IOP Conf. Series: Materials Science and Engineering **263** (2017) 042114 doi:10.1088/1757-899X/263/4/042114

Optimizing fuzzy manufacture inventory models for non-deteriorating items with an invariable demand rate and completely backlogged

P Pandian and M Dhivya Lakshmi
Department of Mathematics, School of Advanced Sciences, VIT University, Vellore-632014, India

E-mail: dhivyalakshmi.m2015@vit.ac.in

Abstract. In this paper, a production inventory model for non-deteriorating items with a constant demand rate and completely backlogged in a fuzzy environment is presented. The production rate of the model is assumed to be a constant and to be proportional to demand rate. Each cycle of the developed model is considered in four different types of situations. The optimal average fuzzy total inventory cost of a cycle and optimal fuzzy time for each situation are obtained. Numerical example is presented to illustrate the proposed model.

1. Introduction

Economical valuable items that are detained by an organization for future use can be referred as an inventory. An inventory system in an organization is the set of policies and controls that monitor levels of inventory, so as to optimize the total cost and to guarantee to run the business smoothly, effectively and beneficially. In last few decades, mathematical models have been developed for different areas of actual situations, particularly for inventory control. The modeling of an inventory problem is based on the information obtained from an organization which has several parameters. In general, four kinds of inventory control models based on demand are considered namely, (a) constant demand (b) time-dependent demand (c) probabilistic demand and (d) stock-dependent demand. In addition, when the shortages occur in an organization, it is assumed that it is completely backlogged, partially backlogged, partially lost or completely lost. But in real life situations, some clients desire to wait for back order and others would turn to purchase from other organizations. In the literature, a number of authors have studied various types of crisp inventory models with partial or complete backlogging.

Many researchers have studied different kinds of crisp inventory control problems under various conditions. Valliathal and Uthayakumar [16] discussed the manufacture inventory model with time condition on production and demand. Tripathi and Neha Sang[14] developed an EOQ model for invariable demand rate with fully backlogged and also, they optimized the average total inventory cost of the inventory system.

In real life situation, all information obtained from the organization may not be certain also, not necessary to follow a theoretical distribution. Because of this reason, some or all parameters in the inventory model may be imprecise. In such case, an inventory model in uncertain environment takes an important place. In recent years, inventory models in the fuzzy environment have received much attention since it provides more information than crisp models and handles the imprecise data. The

Content from this work may be used under the terms of the Creative Commons Attribution 3.0 licence. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.
Published under licence by IOP Publishing Ltd

theory of fuzzy sets introduced by Zadeh [19] helps to study fuzzy inventory models since it has an ability to quantify vagueness and imprecision without using randomness.

The analysis of an inventory control model in the fuzzy environment was initiated by Lee and Yao[6]. Following Lee and Yao[6], many researchers have studied various classes of fuzzy inventory models. A periodic review fuzzy inventory model with variable lead time was studied by Yu-Jen Lin[18]. Mandal et al.[9] optimized a manufacture inventory model with fuzzy period and fuzzy costs for faulty items under fuzzy space constraint. A full-fuzzy inventory control model was discussed by Kazemi et al.[5]. Liang-YuhOuyanget al.[7] studied a fuzzy inventory control model for deteriorating items under vendor credits linked to ordering quantity. A continuous production inventory model for deteriorating items in the fuzzy environment was analyzed by Nagoor Gani and Palaniammal[11].Bharat Chede et al.[1] proposed an approach for solving fuzzy inventory control problems based on the currently available information. A fuzzy inventory control model for deteriorating items with shortages was discussed by Dutta and PavanKumar[2].Ranganathan and Thirunavukarasu[12] analyzed a fuzzy inventory model for logarithmic demand rate and static deterioration items under fully backlogged condition. An inventory model in the fuzzy environment for deteriorating items with variable demand and shortages was studied by Harish Nagar and Priyanka Surana[4].Majumderet al.[8] analyzed production quality models with crisp and fuzzy demand for deteriorating items under partial trade credit policy. A fuzzified economic order quantity model having shortage and deterioration was optimized by Mishra et al.[10].Uthayakumar and Karuppasamy [15]have determined the economic order quantity in a Harris - Wilson inventory model where all cost parameters are fuzzy.

