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Unsteady MHD nanofluid flow over a stretching sheet with chemical reaction

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Abstract. The unsteady magnetohydrodynamic (MHD) mass transfer flow of an incompressible, electrically conducting nanofluid past a porous stretching sheet has been considered. In this study we considered three different types of water based nanofluids viz., copper (Cu), aluminum oxide Al_2O_3, and titanium dioxide (TiO_2). By using appropriate similarity transformations, the governing partial differential equations of momentum and concentration are converted into a set of non-linear ordinary differential equations. The resulting equations are then solved numerically by using shooting method along with R-K fourth order scheme. The effects of diverse flow physical parameters on the flow fields and mass transfer areillustrated graphically.It is found that the influence of unsteady parameter on skin friction coefficient and Sherwood number is opposite.

1. Introduction

The term “nanofluid” refers homogenous mixture of base fluid and nanoparticles. These fluids are characterized by suspension of particle copper which the diameter is less than 100nm. In general, nanofluids possess to improve the thermo-physical properties such as melting diffusivity, consistency, melting potential and heat conduction characteristics as compared with the common base fluids like water, engine oils and lubricants, polymeric solution, bio-fluids and other common liquids. In fact, the main concept of using nanoparticles is to improve the melting potential of the based fluid. Many researchers [1-5] investigated the flow of a nanofluid over various flow fields due to its extensive use in engineering applications. Beg et al. [6]found the unsteady mixed convective nanofluid over a porous exponentially stretching sheet by usingexplicit finite difference scheme. Kumar Nandy et al. [7]examined the influence of Brownian motion and thermophoresis over unsteady permeable shrinking sheet. Freidoonimehr et al. [8] investigated numerically an unsteady magnetohydrodynamic laminar stream of a nanofluid past a perpendicular stretched layer with porous medium. Recently, Zin et al. [9] addressed the effect of thermal energy on unsteady magnetohydrodynamics flow over a permeable medium. Heat transfer analysis on Cu-water nanofluid stream past a stretchinglayer with slip is investigated by Pandey et al. [10]. An extensive variety of survey papers on nanofluids and different utilizations are make in [11-13].

In recent years, several researchers focused very much attention to analyze the unsteady flows over a stretching surface due to its uses in geophysics, paper production, polymer preparing of a biochemical mechanical plant that incorporate both polymer and metal sheets and so on. The chemical reaction effects in boundary layer flows are highly significant for their various reasonable applications within the sight of homogeneous and heterogeneous mass exchange responses happens by diffusive operations which include the atomic dispersion of species. Harish Babu and Satya Narayana[14] investigated the joule heating on magnetohydrodynamic heat exchange of an incompressible fluid with heat source over stretching plate. Chamkha et al. [15] inspected the unsteady limit layer stream because of an extending surface installed in a permeable medium generation/absorption were acquired. Manjula et al. [16] addressed theQualities of heat and mass exchange of an electrically directing and incompressible and viscous liquid through a permeable medium. Mishra et al. [17] proposed the soret effects on hydromagnetic micropolar stream along extending layer in the nearness of the chemical reaction. Satya Narayana et al. [18]analyzed the consequence of chemical response on magnetohydrodynamic heat and mass transfer of a non-Newtonian over extendinglayer. Daba and Devaraj[19] presented the chemically reacting varied convection flow through a permeable unsteady stretching surface with slip and thermal radiation. Recently, some of them studied MHD flows over a stretching sheet see ref. [20-22].

The objective of this model is to analyze the influence of three distinctkinds of nanoparticles (Cu, Al_2O_3, and TiO_2) over aextending layer in the presence of magnetic field and chemical response. By using similar transformations, the momentum and mass transfer equations, under boundary layer assumptions reduce to the set of nonlinear ODE. The resulting ODE's are calculatedstatistically by using R-K rule along with shooting technique see ref. Matausek [23]. The results for emerging parameters are being illustrated through graphs and discussion. Physical behaviors of reduced skin-friction coefficient and Sherwood number are discussed through graphs.

