandtl number  $Pr$ :** Figs.11-13 gives the effects of prandtl number over the velocity, temperature and concentration are observed, the increase in the prandtl number of the fluid is due to enhancement of the viscosity of the fluid or decrease in the thermal diffusivity of the fluid, this will result a gradual reduction in the flow of velocity of the fluid. The percentage of decrement of the velocity is more in the case of PSTC compared to that of PWHCF case because of boundary conditions maintained at the accelerating surface. The velocity decay is in the form of exponential decay profile

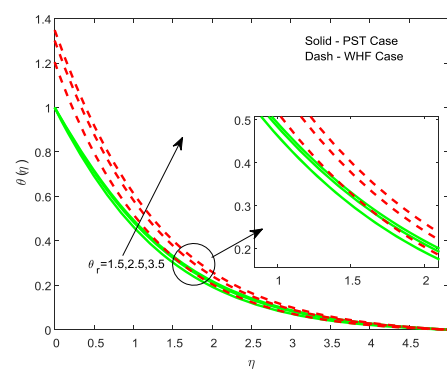


which depicted in the Fig.11. For the same enhancement of the prandtl number then is a gradual decrease of temperature is observed in the Fig.12 due to increase of thermal diffusivity temperature decreases both in the case of PSTC and PWHCF but the rate decreases of temperature of the fluid flow is more in the case of PSTC compare to that of PWHCF case because of adiabation nature of temperature is maintained at the accelerating wall. An opposite behavior is observed in the Fig.13 with the case of concentration profile for different values of prandtl number due to less movement of fluid flow the acceleration of the concentration will gradually increase for enhancement of viscosity but the effect of prandtl number on concentration for both cases is very less compare to that of velocity and temperature. Our results for the variation of prandtl number have good agreement with the earlier works.

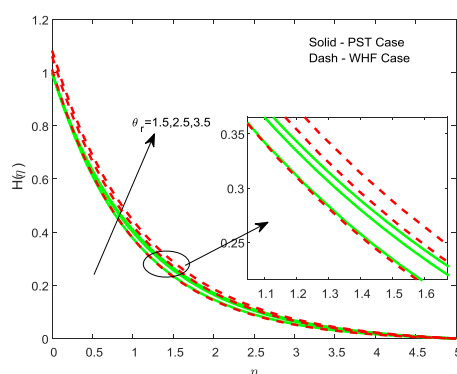
**Effect of Soret number  $Sr$  and Dufour number  $Df$ :** Figs.16-17 shows variations of the Soret number over the temperature and concentration profiles respectively. As increase in the Soret number both dimensionless temperature and concentration will increase. Similarly we can also observe the behavior of velocity, temperature and concentration profiles for various values of Dufour number from the Figs.18-20 respectively. The increase of Dufour number leads to raise the non-dimensional velocity and as well as temperature of the fluid, which can be depict in the Fig.18 and Fig.19 respectively. Where as an opposite behavior can be gleaned from the Fig.20 in the case of concentration profile i.e. an increase of Dufour number helps in reducing the dimensionless concentration.



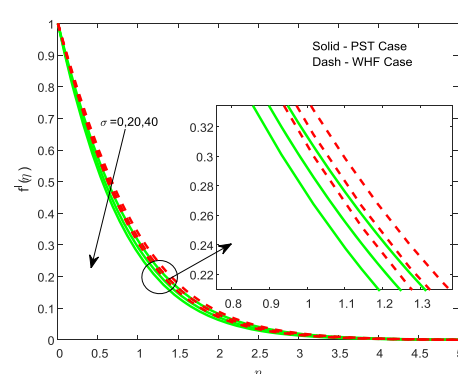
**Fig.2: Velocity profile for different values of viscosity parameter.**



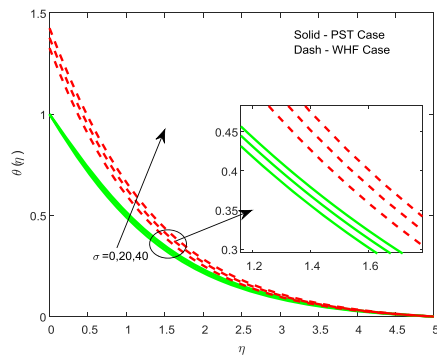
**Fig.3: Temperature profile for different values of viscosity parameter.**



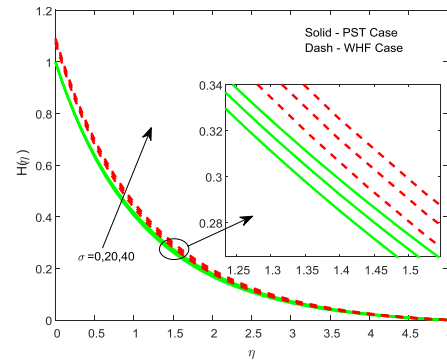
**Fig.4: Concentration profile for different values of viscosity parameter.**