This article is presented as follows: In Section 2, real intervals and its arithmetic operations, fuzzy set theory and the DSW algorithm are given. A crisp production inventory model for non-deterioration items with an invariable demand rate and backlogs is considered and optimized in Section 3. In Section 4, a fuzzy production inventory model with an invariable demand rate and backlogs where deterioration rate and lead time are fuzzy zero, the production rate is propositional to demand rate is discussed and optimized. In Section 5, a numerical example for the developed fuzzy production inventory model is given and optimal fuzzy times and the average fuzzy total inventory cost of the inventory system are computed using DSW algorithm and finally, the conclusion is given in Section 6.

The proposed model can be applied to an industry where the production starts after collecting orders from the buyers. This model finds applications in Printing industries, Oil industries, Food Product industries, Fireworks industries, etc.

2.Preliminaries

We use the following basic definitions on interval theory and fuzzy set theory in this paper which can be found in George J. Klirand Bo Yuan[3].

Let I denote the set of all closed bounded intervals on the real line R. That is, $I = \{ [a,b] : a \le b, a \text{ and b are in } R \}$.

Definition 2.1:

Let $A = [a_1, a_2]$ and $B = [b_1, b_2]$ be in I. Then,

(i) $A \oplus B = [a_1 + b_1, a_2 + b_2]$;

(ii) $kA = [ka_1, ka_2]$ if k is a positive real number;

(iii) $A \otimes B = [p, q]$, where $p = \min\{a_1b_1, a_1b_2, a_2b_1, a_2b_2\}$ and $q = \max\{a_1b_1, a_1b_2, a_2b_1, a_2b_2\}$;

(iv) $\frac{1}{B} = \left[\frac{1}{b_2}, \frac{1}{b_1}\right]$, where $0 \notin [b_1, b_2]$ and

(v) $\frac{A}{B} = A \otimes \left(\frac{1}{B}\right)$, where $0 \notin [b_1, b_2]$.

Definition 2.2:
Let $A = [a_1, a_2]$ and $B = [b_1, b_2]$ be in *I*. Then,
(i) $A \leq B$ if $a_1 \leq b_1$ and $a_2 \leq b_2$ and (ii) $A = B$ if $a_1 = b_1$ and $a_2 = b_2$.

Definition 2.3:
Let $A = [a_1, a_2] \in I$. Then, A is said to be positive denoted by $A \geq 0$ if $a_1 \geq 0$.

Definition 2.4:
If $A = [a_1, a_2] \geq 0$, the square root of A, $\sqrt{A}$ is defined as, $\sqrt{A} = [\sqrt{a_1}, \sqrt{a_2}]$.

Definition 2.5:
Let Y be a subset of a classical non-empty set X and a set $A \subseteq X$. A fuzzy set $\tilde{Y}$ with the membership function $\phi(x)$ is defined as $\tilde{Y} = \{ (x, \phi(x)) : x \in X \}$ where $\phi(x)$ is a function from X to [0,1].

Definition 2.6:
The α-cut of a fuzzy set $\tilde{Y}$ where $\alpha \in [0,1]$ is defined as the set $\left[\tilde{Y}\right]^{\alpha} = \{x : \phi(x) \geq \alpha, x \in X\}$. It is also represented an interval as $\left[\tilde{Y}\right]^{\alpha} = [Y^L, Y^U]$, $\alpha \in [0,1]$ where Y^L and Y^U are the lower and upper bounds of the closed interval.