2. Mathematical formulation

We are taken an unsteady magnetohydrodynamic laminar stream through an extending layer in a H_2O based incompressible nanofluid containing *3*unique sorts of nanoparticles, as appeared in Figure 1. Likewise, it is accepted that the base liquid and the nanoparticles are in current harmony and no slip condition exists between them. It is additionally gathered attractive Reynolds number is minor. In this way, it is possible to disregard the incited attractive field in contrast with the connected attractive field. Consider C_w a chance to fixation at the plate and the focus from the layer is C_∞ additionally, let the response of the species a chance to primary request homogeneous synthetic response of rate k_1 which changes with time. We expect that for time t<0, the liquid and mass streams are relentless. The insecure liquid and mass streams begin at t =0. The layer rises out of an opening at beginning (x = 0, y= 0) and moves with non-uniform $f'(\eta)$ along the x-axis. The speed of the mass exchange opposite to the extending surface is $V_w(t)$.

The governing equations of such kind of flow see ref. [24] and mass transfer has usual notation,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\rho_{nf}\left(\frac{\partial u}{\partial t}+u\left(\frac{\partial u}{\partial x}\right)+v\left(\frac{\partial u}{\partial y}\right)\right)=\mu_{nf}\left(\frac{\partial^2 u}{\partial y^2}\right)-\sigma B^2 u \tag{2}$$

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-k_1\left(C-C_\infty\right) \tag{3}$$

Here , u&v are the velocity components along with x and y-Coordinates, individually, t indicates time, ρ_{nf} & μ_{nf} are indicates density and the dynamic viscosity of the nanofluid, individually, μ_{nf} was suggested by Brinkman [25], σ bethe electrical conductivity, magnetic field $B=B_0/\sqrt{(1-ct)}$ imposed along the y-axis, the diffusion coefficient$D$ of the diffusing species in the liquid, the time-dependent reaction rate $k_1(t)=\frac{k_0}{(1-ct)}$; $k_1>0$represent for destructive response whereas $k_1<0$indicate for constructive response, k_0 is a constant, the relaxation time of the period $\lambda=\lambda_0(1-ct)$, λ_0 is a constant.

$$\rho_{nf}=(1-\phi)\rho_f+\phi\rho_s; \qquad \mu_{nf}=\frac{\mu_f}{(1-\phi)^{25/10}} \tag{4}$$

Here, ϕ is nanoparticle volume fraction parameter, f & s subscripts is refer to fluid and solid fraction properties. The properties of thermos-physical base fluid (H_2O) and variety of nanoparticles are given in **Table 1**

The proper boundary conditions for the problem are given by

$$u=U_w(x,t)=\frac{ax}{(1-ct)},\ v=v_w(x,t)=\frac{v_0}{\sqrt{1-ct}},\ C=C_w=C_\infty+\frac{ax}{(1-ct)^2} \text{ at } y=0$$
$$u\to 0,\ C\to C_\infty \text{ as } y\to\infty \tag{5}$$

where, a and $b$constants (where a>0 and c≥0, with ct<1). These two constants have t^{-1} measurement. Additionally, a is the underlying stretching rate and powerful stretching rate $a/(1-ct)$ which is creating with time. The accompanying dimensionless capacities $f(\eta),\phi(\eta)$ are the comparability factors are utilized as:

$$\eta=\left(\frac{a}{\nu_f(1-ct)}\right)^{\frac{1}{2}},\ \psi(x,y)=\left(\frac{\nu_f a}{1-ct}\right)^{\frac{1}{2}} f(\eta),\ \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty} \tag{6}$$

where, $\psi(x,y)$ is the free stream function which fulfils the continuity Eq. (1) With

$$u=\frac{\partial\psi}{\partial y}=\frac{ax}{1-ct}f'(\eta)\ \ v=-\frac{\partial\psi}{\partial x}=-\left(\frac{\upsilon_f a}{1-ct}\right)^{1/2} f(\eta) \tag{7}$$

