

Time Analysis of Building Dynamic Response Under Seismic Action. Part 1: Theoretical Propositions

E M Ufimtcev

Department of Construction Industry and Theory of Structures, South Ural State University, 76, Lenin Avenue, Chelyabinsk 454080, The Russian Federation

E-mail: ufimtcevem@susu.ru

Abstract. The first part of the article presents the main provisions of the analytical approach – the time analysis method (TAM) developed for the calculation of the elastic dynamic response of rod structures as discrete dissipative systems (DDS) and based on the investigation of the characteristic matrix quadratic equation. The assumptions adopted in the construction of the mathematical model of structural oscillations as well as the features of seismic forces' calculating and recording based on the data of earthquake accelerograms are given. A system to resolve equations is given to determine the nodal (kinematic and force) response parameters as well as the stress-strain state (SSS) parameters of the system's rods.

1. Introduction

Earthquakes are underground tremors and vibrations of the Earth's surface caused by natural or (more rarely) artificial causes [1]. Earthquakes for their devastating consequences, the number of victims and destructive effects on human habitat occupy one of the first places among other natural disasters [1-3]. Annually on the whole Earth there are occur more than one million earthquakes [1]. Most of them are insignificant in strength and not accompanied by catastrophic consequences, however, sometimes (up to several times a year) there occur earthquakes accompanied by destruction and human victims.

In addition, earthquakes are also capable of causing other equally dangerous impacts, such as landslides, avalanches, tsunamis, etc.

The current state of science and technology does not allow to set the task of preventing destructive earthquakes. Moreover, the forecast of earthquakes in the broad sense of the word is a difficult task. Nevertheless, at the present time the problem of development of antiseismic measures is posed, consisting of two main parts [3]:

1) development of methods for predicting the location of the expected destructive earthquake and assessing its intensity;

2) development of seismic resistant structures, i.e. facilities possessing high technical and economic indicators and capable of perceiving an expected earthquake with minimal damage.

The problem of increasing the seismic resistance of buildings and structures is currently very relevant. In this area, many Russian [3-11] and foreign [12-17] specialists work.

The second direction of development of anti-seismic measures includes, firstly, the development of effective methods and techniques for calculating structures for seismic forces and, secondly, the creation of special devices (vibration dampers, energy absorbers, seismic insulators, etc.) designed to improve reliability and the survivability of buildings in an earthquake.



As the main methods of calculation of buildings and structures under seismic actions are the normative method, the decomposition method in terms of eigenmodes, as well as the finite element method.

This paper considers application of one of the analytical approaches – the time analysis method (TAM) – to the calculation of structures under the action of seismic forces. This method is based on the study of the characteristic matrix quadratic equation (MQE) [18,19]. The process of calculating of the dynamic response is divided on several quasilinear time intervals $t \in [t_i, t_{i+1}]$, at which the stiffness, damping and inertia parameters of the design dynamic model (DDM) are constant. The TAM solution at each of these intervals is constructed in the closed form of the Duhamel's integral.

The possibility of using this method in the calculation of systems under complex dynamic effects, as well as taking into account the nonlinear work of the material and the design, is due to its universality and its ability to modify and create such computational schemes in which the initial problem is reduced to a sequence of linear problems. The TAM does not require spectral decomposition of solution and can be realized for any type of damping within the viscous friction model of the material.

The usage of TAM is illustrated by the example of calculation of a 3-storey building modeled as vertical cantilever rod with 3 degrees of freedom under the action of the seismic load given by the earthquake accelerogram.

2. The equation of motion of a discrete system

The differential equation of motion (1) of the system with n degrees of freedom under the action of a seismic load simulated by an accelerogram in the process of elastic oscillations on any quasilinear motion interval $t \in [t_i, t_{i+1}]$ together with the initial conditions of problem (2):

$$\begin{cases} M\ddot{Y}(t) + C\dot{Y} + KY(t) = -M\ddot{\Delta}(t), \\ Y_0 = Y(t_i), \dot{Y}_0 = \dot{Y}(t_i). \end{cases} \quad (1,2)$$

where $M, C, K \in M_n(\mathbf{R})$ – respectively, matrices of mass, damping and rigidity of the system; $Y(t)$ – vector of the required nodal displacements of the DDS; $\ddot{\Delta}(t)$ – acceleration of the base of the building (accelerogram).