Definition 2.7:
A fuzzy number $\tilde{a}$ is a triangular fuzzy number represented as (p, q, r) where p, q and *r* are real numbers and its membership function $\phi_{\tilde{a}}(x)$ is given below.

$$\phi_{\tilde{a}}(x) = \begin{cases} (x-p)/(q-p) & : \text{for } p \leq x \leq q \\ 1 & : \text{for } x = q \\ (r-x)/(r-q) & : \text{for } q \leq x \leq r \\ 0 & : \text{otherwise} \end{cases}$$

Definition 2.8:
The α-cut of $\tilde{a} = (p, q, r)$ is given as $\left[\tilde{a}\right]^{\alpha} = [p + \alpha(q-p), r - \alpha(r-q)]$, where $\alpha \in [0,1]$

The DSW algorithm introduced by Timothy Ross [13] is an approximate method which is used in defining membership function of a fuzzy set. It is based on the interval analysis that a fuzzy set having continuous membership function can be represented by a series of an α-cut interval, $\alpha \in [0,1]$. The solution procedure of the DSW algorithm are as follows:

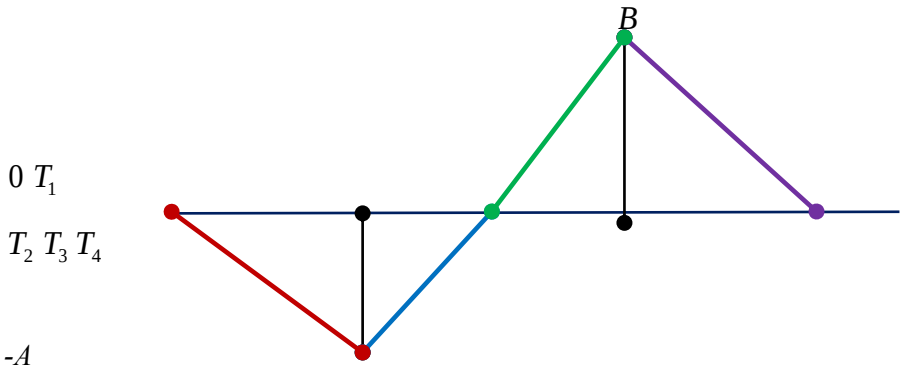
1. Select a α-value where $\alpha \in [0,1]$.
2.Find the interval(s) for the input membership function(s) that correspond to the selected α.
3. Using interval the or compute the interval for the output membership function.
4. Repeat Step 1to Step 3foreach value of α, $\alpha \in [0,1]$ to complete the

α -cut of the fuzzy output.

3. Crisp inventory models with an invariable demand rate and backlogs

Now, we consider a crisp manufacture inventory model in which demand rate D is invariable, items are non-deteriorated, the production rate is $P=\lambda_0 D$ where $\lambda_0(>1)$ is a constant., lead time is zero and shortages are permitted and are totally backlogged. Let C_1 be the carrying cost per unit per unit time, C_2 be the shortage cost per unit per unit time and C_3 be the setup cost per production run. Let us assume that C_1, C_2 and C_3 are known and unchanged during a production cycle.

At the time $t=0$, the inventory level is zero. The shortage begins at $t=0$ and accumulates up to the level A at time $t=T_1$. The production be gins at $t=T_1$ and backlogs are totally cleared at $t=T_2$. The stock - level reaches the level B at $t=T_3$. The production stops at level $t=T_3$. Then, the inventory level decreases steadily due to demand and reaches zero at $t=T_4$. The production cycle is ended at $t=T_4$ and it repels itself. Let $I(t)$ be the inventory level at any time t $(0\le t\le T_4)$.



Manufacture inventory model within variable demand rate and backlogs

This above said crisp production model was studied in [14] and the optimal average total inventory cost (C^*), the first stage optimal time (T_1^*), the second stage optimal time (T_2^*), the third stage optimal time (T_3^*), and optimal time for the fourth stage (T_4^*) were obtained as given below:

$$C^* = \frac{D(\lambda_0 - 1)\left[C_2 T_2^{*2} + C_1\left(T_4^* - T_2^*\right)^2\right] + 2\lambda_0 C_3}{2\lambda_0 T_4^*}; \quad (1)$$

$$T_2^* = \sqrt{\frac{2\lambda_0 C_1 C_3}{C_2 D(C_1 + C_2)(\lambda_0 - 1)}}; \quad (2)$$