ExchangeEq's. (6) and (7) in Eqs. (2), (3) and (5), the below ordinary differential equations are getting:

$$\frac{1}{(1-\phi)^{1/2}}f'''(\eta)-\left(1-\phi+\phi\left(\frac{\rho_s}{\rho_f}\right)\right)\begin{Bmatrix} f'^2(\eta)-f(\eta)f''(\eta) \\ +A\left(f'(\eta)+\frac{1}{2}\eta f''(\eta)\right)\end{Bmatrix}-Mf'(\eta)=0 \tag{8}$$

$$A\left(\frac{1}{2}\eta\phi'(\eta)+2\phi(\eta)\right)+f'\phi(\eta)-f\phi'(\eta)=\frac{1}{Sc}\phi''(\eta)-\gamma\phi(\eta) \tag{9}$$

The transformed boundary conditions become

$$f(\eta)=f_w,\ \ f'(\eta)=1,\ \ \phi(\eta)=1 \ \text{ at } \eta=0$$
$$f'(\eta)\to 0,\ \phi(\eta)\to 0 \ \text{ as } \ \eta\to\infty \tag{10}$$

Here, separation as for η in prime demonstrated, $A=c/a$ is the unsteasiness parameter, $M=\sigma B_0^2/a\rho_f$ is the magnetic parameter, Schmidt number is $Sc=\nu/D$, $f(w)=-v_0/\sqrt{v_f a}>0$is the suction parameter, chemical reaction parameter is $\gamma=k_0/b$. Here, γ >0 speaks to the destructive response, γ =0 relates to no response and γ <0 see for the generative response and furthermore Eqs. (7) and (8) alongside limit conditions (10) were understood numerically by applying shooting design. C_f , and Sh_x are given by

$$C_f = \frac{\tau_w}{\rho_f U_w^2} \text{ , } Sh_x = \frac{x q_m}{D(C_w - C_\infty)} \tag{11}$$

Where, $\tau_w = \mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0}$, $q_m = -D\left(\frac{\partial C}{\partial y}\right)_{y=0}$ (12)

Implement the non-dimensional transformations to (11), we obtain

$$C_f \operatorname{Re}_x^{1/2} = \frac{1}{(1-\varphi)} f''(0) \text{ , } Sh \operatorname{Re}_x^{-\frac{1}{2}} = -\phi'(0) \tag{13}$$

where local Reynolds number is $\operatorname{Re}_x = U_w x / \upsilon_f$.

3 Results and Discussion

The nonlinear ODE (8) &(9) subjected to the boundary conditions (10) are calculated numerically by testing shooting technique. Complete numerical calculations are solved for distinct values of magnetic field parameter (M), unsteadiness parameter (A), Suction parameterf_w, nanoparticle volume fraction ϕ with three variouskinds of nanofluids. The numerical results of $f'(\eta)$ and$\phi(\eta)$ distributions as well as $\phi'(0)$ and $C_f \operatorname{Re}_x^{1/2}$ are presented in Figures 2-6.

