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Geometric Modeling of Construction Communications with Specified Dynamic Properties

V A Korotkiy, E A Usmanova, L I Khmarova

Department of Engineering and Computer Graphics, South Ural State University, 76, Lenin Ave., Chelyabinsk 454080, Russia

E-mail: korotkiiva@susu.ru

Abstract. Among many construction communications the pipelines designed for the organized supply or removal of liquid or loose working bodies are distinguished for their functional purpose. Such communications should have dynamic properties which allow one to reduce losses on friction and vortex formation. From the point of view of geometric modeling, the given dynamic properties of the projected communication mean the required degree of smoothness of its center line. To model the axial line (flat or spatial), it is proposed to use composite curve lines consisting of the curve arcs of the second order or from their quadratic images. The advantage of the proposed method is that the designer gets the model of a given curve not as a set of coordinates of its points but in the form of a matrix of coefficients of the canonical equations for each arc.

1. Introduction

Among many construction communications, pipelines designed for the organized supply or removal of liquid or loose working bodies are distinguished for their functional purpose. These are water supply and drainage networks, steam pipelines, gas-oil pipelines, gravity pipes of round or rectangular cross-section for feeding grain and flour in granaries and elevators etc. Such communications must have certain geometric properties that allow transportation of the working fluid with minimal losses to friction and vortex formation. For example, it is inadmissible to use angular transitions in channels intended for the transport of bulk bodies, since a stagnation zone appears in the corners. The centerline of such a channel must not contain kinks. From a geometric point of view, this means that the centerline must have a first (minimal) degree of smoothness [1,2].

More significant requirements are imposed on pipelines providing the supply of a working fluid under pressure to the executive body. In these cases, it is not sufficient to ensure the absence of kinks in the center line, and it is required to exclude abrupt changes in the curvature of the streamlines. The channel axis must have an improved smoothness (second-order smoothness). Geometrically, this means that the second derivative of the center line along its entire length should change without jumps [3].

In some cases, it is not enough to ensure the smoothness of the second order. For example, special overpasses (sledge-bobsled trails, expressways, etc.) are designed in such a way as to ensure a smooth change of the second derivative along the center line of the route. In this case, the graph of the second derivative along the axis does not contain jumps or breaks. Such an overpass has third-order smoothness [4,5].

From the point of view of geometric modeling, the given dynamic properties of the projected communication mean the required degree of smoothness of its center line. Obviously, the technique of designing communication depends on the requirements imposed on its dynamic properties.

2. Relevance
The provision of the specified dynamic properties of building communications takes on special significance in conditions of energy saving requirements. The correct choice of the geometric parameters of the channels for supplying the working fluid ensures minimization of transport losses for friction, vortex formation and heat transfer.

3. Formulation of the problem
In the plane or in space, a number of reference points are set through which the axial line of the projected channel or overpass must pass. It is required to design a plane or spatial centerline of the channel passing through these points and having the given dynamic properties. The projected curve can be composed of sections of algebraic curves docked together with the given degree of smoothness. Such a compound curve is called the dynamic contour [6,7]. In contrast to the known computational methods, in this paper it is proposed to use arcs of curves of the second order or their quadratic images to design dynamic contours. The advantage of the proposed method is that the designer gets the model of the given curve not as the set of coordinates of its points, but in the form of a matrix of coefficients of the canonical equations of each arc.

4. Geometric conditions for conjugation of arcs of curves of the second order
Two curves of the second order (CSO) intersect at four points (real or imaginary, different or coincident). When two intersection points coincide, a common tangent appears on the curves. We obtain a first-order dynamic connection of smoothness. The coincidence of the three intersection points means the presence at this point of a common circle of curvature and a connection of the second order of smoothness [8].

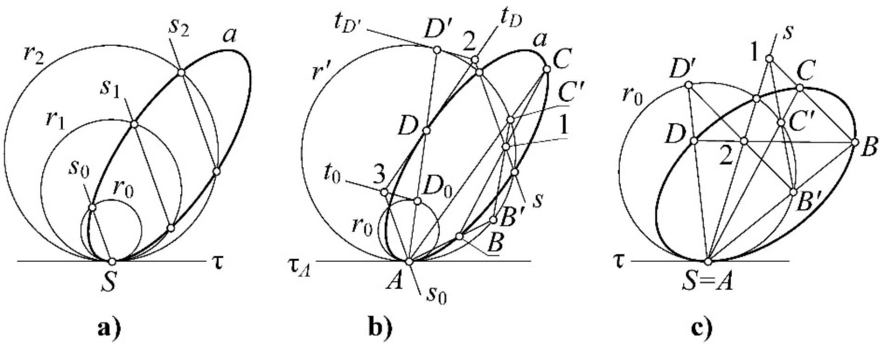

Figure 1. Smooth connection of the conic sections.

