

A Spectrum Sensing Scheme Based on Subspace Filtering

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Abstract. Spectrum sensing (SS) has attracted much concern of researchers due to its significant contribution on the spectral efficiency. Energy Detection (ED) has been a critical method for Spectrum Sensing in Cognitive Radio Networks (CRNS) due to its low complexity and simple implement. However, noise uncertainty in ED greatly degrades the detection performance, especially under a low Signal-to-Noise Ratio (SNR). To remove noise uncertainty as much as possible, a scheme based on subspace decomposition is proposed for SS, where the received signal is decomposed into two parts: noise subspace and signal-plus-noise subspace. Then the closed-form solution of the detection and false alarm probabilities is given on the basis of the signal-plus-noise subspace in Rayleigh fading channel. The energy of the remainders after removal of noise subspace and noise contribution in signal-plus-noise subspace is used to decide whether the primary user (PU) exists by a comparison with a redesigned threshold. Eventually, some simulations based on MATLAB platform is made to validate the proposed method.

1. Introduction

Cognitive Radio (CR) [1] has been widely accepted as a vital way to relieve the limited spectrum resource and inefficient efficiency of spectrum utilization. One core technology behind a CR is Spectrum sensing (SS) [2]. SS works to acquire the spectrum usage by signal detection and processing in wireless propagation channel.

The performance of sensing spectrum holes is evaluated by the detection probability and the probability of false alarm [3]. A miss in the detection will cause the interference with PU and a false alarm will reduce the spectral efficiency [4]. The optimal detection performance occurs in the case that the detection probability is maximized subject to the constraint of the false alarm probability. According to the standard of wireless regional area networks (WRAN) [5] based on CR of 802.22 working group [6], the detection probability of propagable CR is at least for 90% while the probability of false alarm is below 10%. Practically, to the best of our knowledge, this is a hard work in the state of art considering the computational and implementation complexity.

In the literature of SS, a number of algorithms have been proposed to identify the presence of the primary signal as well as to improve the detection efficiency by inhibiting the disgusting interference in the sensing environment. Some examples of the existing proposals include Energy Detector (ED) [7], Matched Filtering (MF) [8], Covariance-Based Detection (CBD) [9] and Cyclostationary Feature Detection (CFD) [10]. MF works based on the prior knowledge of the signal and needs well time synchronization. Although the CBD based method outperforms than the others on detection performance due to its nearly independence of noise uncertainty, the required sampling frequency is much higher than the normal conditions, leading to a higher complexity of implementation.



Energy Detector (ED) is a classical blind spectrum sensing without the prior knowledge of signal, time synchronization and a higher sampling frequency. Moreover, ED is an optimal blind detector. However, its main drawback consists in the noise uncertainty incurred by the estimation of noise variance, especially under a lower signal-noise-ratio (SNR), where an accuracy estimation of noise variance is difficult. For a possible removal of noise uncertainty in ED, a good number of corresponding schemes are proposed, such as cooperative spectrum sensing, etc. Cooperative spectrum sensing (CSS) [11] lowers the volatility of noise variance and considers fading and multipath effect in wireless propagation channel.

To the best knowledge of the author, most of ED based methods are devoted to eliminating noise uncertainty in an indirect way and the suppression effect of noise uncertainty is not excellent enough. As a result, a spectrum sensing scheme is proposed in this paper on the basis of subspace filtering (SSF), where the received signal at the receiver passes a subspace filter [12]. The subspace filter firstly decomposes the received signal into two orthogonal subspaces, noise subspace and signal-plus-noise subspace. Noise subspace is filled with noise and sign-plus-noise subspace contains the whole signal and segmental noise. Then the noise subspace is moved away together with the noise contribution in the signal-plus-noise subspace. After subspace filtering, background noise is filtered greatly and the energy of the remainders after removal of noise subspace and noise contribution in signal-plus-noise subspace is used to decide whether the primary user (PU) exists by a comparison with a redesigned threshold. It is voted that the sensing performance will have a distinct improvement due to the filtering of environmental noise. As a supplement, simulations based on MATLAB platform verify the proposed method.