$$T_4^* = \frac{(C_1 + C_2)}{C_1} T_2^*; \quad (3)$$

$$T_1^* = \frac{(\lambda_0 - 1)T_2^{**}}{\lambda_0}; \quad (4)$$

and

$$T_3^* = \frac{(\lambda_0 - 1)T_2^* + T_4^*}{\lambda_0}. \tag{5}$$

4. Fuzzy Inventory Models with constant demand and shortages

Consider a fuzzy inventory model where fuzzy demand rate $\tilde{D}$ is invariable, items are non-deteriorated, production rate $\tilde{P} = \lambda_0 \tilde{D}$ where $\lambda_0 (>1)$ is a crisp constant, lead time is zero and shortages are permitted and are totally backlogged. Let $\tilde{C}_1$, $\tilde{C}_2$ and $\tilde{C}_3$ be the fuzzy carrying cost per unit per unit time, the fuzzy shortage cost per unit per unit time and the fuzzy setup cost per production run respectively. Let us assume that the fuzzy quantities $\tilde{C}_1, \tilde{C}_2$ and $\tilde{C}_3$ are known and unchanged during a production cycle.

At the fuzzy time $\tilde{t} = \tilde{0}$, the inventory level is a fuzzy zero. The shortage begins at $\tilde{t} = \tilde{0}$ and accumulates up to the level $\tilde{A}$ at the fuzzy time $\tilde{t} = \tilde{T}_1$. The production begins at $\tilde{t} = \tilde{T}_1$ and the backlog is totally cleared at the fuzzy $\tilde{t} = \tilde{T}_2$. The stock - level reaches the level $\tilde{B}$ at the fuzzy time $\tilde{t} = \tilde{T}_3$. The production stops at $\tilde{t} = \tilde{T}_3$. Then, the inventory level decreases steadily due to demand and becomes zero at the fuzzy time $\tilde{t} = \tilde{T}_4$. The cycle is ended and repels itself. Let $\tilde{I}(\tilde{t})$ be the fuzzy inventory level at any fuzzy time $\tilde{t}$ $\left(0 \le \tilde{t} \le \tilde{T}_4\right)$.

Now, by DSW algorithm and using (1) to (5) in the Section 3., the optimal values of $\tilde{T}_1^*$, $\tilde{T}_2^*$, $\tilde{T}_3^*$, $\tilde{T}_4^*$ and $\tilde{C}^*$ can be determined using the following α-cut formulae:

For $0 \le \alpha \le 1$,

$\left[\tilde{T}_2^*\right]^\alpha = \left[T_2^{*L}(\alpha), T_2^{*U}(\alpha)\right]$

where

$$T_2^{*L}(\alpha) = \sqrt{\frac{2\lambda_0 C_1^L(\alpha) C_3^L(\alpha)}{(\lambda_0 - 1)C_2^U(\alpha) D^U(\alpha)\left(C_1^U(\alpha) + C_2^U(\alpha)\right)}} \text{ and}$$

$$T_2^{*U}(\alpha) = \sqrt{\frac{2\lambda_0 C_1^U(\alpha) C_3^U(\alpha)}{(\lambda_0 - 1)C_2^L(\alpha) D^L(\alpha)\left(C_1^L(\alpha) + C_2^L(\alpha)\right)}}.$$

$$\left[\tilde{T}_1^*\right]^\alpha = \left[\left(\frac{\lambda_0 - 1}{\lambda_0}\right)T_2^{*L}, \left(\frac{\lambda_0 - 1}{\lambda_0}\right)T_2^{*U}\right];$$

$$\left[\tilde{T}_4^*\right]^\alpha = \left[\frac{\left(C_1^L + C_2^L\right)T_2^{*L}(\alpha)}{C_1^U}, \frac{\left(C_1^U + C_2^U\right)T_2^{*U}(\alpha)}{C_1^L}\right];$$

$$\left[\tilde{T}_3^*\right]^\alpha = \left[\frac{(\lambda_0 - 1)T_2^{*L}(\alpha) + T_4^{*L}(\alpha)}{\lambda_0}, \frac{(\lambda_0 - 1)T_2^{*U}(\alpha) + T_4^{*U}(\alpha)}{\lambda_0}\right];$$

and $\left[\tilde{C}^*\right]^\alpha = \left[C_1^{*L}, C_2^{*U}\right]$, where

$$C_1^{*L} = \frac{D^L(\lambda_0 - 1)\left[C_2\left(T_2^{*L}\right)^2 + C_1\left(T_4^{*L} - T_2^{*U}\right)^2\right] + 2\lambda_0 C_3^L}{2\lambda_0 T_4^{*U}} \text{ and}$$

$$C_1^{*U} = \frac{D^U(\lambda_0 - 1)\left[C_2\left(T_2^{*U}\right)^2 + C_1\left(T_4^{*U} - T_2^{*L}\right)^2\right] + 2\lambda_0 C_3^U}{2\lambda_0 T_4^{*L}}.$$