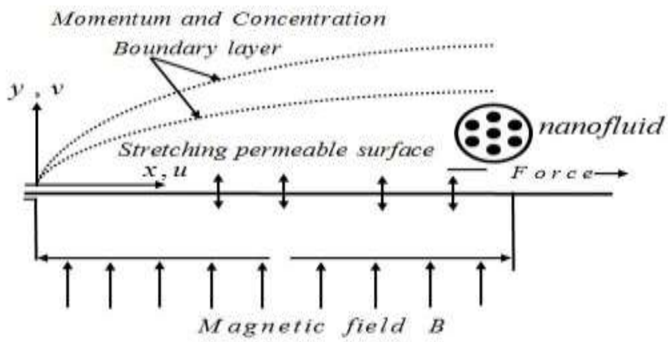

Figure1. The geometry of physical model

The effect of M on $f'(\eta)$ and $\phi(\eta)$are depicted in Figures2(a) and 2(b)individually. It is observed that the $f'(\eta)$ portraitdwindle with ascending of M for different types of nanofluids. Physically, Lorentz force which is a drag like power is made by depending upon infliction of the perpendicular M to the electrically conducting liquid. This force results in dwindle of$\phi(\eta)$of the stream through the extending layer. In addition, M shows same to on concentration distributions. Also, here seen that $\phi(\eta)$ distribution and the velocity boundary layer thickness are larger in case of Cu than to Al_2O_3 nanofluids.

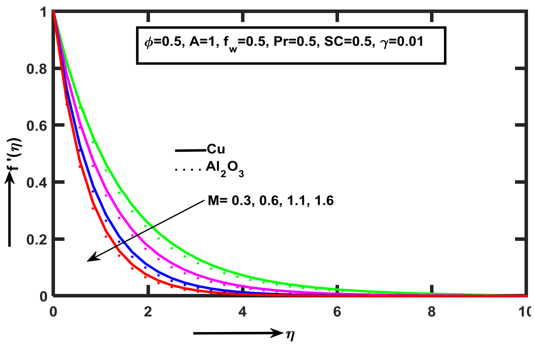

Figure 2(a). Portrait $f'(\eta)$for distant values of M

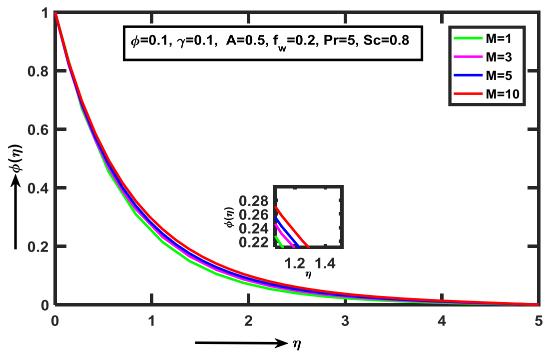

Figure 2(b). Portraiton$\phi(\eta)$for distant values of M

Figures3(a) &3(b) respectively present the consequence of A on $f'(\eta)$&$\phi(\eta)$portrait. It is recognized that, both $f'(\eta)$&$\phi(\eta)$ figures dwindle with raising of A. As the portrait A ascending, less mass is exchanged from the liquid to the layer; hence, the focus dwindles. Since the liquid stream is caused just by the extending region and the territory surface fixation is bigger than free stream focus, the figure $\phi(\eta)$ dwindle with ascending η. Also the fluid $f'(\eta)$&$\phi(\eta)$distributions are more when fluid is away from the wall.