The rest of this paper is organized as follows. Section 2 presents the primary principle of subspace filtering. The main contribution of this paper is shown in Section 3, which consists in a spectrum sensing scheme based on subspace filtering. Simulation experiments and result analysis are accomplished in Section 4. Finally, Section 5 concludes this paper.

2. Subspace filtering

2.1. Model and motivation

Noise in CR communication systems is an aggregation of various independent sources including receiver device noise and environment noise [13]. Receiver device noise includes: (1) non-linearity components; (2) non-uniform and time-varying thermal noise. Environment noise includes: nearby unintended emissions and weak signals from other transmitters far away. By appealing to the Central Limit Theorem (CLT) [14], one usually assumes that the noise at the receiver is a Gaussian random variable.

Let us denote the clean primary signal at the receiver after a bass-pass filter as $s(t)$ and assume $s(t)$ is a random process of independence and identical distribution with bandwidth B , central frequency f_0 and variance σ_s^2 . As discussed above, the background noise $u(t)$ is usually modeled as Gaussian, independent and identically distributed random process and $u(t)$ is independent of $s(t)$. In addition, spectrum sensing is an issue of binary hypothesis [15], as a result, the received signal at the receiver could be formulated as

$$H_1 : r(t) = s(t) + u(t) \quad (1)$$

$$H_0 : r(t) = u(t) \quad (2)$$

where H_1 denotes the case that the primary signal $s(t)$ exists and H_0 indicates the radio band of interest is unoccupied.

After a sampler with sampling frequency f_s at the receiver, the observed signal of two scenarios could be rewritten as

$$H_1 : r(n) = s(n) + u(n) \quad (3)$$

$$H_0 : r(n) = u(n) \quad (4)$$

2.2. Subspace filterin

Subspace filtering is an active noise reduction method on speech enhancement and other aspects of signal processing due to the optimality in theory. The essence of subspace method consists in the construction of a linear estimator H , after which the received signal $r(n)$ is decomposed into two orthogonal subspaces: signal-plus-noise subspace $Hy(n)$ and noise subspace $(I - H)r(n)$. Noise subspace only contains noise and signal-plus-noise subspace contains segmental noise and the whole signal. After removal of noise subspace and noise contribution in the signal-plus-noise, the remainders $\hat{y}(n)$ could be formulated as

$$\hat{y}(n) = Hy(n) \quad (5)$$

Consequently, the main work for an optimal subspace based filtering consists in how to acquire an optimal linear estimator H_{opt} . According to the corresponding research on subspace filtering, the optimal linear estimator H_{opt} could be obtained by minimizing the signal distortion and keeping the noise residual noise under a preset threshold Δ_{th} ,

$$\begin{cases} \min \overline{\mathcal{E}_s^2} \\ \overline{\mathcal{E}_u^2} < \Delta_{th} \end{cases} \quad (6)$$

where R_s and R_u are the covariance matrix of $x(n)$ and $u(n)$, respectively. Δ_{th} is always some a number below threshold of audibility.

Due to the fact that the background noise is white, the covariance matrix of the received signal R_r could be denoted as a sum of the signal covariance matrix R_s and noise covariance matrix R_u ,

$$R_r = R_s + R_u \quad (7)$$

and they share the same eigenvector matrix V . Taking eigendecomposition of matrix R_r , we shall have

$$R_r = V \Lambda_r V^T \quad (8)$$

where $\Lambda_r = \text{diag}[\lambda_1, \dots, \lambda_N]$ contains all the eigenvalues of matrix R_r . If the eigenvalues are arranged in the descending order, i.e., $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_N$, then we have $\lambda_k > \sigma^2$ for $k = 1, 2, \dots, K$ and $\lambda_{K+1} = \dots = \lambda_N = \sigma^2$. The part $\lambda_k > \sigma^2$ represents the signal subspace and the remainders are the noise subspace. Consequently, the signal covariance matrix R_s and noise covariance matrix R_u in Eq.7 could be respectively estimated as