Now, since the performance measures are described by series of α-cut intervals, the optimal average fuzzy total inventory cost and the optimal fuzzy time variable at each stage are fuzzy sets. However, the practitioners would like only crisp value for the inventory system characteristics rather than a vague value. For defeating this issue, the fuzzy values of the system characteristics are transformed into crisp values by using defuzzified methods. The crisp/defuzzified value of a fuzzy set $\tilde{a}$, $O(\tilde{a})$ by Yager's ranking index method [17] is given below:

$$O(\tilde{a}) = \int_0^1 \left(\frac{a^L(\alpha) + a^U(\alpha)}{2}\right) d\alpha;$$

where $[\tilde{a}]^\alpha = [a^L(\alpha)\ \ a^U(\alpha)]$, $0 \le \alpha \le 1$.

Now, $O(\tilde{a})$ can also be, calculated using the following summation formula**:**

$$O(\tilde{a}) = \sum_{\alpha=0}^{1} \left(\frac{a^L(\alpha) + a^U(\alpha)}{2}\right) \Delta\alpha, \qquad (6)$$

where $\Delta\alpha$ is the width of the each subinterval of the interval $[0,1]$ and it is very close to zero.

Using MATLAB software, all system characteristics and its defuzzified values can be computed.

5. Numerical example

Consider a fuzzy production inventory model in which demand rate is $\tilde{D} = (100,105,110)$, the production rate is $\tilde{P} = \lambda_0 \tilde{D}$ where $\lambda_0 = 1.8$ is a constant and shortages are permitted and are backlogged. Let $\tilde{C}_1 = (15,20,25)$, $\tilde{C}_2 = (25,30,35)$ and $\tilde{C}_3 = (35,40,45)$ also, let us assume that $\tilde{C}_1$, $\tilde{C}_2$ and $\tilde{C}_3$ are fixed during a production cycle. Now, using MATLAB software, the optimal fuzzy time for each location an interval and optimal average fuzzy total inventory cost as an interval for $\alpha = 0(0.1)1$, are computed and they are summarized below in a table form:

Time		α-*value*										
		0	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1.0**
$\left[\tilde{T}_1^*\right]^\alpha$	**L**	0.045	0.047	0.049	0.0508	0.053	0.055	0.057	0.060	0.062	0.065	0.067
	U	0.100	0.096	0.092	0.0886	0.085	0.082	0.079	0.076	0.073	0.070	0.067
$\left[\tilde{T}_2^*\right]^\alpha$	**L**	0.101	0.105	0.110	0.1144	0.119	0.124	0.129	0.134	0.140	0.145	0.151
	U	0.225	0.216	0.208	0.1994	0.192	0.184	0.177	0.170	0.164	0.157	0.151
$\left[\tilde{T}_3^*\right]^\alpha$	**L**	0.135	0.145	0.156	0.1671	0.180	0.193	0.207	0.223	0.240	0.258	0.277
	U	0.600	0.553	0.510	0.4714	0.436	0.405	0.374	0.341	0.322	0.298	0.277
$\left[\tilde{T}_4^*\right]^\alpha$	**L**	0.162	0.176	0.192	0.2093	0.228	0.248	0.270	0.294	0.319	0.347	0.378
	U	0.900	0.823	0.752	0.6889	0.631	0.579	0.531	0.488	0.448	0.411	0.378
$\left[\tilde{C}^*\right]^\alpha$	**L**	46.7	51.5	57.3	64.4	73.16	84.21	98.31	116.48	140.09	171.00	211.66
	U	2956.1	2212.7	1662.9	1255.2	952.11	726.26	557.58	431.33	336.62	265.36	211.66