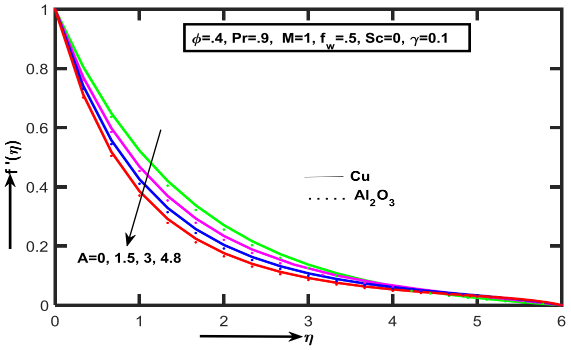

Figure 3(a). Portrait $f'(\eta)$ for distant values of A

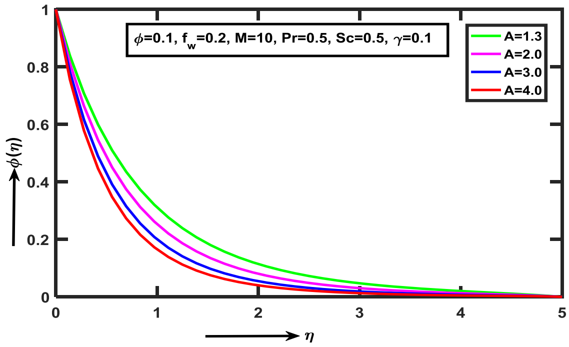

Figure 3(b). Portraiton $\phi(\eta)$ for distant values of A

Figures 4(a), 4(b) &4(c) respectively depict $\phi(\eta)$profile for distinct values of ϕ, Sc and γ. It is recognized that, $\phi(\eta)$ is dwindle with expanding of ϕ, Sc and γ. This is because of certainty that the solutal boundary layer thickness enhances with higher values of ϕ.

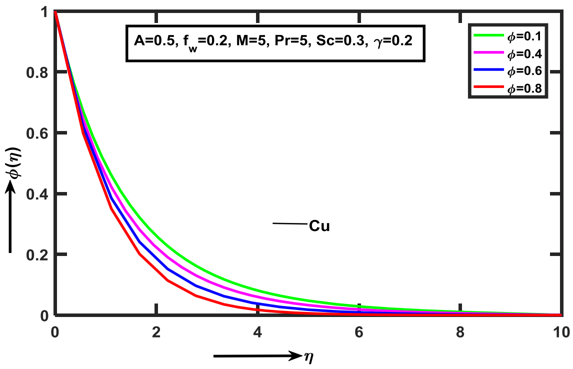

Figure 4(a). Portraiton $\phi(\eta)$ for distant values of ϕ

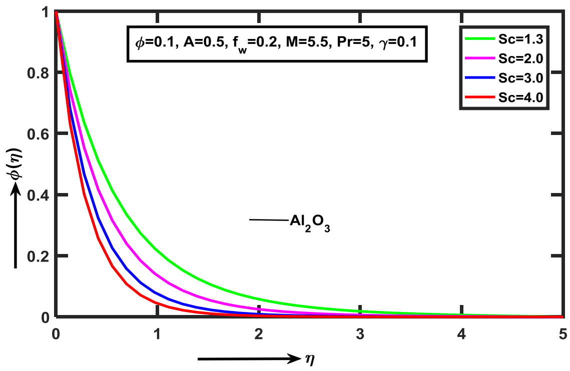

Figure 4(b). Portrait on $\phi(\eta)$ for distant values of Sc

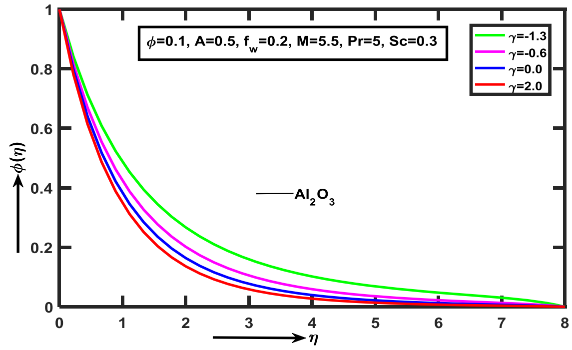

Figure 4(c). Portraiton $\phi(\eta)$ for distant values of γ

Figures 5(a)&5(b) present the response of f_w and ϕ against A on $C_f \operatorname{Re}_x^{1/2}$ respectively. It is observed that $C_f \operatorname{Re}_x^{1/2}$ be dwindle of f_w and ϕ. It is realized that $C_f \operatorname{Re}_x^{1/2}$ is more in case of nanofluid (ϕ =0.1) than to the viscous fluid (ϕ =0). Moreover, $C_f \operatorname{Re}_x^{1/2}$ is bigger in *Cu—H₂O* nanofluid related to the Al_2O_3-H_2O nanofluid.