The structure load changes at each step of the analysis t_i when the value $\ddot{\Delta}(t)$ changes. In this case, it is convenient to represent the external action in the form of a rectangular pulse of length equal to the time analysis step $t_a = t_{i+1} - t_i$, and the amplitude:

$$P_0 = -m_{kk}\ddot{\Delta}(t_i) \quad (3)$$

where k – number of the degree of freedom of the system. When t_i of the initial condition (2) is also corrected.

Integration of a homogeneous ODE corresponding to (1) is connected with the construction of a fundamental matrix $\Phi(t) = e^{St}$, where $S \in M_n(\mathbf{C})$ – matrix, which is the solution (root) of the MQE:

$$M S^2 + C S + K = 0 \quad (4)$$

The spectrum of the matrix S contains the internal dynamic characteristics of the structure: eigenfrequencies, damping coefficients, and modes of natural oscillations [19].

3. Dynamic response parameters of the system

In constructing the resolving equations for the problem (1), (2), the following assumptions were made:

1) the whole process of calculating the dynamic response is divided into quasilinear time intervals $t \in [t_i, t_{i+1}]$, on which the DDM parameters are constant;

2) boundary (critical) points of intervals (for t_i) are the time points at which the accelerations $\ddot{\Delta}(t_i)$ changes;

3) the transition of the system to the i -th state is considered instantaneous.

The system of equations for the dynamic response of the structure under seismic action, modeled as described in paragraph 2 of the paper, has the form [19]:

$$\begin{cases} Y(t) = 2\text{Re}\{Z(\tilde{t})\}, & \dot{Y}(t) = 2\text{Re}\{S Z(\tilde{t})\}, & \ddot{Y}(t) = 2\text{Re}\{S^2 Z(\tilde{t})\} + M^{-1}P(t), \\ Z(\tilde{t}) = \Phi(\tilde{t})U^{-1}M[-\bar{S}Y_0 + \dot{Y}_0] + [\Phi(\tilde{t}) - E](US)^{-1}P(t). \end{cases} \quad (5)$$

here: $\tilde{t} = t - t_i$; $\Phi(\tilde{t}) = e^{S\tilde{t}}$; $U = MS + S^T M + C$; $P(t) = -M\ddot{\Delta}(t)$. This system makes it possible to determine from a unified position the response of a quasilinear DDS on the interval $t \in [t_i, t_{i+1}]$ when solving the dynamic problem (1), (2).

The expressions (5) determine the nodal kinematic parameters of the system: displacement, velocity and acceleration. On their basis, the force characteristics (respectively, the vectors of restoring, dissipative and inertia forces):

$$R(t) = K Y(t), \quad F(t) = C\dot{Y}(t), \quad I(t) = -M\ddot{Y}(t) \quad (6)$$

With the help of nodal displacements $Y(t)$, it is possible to calculate the parameters of the stress-strain state (SSS) of rods of the construction such as the bending moments $M(t)$ and normal stresses $\sigma(t)$ at the given calculation cross sections. This will allow the analysis of the strength of the structure.

4. Conclusions

The foundations of the time analysis method, which allows to construct in a closed form a dynamic DDS response (node kinematic and force parameters of the reaction, SSS parameters of the system elements) in the process of elastic oscillations caused by the action of seismic forces are presented.

The second part of the article will illustrate the application of the time analysis method by the example of the calculation of a 3-storey building under seismic action. The task is implemented in MATLAB [20].

Acknowledgments

The work was supported by Act 211 Government of the Russian Federation, contract № 02.A03.21.0011.

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