Let the parabolic bundle of circles with the base point S and the axis τ be indicated. Draw an arbitrary conic a that has a contact at the point S with the axis τ (figure 1, a). The conic a is connected by homological correspondence with any circle r_i of the bundle. The homology axes s_i are mutually parallel. Note that some circle r_0 from the bundle r_i is a circle of curvature of the conic a at the point S. The axis s_0 of the homology connecting the conic a and the circle r_0, passes through the center S. Let the conic a be given by the points A, B, C, D and the tangent t_D at the point D (figure 1, b). It is required to build at the point A the circle of curvature r_0. Through the point A an arbitrary cirlce r' is drawn, having with the conic a a common tangent τ_A. The conics r' and a are related by homology to the center A. Find the axis s of the homology $r'{\sim}a$, using the relevant points $B{\sim}B'$, $C{\sim}C'$, $D{\sim}D'$. The axis s passes through the points $1{=}BC{\cap}B'C'$ and $2{=}t_D{\cap}t_{D'}$, where $t_{D'}$ – is the tangent to the r' at the point D'. Consider the homology connecting the conic a with the circle of curvature r_0. The axis s_0

passes through the point A, and the lines s_0 and s are parallel. The circles r', r_0 form a homology with an improper axis, so the lines t_0 and $t_{D'}$ are parallele. Here t_0 – is the tangent to the circle of curvature at the point D_0 (the points D_0 and D correspond to each other in the homology $a{\sim}r_0$). The tangent t_0 passes through the point $3{=}t_D{\cap}s_0$ and intersects the line DA at the point D_0. The required circle of curvature is given by points A, D_0 and the tangent τ_A.

Let the conic a be given by the points A, B, C. At the point A, the circle of curvature r_0 is indicated (figure 1, c). It is required to construct a conic and determine its principal axes. The axis s of the homology $a{\sim}r_0$ passes through the center $a{\sim}r_0$ and the point $1{=}\ BC{\cap}B'C'$. Mark an arbitrary point D' on r_0 and find the corresponding point D. The desired conic is completely determined by the points A, B, C, D and the tangent τ at the point A.

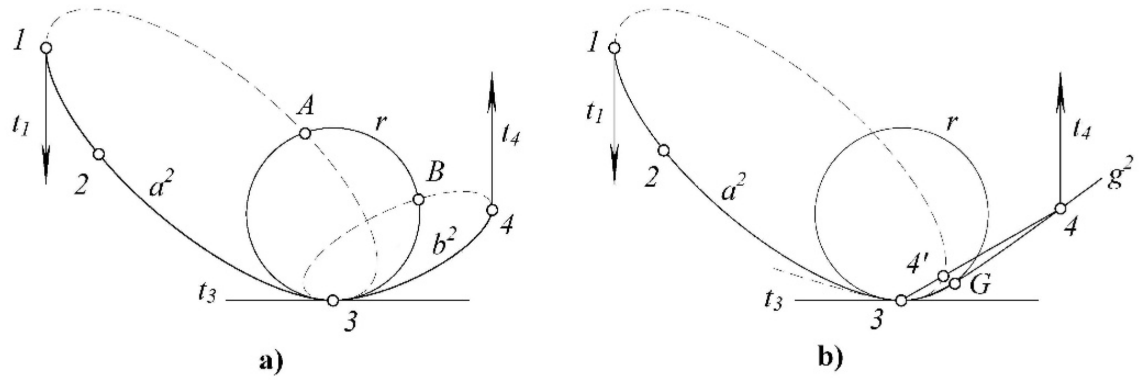

Figure 2. The dynamic compound curves of the second order.

Let us consider the construction of a dynamic contour with continuously varying curvature. Let the first section be given by points 1, 2, 3 and tangents t_1, t_3. The second section is given by point 4 and tangent t_4 (figure 2, a). At point 3, it is required to provide a dynamic connection of the second order of smoothness. Through points 1, 2, 3 there passes a unique conic a satisfying the conditions of contact with lines t_1, t_3. Using the algorithm for solving the analysis problem, we find the circle of curvature r at point 3. The second section b is given by the same circle of curvature r, point 4 and tangent t_4. Solving the synthesis problem, we find the conic b. At point 3, a three-point contact is obtained between the curves a, b. The curve a intersects with the circle of curvature r at the point A and at three infinitesimally close points that coincide with point 3. The curve b intersects the circle r at the point B and at the same three points coinciding with point 3. A dynamic contour of the second order of smoothness is obtained.