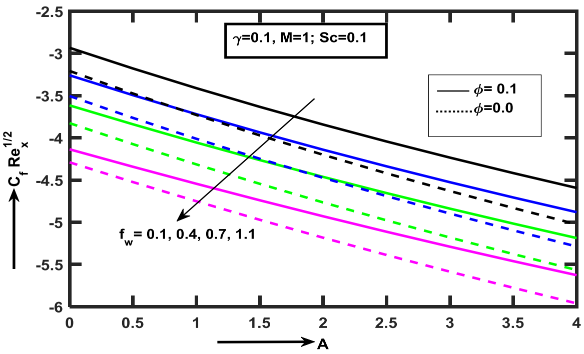

Figure 5(a). Variety of $C_f \operatorname{Re}_x^{1/2}$ against A for distant values f_w

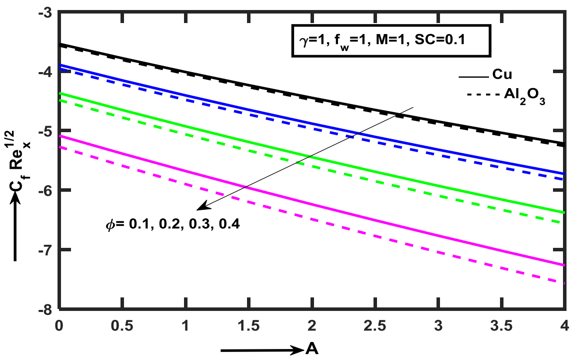

Figure 5(b). Variety of the $C_f \operatorname{Re}_x^{1/2}$ against A for distant values of ϕ

Figures. 6(a) & 6(b) respectively show the reputation of Sc and A on Sherwood number for distinct values of γ. It is recognized that, the $\phi'(0)$ developed with increasing γ as well as Sc and A.

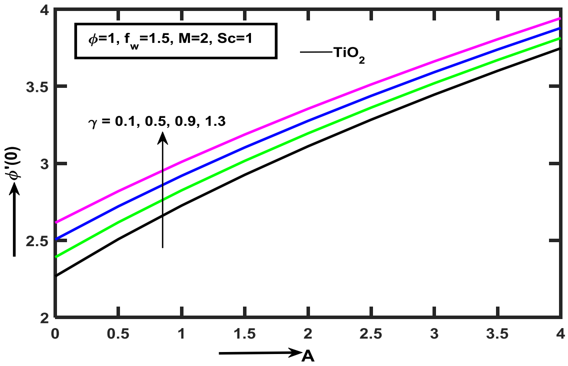

Figure 6(a). Variety of $\phi'(0)$ against A for different values of γ

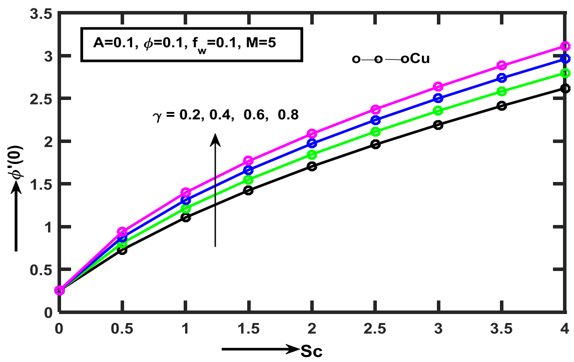

Figure 6(b). Variety of $\phi'(0)$ against Sc for distant values of γ

Table 1. Thermos physical goods of the base liquid and diverse nanoparticles. See ref. **[26]**

Physical properties	Fluid phase(water)	*Cu*	Al_2O_3	TiO_2
C_p(*J/kg K*)	4179	385	765	686.2
P (kg/m^3)	997.1	8933	3970	4250
k (*W/m K*)	0.613	401	40	8.9538

4 Conclusions

The mass move in insure magnetohydrodynamic stream of nanofluids past an extending layer with concoction response is explored numerically for various kinds of nanoparticles. From the present investigation we found that:

- The portrait $f'(\eta)$ distribution reduction with raising the field *M* and *A*.
- The profile $\phi'(\eta)$ decrease for f_w, Sc and *A* for three different kinds of nanofluids (like copper, titanium dioxide, aluminium oxide–H_2Onanofluids).
- By reducing the fields A, ϕ ,*M*, and suction parameter the$C_f \mathrm{Re}_x^{1/2}$decrease in the presence nanofluids.
- The profile $\phi'(0)$ enhances by raising γ for both Cu-H_2O and TiO_2– H_2Onanofluids.