Where L is lower value and U is the upper value of the interval**.**

Now, using MATLAB software, the graph of the optimal average fuzzy total inventory cost (Fig.1) and the graph of the optimal fuzzy time for each stage (Fig.2) are given below:

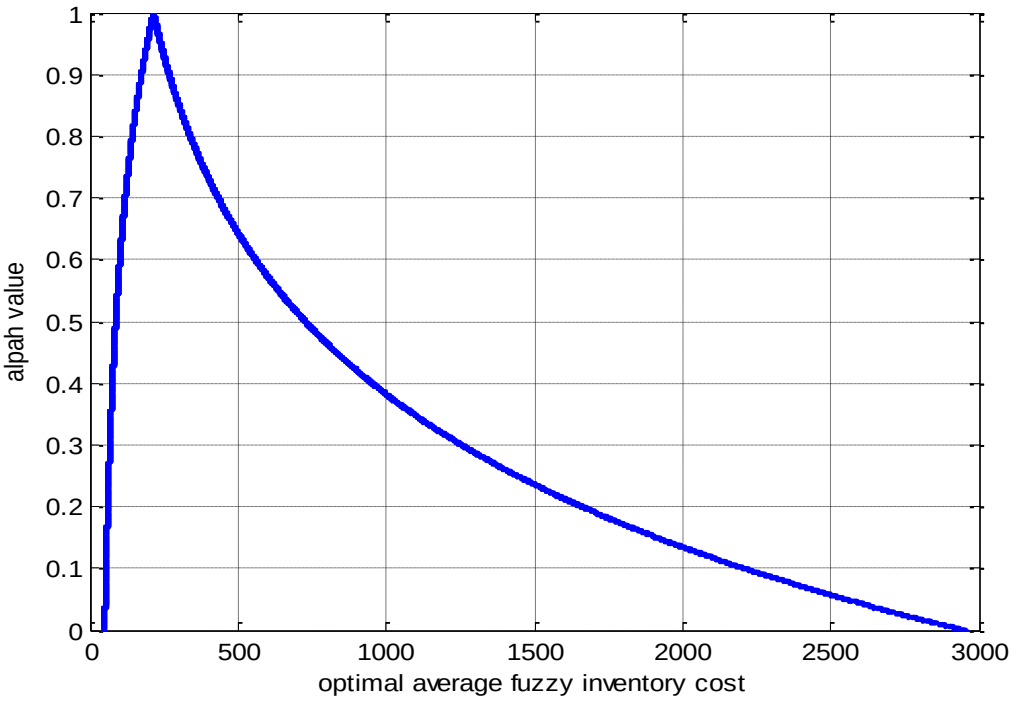


Figure 1.