We require that the curves have at the point 3 a four-point contact (a third-order connection of smoothness). The first section 1-3 coincides with the conic a (figure 2, b). Only one additional condition can be imposed on the second section g of the contour: either incidence to point 4, or touching the line t_4. By preserving the incidence condition and eliminating the touch condition, we obtain the homology $a{\sim}g$ given by the t_3 axis, the center 3 and two corresponding points 4~4 '. In the homology compiled, we determine the five points of the section g. We obtain a hyperbola g, one branch of which is shown in figure 2, b. At the point G, it intersects the circle of curvature r common to the sections a and g of the contour being constructed. The curves a and g touch at the point 3 most closely, forming a third-order connection of smoothness, but the touch condition t_4 is not satisfied. Thus, arcs of curves of the second order can be used to construct everywhere convex dynamic contours of the second order of smoothness.

5. Modeling of flat and spatial contours of the first order of smoothness

The problem of constructing a planar dynamic contour of the first order of smoothness is solved by the engineering discriminant method [8-10]. Let it be required to draw a piecewise smooth centerline

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of the pipeline through given points A^1, A^2, A^3, A^4 with the current lines indicated at these points (figure 3). The control of the shape of the center line is provided by indicating additional control points B^1, B^2, B^3. The sections of the center line are drawn using the software [11,12]. The first section A^1A^2 – arc of hyperbole, the second section A^2A^3 – part of the ellipse, the third section A^3A^4 – arc of parabola.

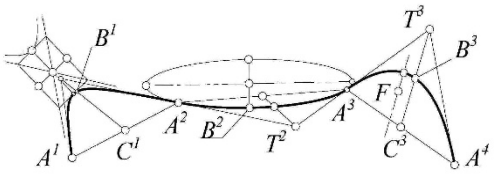

Figure 3. A flat contour.

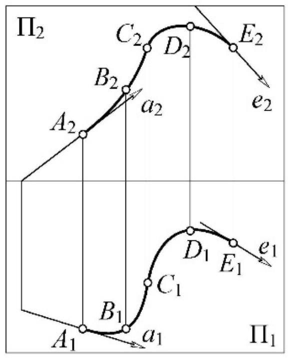

Figure 4. The center line (projections on Π_2 and Π_1).

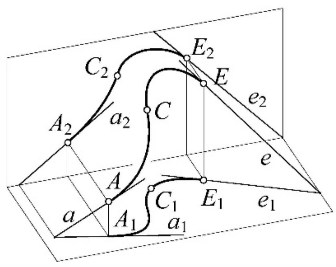

Figure 5. The center line (axonometry).

Simulation of smooth spatial lines is performed using projective methods of descriptive geometry [13,14]. Suppose that the space contains support points A, B, C, D, E, through which the axis of the projected channel must pass. The drawing shows the directions of the streamlines $a(a_1, a_2)$ at the input and $e(e_1, e_2)$ at the output (figure 4). We single out two sections ABC and CDE projected spatial curve. At the joint point C specify the direction c of the current line. Using the engineering discriminant method, we find orthogonal projections $j_1=A_1B_1C_1D_1E_1$ and $j_2=A_2B_2C_2D_2E_2$ spatial axial line j on the planes of projections Π_1 and Π_2. The composite curve j_1 contains an arc $A_1B_1C_1$ hyperbole and arc $C_1D_1E_1$ of the ellipse. At the joint point C_1 both arcs have a common tangent. The projection j_2 contains the arc $A_2B_2C_2$ hyperbole and arc $C_2D_2E_2$ of the ellipse; both arcs at the point C_2 have a common tangent. As a result, we obtain a smooth spatial curve $j(j_1, j_2)$, consisting of two sections ABC and CDE of algebraic biquadratic curves having a common tangent at C. The axonometric image of the spatial axis line j is shown in figure 5.