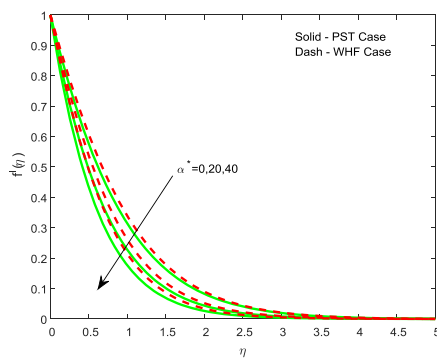
**Fig.5: Velocity profile for different values of Porous parameter.**



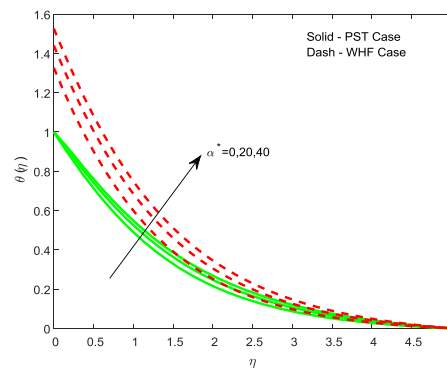
**Fig.6:** Temperature profile for different values of Porous parameter.



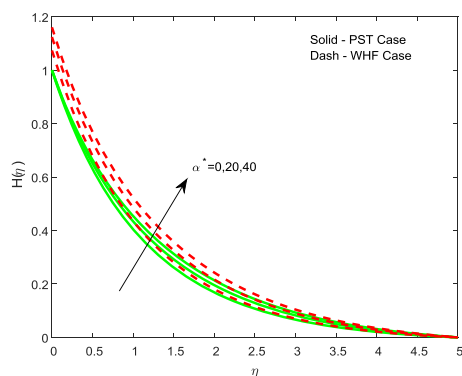
**Fig.7:** Concentration profile for different values of Porous parameter.



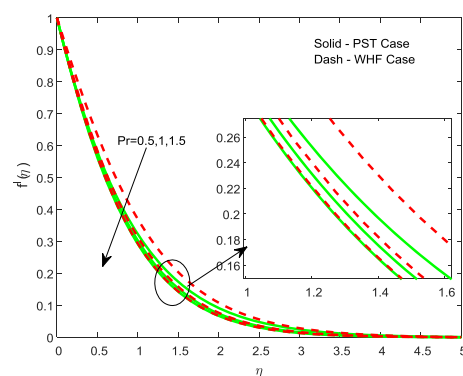
**Fig.8:** Velocity profile for different values of ratios of viscosities.



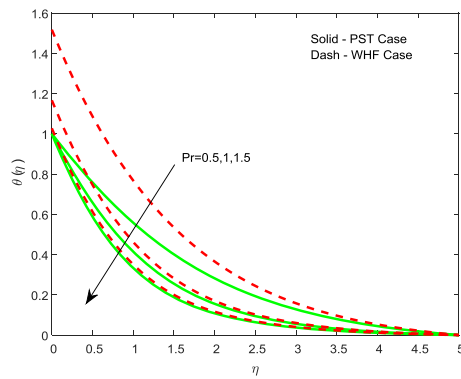
**Fig.9:** Temperature profile for different values of ratios of viscosities.



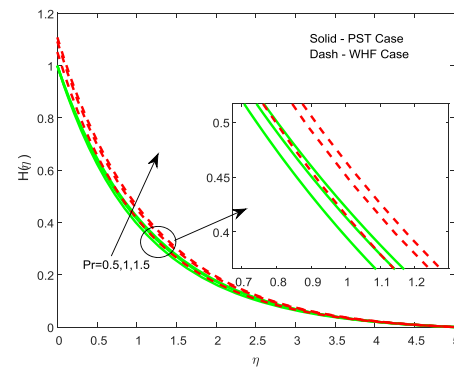
**Fig.10:** Concentration profile for different values of ratios of viscosities.



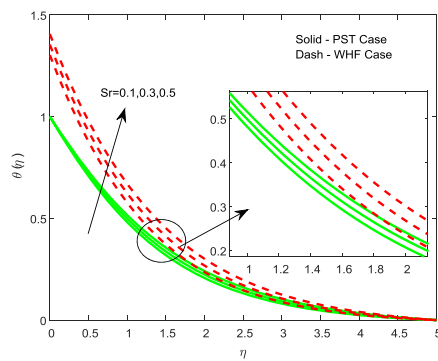
**Fig.11:** Velocity profile for different values of Prandtl number.



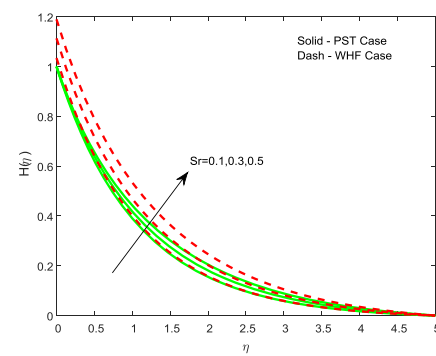
**Fig.12:** Temperature profile for different values of Prandtl number.