References

[1] Mabood F, MastroberardinoA 2015 *J. Taiwan Inst. Chem. Eng.***57,** 62-68
[2] Haq RU, Khan ZH, Hussain ST, Hammouch Z 2016*J. Mol. Liq*. **221**, 298–304
[3] Nadeem S,Riaz S, Ellahi R, Akbar NS 2014*Appl. Nanoscience* **4**, 393–404
[4] Venkateswarlu B, Satya Narayana P V 2015*Appl. Nanoscience* **5,**351–360
[5] Khan M,Azam M, Alshomrani AS 2017 *J. Mol .Liq*. **231,**474-484
[6] Bég O.A, Khan MS, Karim I, AlamMd M,Ferdows M 2014*Appl. Nanoscience* **4**No. 8943-957
[7] Kumar Nandy S, Sidui S, Mahapatra T R 2014 *Alexandria Eng. J*. **53,** 4929-937
[8] Freidoonimehr N, Rashidi M M, Mahmud S 2015 *Int. J. Thermal Sciences***87,** 136-145
[9] Zin NAM, Khan I, Shafie S, Alshomrani A S 2017 *Results Phys*.**7**, 288–309
[10] Pandey AK, Kumar M 2017 *Alexandria Eng. J*.
[11] Satya Narayana PV, Venkateswarlu B, Venkataramana S 2015*Heat Transfer Asian Res*.**44**,1, 1-19
[12] Sheikholeslami M, GorjiBandpy M, Ellahi R, Hassan M, Soleimani S 2014 *J. Magnet. Magn. Mater*. **349,** 188–200
[13] Sheikholeslami M, Gorji-Bandpy M,Ganji DD, Rana P, Soleimani S 2014 *Comput. Fluids***94,** 147–160
[14] Harish Babu D, Satya Narayana P V 2016 *J. Magnet.Magn Materials* **412**, 185–193
[15] Chamkha A J, Aly A M, Mansour M A 2010*Chem. Eng. Commun*. **197,** 846-858
[16] Manjula J, Padma P, Gnaneswara Reddy M, Venakateswarlu M 2015 *Procedia Eng*. **127**,1315 – 1322
[17] Mishra S R, Baag S, Mohapatra D K 2016*Eng. Science Tech. Int. J*. **19,** 1919–1928
[18] Satya Narayana P V, Harish Babu D 2016*J. Taiwan Inst. Chem. Eng*. **59,** 18–25
[19] Daba M, Devaraj P 2016*J.Nigerian MathSoci*. **35**, 245–256
[20] Harish babu D, Venkateswarlu B, Satya Narayana PV 2017 *Frontiers heat mass transfer* **8**, 5
[21] Harish Babu D, Satya Narayana P V 2013*ISRN Thermodynamics* **2013,**1-17
[22] Akbar NS,Raza M, Ellahi R 2016 *Appl. Nanoscience***6,** 3,359–370

[23] Matausek M R 1973*J. Optimization Theory Appl*. **12,** 2, 152-172
[24] Alizadeh-Pahlavan A, Sadeghy K 2009Commun. *Nonlinear Sci. Numer. Simulat*. **14,** 1355-1365
[25] Brinkman H C 1952 *J. Chem. Phys.***20**,571
[26] Abolbashari M H, Freidoonimehr N, Nazari F, Rashidi M M 2014 *Powder Tech.***267**,256-267