6. Application of quadratic transformations in the design of highways

When constructing smooth interfaces of transport highways, there arises the need for conjugation of ring junctions ρ_A, ρ_B. This problem is reduced to constructing a transition curve between circles ρ_A, ρ_B with radii R_A, R_B. At the boundary points A, B tangents are given T_A, T_B (figure 6). The transition curve should provide a smooth change in the curvature of the path when passing from the ring to the transition curve. In the case of automated design, composite curves are used as the transition curves: algebraic splines or Bezier curves [15]. In this paper, it is proposed to perform conjugation with the help of an algebraic curve obtained as the image of a circle in generalized inversion (the quadratic Hyrst transform) [16-18].

Let the generalized inversion be given by the center F_1 and the invariant circle d^2. It is required to build an image r of the circle r_0. Mark the intersection points 1, 1′ of the circle r_0 with the line $j_1=F_2F_3$. All the points of this line, including the marked points 1, 1′, are transformed into the point F_1, that is why F_1, F_2, F_3 – are double points of the curve r. It follows that the image of the circle in the Hyrst transform is a rational algebraic curve of order 4 (figure 7).

To construct a transition curve, one of the boundary points, for example, point B, is taken as the center of the quadratic involution I_2 with coincident F-points $F_1=F_2=F_3=B$. The given circle of curvature ρ_B is assumed as the limiting circle p^2 of the transformation I_2.

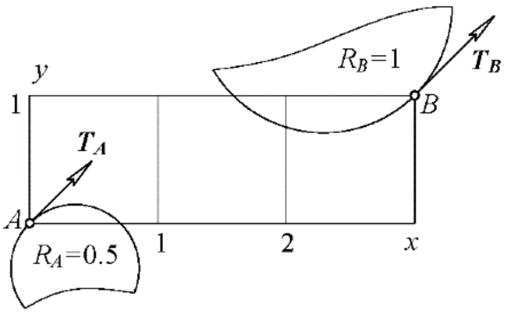

Figure 6. Border conditions.

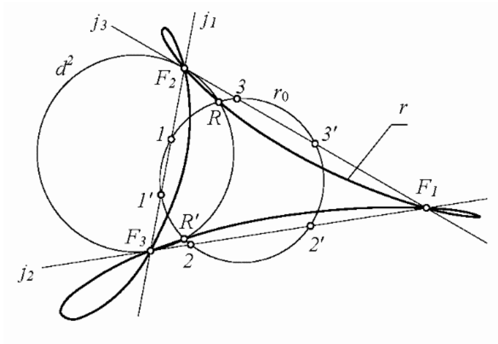

Figure 7. Circle inversion.

In the involution I_2 find the image ρ'_A of the circle of curvature ρ_A. We obtain an algebraic curve of the fourth order ρ'_A. Find the tangent and the circle of curvature c' of the curve ρ'_A at the point A'. We find the inverse image of the circle c' in the generalized inversion $I_2(B, d^2)$. This is an algebraic curve of the fourth order c^4 with a double point B (figure 8). The curve c^4 satisfies the value given in Fig. 6 to geometric boundary conditions. Indeed, both branches of the curve c^4 have at the point B the same circle of curvature $\rho_B = p^2$. The point A', lying on c', in the inversion $I_2(B, d^2)$ returns to the point A. Consequently, the inverse image c^4 of the circle c' passes through the point A. The circleь c' has at the point A' a three-point contact with the image ρ'_A of the circle ρ_A. That is why the inverse images c^4 and ρ_A must have a three-point contact at the point A, since the inversion transformation preserves the incidence [19].

The algebraic curve c^4 contains two different arcs AB. If not take into account the given direction of traffic along the highway, both arcs satisfy the conditions of the problem. Taking into account the direction of motion, one should select the upper branch of the curve c^4 (figure 8).

The smooth conjugation of two circles can be performed using an automated procedure for constructing a spline with given tangents and the radii of curvature at its boundaries [15,20]. In this case, undesirable oscillations can be obtained. For example, in figure 9, a shows a cubic spline consisting of three cubic parabolas with axes parallel to the coordinate axes x, y. The resulting composite curve satisfies the boundary conditions, but can not be used in practice because of undesirable oscillations. For comparison, figure 9, b shows the upper branch of the curve of the fourth order c^4, which is best suited for use as a transition curve.