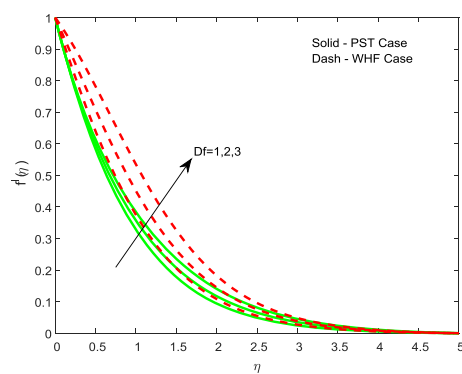
**Fig.13:** Concentration profile for different values of Prandtl number.



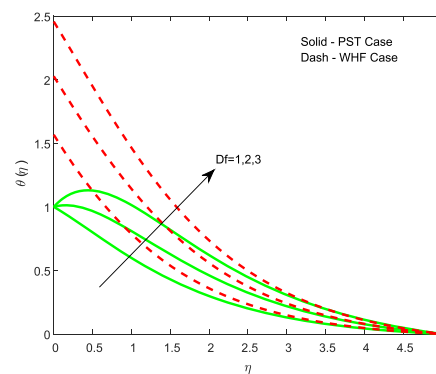
**Fig. 14:** Temperature profile for different values of Soret number.



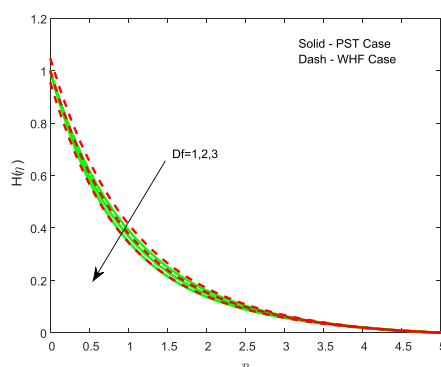
**Fig. 15:** Concentration profile for different values of Soret number.



**Fig. 16:** Velocity profile for different values of Dufour number.



**Fig. 17:** Temperature profile for different values of Dufour number.



**Fig. 18: Concentration profile for different values of Dufour number.**

Our results are compared with the earlier work of Nalinakshi et al [33] for the constant viscosity of the fluid which is shown in the below Table 1. From this table it is observed that the values of the skin friction, Nusselt number and Sherwood number are compared and found to be an excellent agreement for the accuracy of  $10^{-6}$ .

**Table 1.** Results for  $f''(0)$ ,  $\theta'(0)$  and  $\phi'(0)$  for  $P_r = 0.71$ ,  $S_c = 0.22$ ,  $E = 0.1$ ,  $\epsilon_0 = 0.4$  for variable

Permeability.

$\sigma^*$	$\frac{G_r}{Re^2}$	$\frac{\alpha^*}{\sigma Re}$	Nalinakshi et al[33]			Present value		
			$f''(0)$	$\theta'(0)$	$\phi'(0)$	$f''(0)$	$\theta'(0)$	$\phi'(0)$
2	0.0	0.0	0.363800	0.282750	0.280750	0.364000	0.284760	0.280340
		0.1	0.435870	0.325750	0.328590	0.435660	0.326753	0.328689
		0.5	0.687800	0.400580	0.400780	0.686700	0.410080	0.400790
	0.2	0.0	0.425800	0.400990	0.401205	0.426700	0.400678	0.403405
		0.1	0.538500	0.541456	0.545672	0.539500	0.542456	0.547672
		0.5	0.775600	0.561578	0.567652	0.777800	0.560478	0.565552
	2.0	0.0	1.346070	0.781453	0.794323	1.345570	0.780353	0.796723
		0.1	1.379100	0.881132	0.901256	1.378500	0.881130	0.901306
		0.5	2.004900	0.980023	0.988976	2.005300	0.980076	0.989000
4	0.2	0.1	0.552345	0.584573	0.571562	0.552465	0.584784	0.571775

## 5. Conclusions

Numerical computation has been analyzed for the study of highly non-linear coupled partial differential equations for combined effects of Soret, Dufour and variable fluid properties like variable porosity, viscosity, solutal diffusivity, thermal diffusion, and variable permeability on thermal and diffusion mixed convection flow over an accelerating surface in the presence of porous media. The study has been carried out for other additional terms like viscous dissipation and ohmic effect due to the presence of viscous and porous media. The main conclusions are:

- The velocity profile decreased for an increase in viscosity parameter  $\theta_r$  whereas temperature and concentration profiles increased for an increase in viscosity parameter  $\theta_r$ .
- As an increase in porous parameter  $\sigma$  results decrease the velocity profile but increase the temperature and concentration profiles.

- The velocity profile increased for an increase in ratios of viscosities parameter  $\alpha^*$  whereas the temperature and concentration profiles are increased for an increase in ratios of viscosities  $\alpha^*$ .
- An excellent agreement is found for the constant viscosity with the earlier work of Nalinakshi et al [33].

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