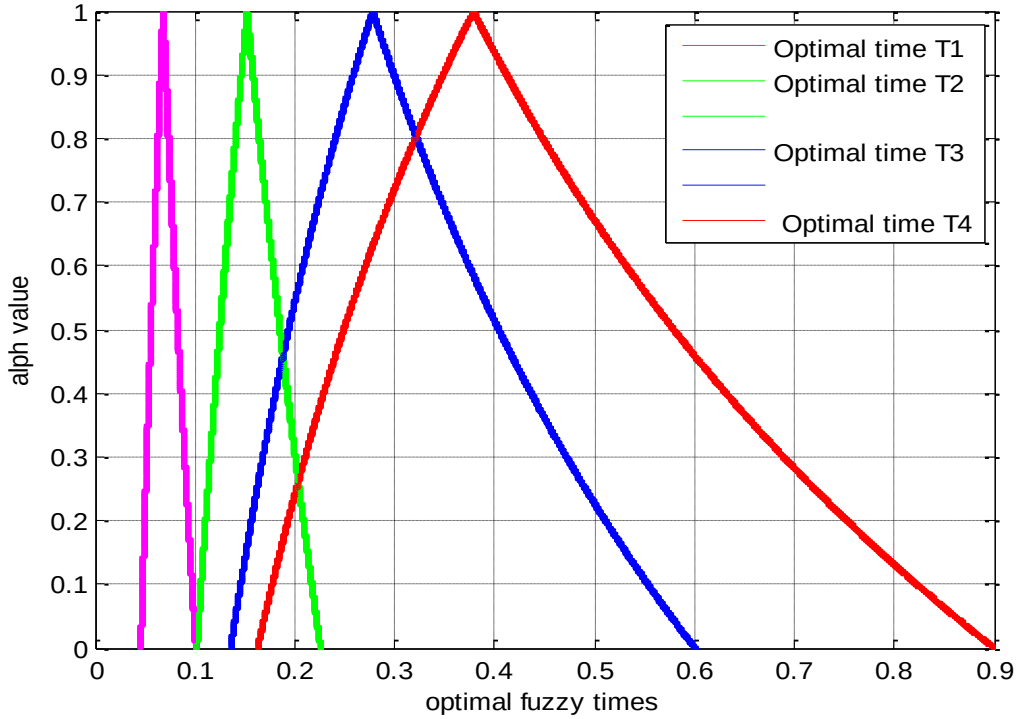


Figure 2.

Now, using (6), the crisp/ defuzzified values of $\tilde{T}_1^*$, $\tilde{T}_2$, $\tilde{T}_3^*$, $\tilde{T}_4^*$ and $\tilde{C}^*$ are given below:

$O(\tilde{T}_1) = 0.069$; $O(\tilde{T}_2) = 0.155$; $O(\tilde{T}_3) = 0.306$; $O(\tilde{T}_4) = 0.427$ and $O(\tilde{C}) = 545.766$.

6. Conclusion

A fuzzy manufacture inventory model has been discussed where the demand rate of a product is invariable, the rate of production is larger than demand rate and is proportional to the rate of demand, shortages are backlogged, the replenishment rate is infinite and items are non-deteriorating in this paper. In the model, a cycle has been divided into four different stages. The optimal average fuzzy total inventory cost of a cycle and optimal fuzzy time for each stage for the developed production model are obtained. The defuzzified values of the system characteristics have been computed using Yager's ranking index method. Numerical example has been presented to illustrate the developed fuzzy production inventory model.

References

[1] BharatChede, Jain C K, Jain S K andAparnaChede2012*Ind Eng Lett* **2** 13-21
[2] Dutta D and Pavan Kumar 2013 *Int J Soft ComputEng* **3** 393-398
[3] George J Klir and Bo Yuan 2008*Fuzzy Sets and FuzzyLogic: Theory and Applications*(New Jersey : Prentice-Hall)
[4] HarishNagar andPriyankaSurana 2015*J Comput MathSci* **6** 55-66
[5] Kazemi N, Ehsani E and Jaber M Y 2010 *Int J ApprReason* **51** 964-972
[6] Lee H M and Yao J S 1998*Eur J OperRes***109**203-211
[7] Liang-YuhOuyang, Jinn-TsairTeng and Mei-Chuan Cheng 2010 *JInfSci Eng***26** 231-253
[8] Majumder P,Bera U K and Maiti M 2015*Procedia ComputSci* **45** 780-789
[9] Mandal S, Maity K,Mondal S and Maiti M 2009*Appl Math Model* **34** 810–822
[10] Mishra S S, Gupta S,Yadav S K and Rawat S 2015*Am J Oper Res.***5**103-110

[11] NagoorGani A and Palaniammal P 2011*Appl MathSci* **5** 233 - 241
[12] Ranganathan V andThirunavukarasu P 2010 *Int J Fuz Math and Syst* **4**17–26
[13] Timothy J Ross 2010 *Fuzzy Logic with Eng. App.* (Chichester : A John Wiley and Sons Ltd)
[14] Tripathi R P and Neha Sang 2012 *Appl Comput Math* **1** 1-4
[15] Uthayakumar R andKaruppasamy S K 2016 *Oper Res Appl : Int J* **3**17-29
[16] Valliathal M and Uthayakumar R 2010 *Appl MathSci* **4** 289 – 304
[17] Yager R R1981 *InfSci* **24**143-161
[18] Yu-Jen Lin 2007*ComputInd Eng* **54** 666–676
[19] Zadeh L A 1965 *Information and Control* **8**338-353