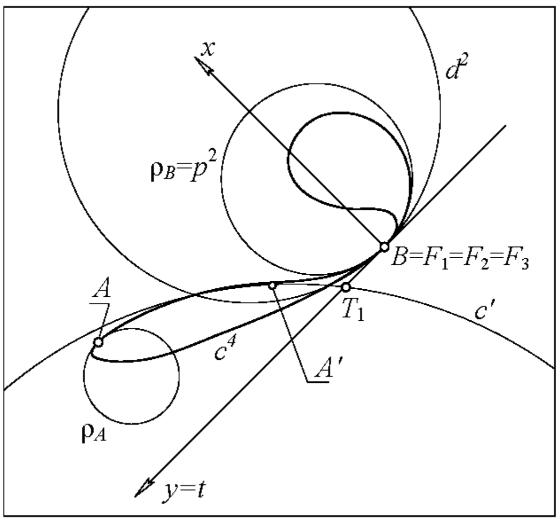

Figure 8. The inverse imagec^4 of the circle c'.

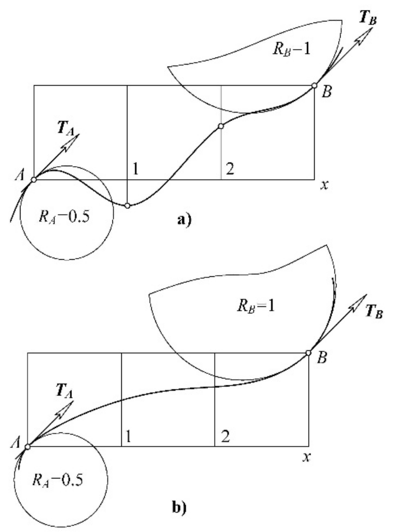

Figure 9. Comparison of the models of transition curves.

7. Conclusion

In the practice of designing construction communications, not only automated CAD tools (splines, Bezier curves, etc.) can be effectively used, but also classical computational and graphical methods of descriptive and projective geometry. In specialized CAD, a composite curve is represented by its graph and a two-dimensional digital array of coordinates of the points of this curve. In contrast to the known methods, it is proposed to use arcs of curves of the second order and their quadratic images to construct the axial lines of transport communications. The advantage of the proposed method is that the designer gets the model of the given curve not as the set of coordinates of its points, but as a matrix of coefficients of the canonical equations of the sections of the curve. Due to this, the geometric model of the projected communication can be presented in an analytical form, which is necessary for subsequent dynamic or strength calculations.

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References

[1] Atanasyan L S and Bazylev V T 2015 *Geometry* (Moscow: Knorus) p 424
[2] Prasolov V V and Tikhomirov V M 2013 *Geometry* (Moscow: MCCME) p 336
[3] Byshgens S S 2008 *Differential geometry* (Moscow: Publishing house LCI) p 304
[4] Savelov A A 2010 *Flat curves* (Moscow: LIBROKOM) p 296
[5] Walker R 2009 *Algebraic curves* (Moscow: LIBROKOM) p 240
[6] Ivanov G S 1995 *Descriptive geometry* (Moscow: Mechanical Engineering) p 224
[7] Fox A and Pratt M 1982 *Computational geometry. Application in design and production* (Moscow: Mir) p 304
[8] Glagolev N A 1963 *Projective geometry* (Moscow: Higher School) p 244
[9] Ivanov G S 1998 *Theoretical bases of descriptive geometry* (Moscow: Mechanical Engineering) p 157
[10] Boykov V N, Purkin V I and Fedotov G A 2005 *Automated design of highways* (Moscow: Izd-vo MADI) p 224
[11] Korotkiy V A 2014 Synthetic algorithms for constructing a second-order curve Bulletin of Computer and Information *Technologies* vol 11 pp 20–24
[12] Korotkiy V A and Khmarova L I 2016 Universal Computer Conicograph GraphiCon *Proc of 26-th Int Conf* (Nizhny Novgorod) pp 347–351
[13] Koroyev Yu I 2015 *Descriptive geometry* (Moscow: KNORUS) p 422
[14] Korotkiy V A, Khmarova L I and Butorina I V 2014 *Descriptive geometry. Lecture notes* (Chelyabinsk: Publishing Center of SUSU) p 191
[15] Shikin E V and Plis A I 1996 *Curves and surfaces on the computer screen* (Moscow: DIALOG-MIFI) p 240
[16] Ivanov G S 1987 *Construction of technical surfaces* (Moscow: Mechanical Engineering) p 192
[17] Hudson H 1921 *Cremona transformations in plane and space* (Cambridge) p 433
[18] Smogorzhevsky A S and Smolova E S 1961 *Handbook of the theory of plane curves of the third order* (Moscow: GIFML) p 263
[19] Efimov N V 2003 *Higher Geometry* (Moscow: Fizmatlit) p 584
[20] Golovanov N N 2012 *Geometric modeling* (Moscow: Publishing House of Physics and Mathematics) p 472