$$\begin{aligned} R_s &= V \Lambda_s V^T \\ &= V \begin{bmatrix} \lambda_1 & & & \\ & O & & \\ & & \lambda_K & \\ & & & 0 \end{bmatrix} V^T \end{aligned} \quad (9)$$

$$\begin{aligned} R_u &= V \Lambda_u V^T \\ &= V \begin{bmatrix} 0 & & & \\ & \lambda_{K+1} & & \\ & & O & \\ & & & \lambda_N \end{bmatrix} V^T \end{aligned} \quad (10)$$

Then an approximatively equivalent expression of H_{opt} could be shown as

$$H_{opt} = V \Lambda_s (\Lambda_s + \mu \Lambda_u)^{-1} V^{-1} \quad (11)$$

In practice, the covariance matrix above-mentioned is impossible to be acquired. As a result, it is replaced with the sample covariance matrix, just as follows

$$R_r \approx \frac{1}{N-1} \sum_{i=1}^N r(i)r(i)^T \quad (12)$$

In theory, after removal of background noise by subspace based filtering, the remainders could be expressed as

$$\begin{cases} \hat{r}(n) = H_{opt} r(n) \approx s(n), H_1 \\ \hat{r}(n) \approx 0, H_0 \end{cases} \quad (13)$$

3. Spectrum sensing based on subspace filtering

In this part, the proposed spectrum sensing scheme based on subspace filtering is mainly discussed.

Let us define the energy of $s(n)$ as

$$E_s = E(|s(n)|^2) \approx E(|H_{opt} s(n)|^2) \quad (14)$$

Then the detection probability and false alarm probability could be formulated as

$$P_d = P(E(|H_{opt} r(n)|^2) > 0 | H_1) \quad (15)$$

$$P_f = P(E(|H_{opt} r(n)|^2) > 0 | H_0) \quad (16)$$

Eq.15-16 works well in ideal conditions when the background noise is completely removed after subspace based filtering. However, it is noted that the residual signal after subspace filtering still exists some environmental noise. Obviously, much error leaves behind to estimate the detection and false alarm probabilities within Eq.15-16.

Let us also assume the residual noise after subspace filtering is Gaussian, independent and identically distributed due to CLT. The distinction of noise distribution between the noise before subspace filtering comes with the variance and average value of Gaussian distribution. Then the revised detection probability could be expressed as

$$\begin{aligned} P_d(\varepsilon, \tau) &= P\left(\frac{1}{N} \sum_{n=1}^N |y(n)|^2 > \varepsilon | H_1\right) \\ &= Q\left(\left(\frac{\varepsilon}{\sigma_u^2} - 1 - \gamma\right) \sqrt{\frac{\tau f_s}{2\gamma + 1}}\right) \end{aligned} \quad (17)$$

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp(-t^2/2) \quad (18)$$

where τ is the sensing time, f_s is the sampling frequency of SU, $N = \tau f_s$ is the sampling number of

SU, $\gamma = \frac{\sigma_s^2}{\sigma_u^2}$ (σ_u^2 is the variance of noise after subspace filtering) and ε is the predetermined energy threshold.

The corresponding false alarm probability could be described as

$$\begin{aligned} P_f(\varepsilon, \tau) &= P\left(\frac{1}{N} \sum_{n=1}^N |u_1(n)|^2 > \varepsilon | H_0\right) \\ &= Q((\varepsilon - \mu_0) / \sqrt{\sigma_0^2}) \end{aligned} \quad (19)$$

where $\mu_0 = \sigma_u^2$, $\sigma_0^2 = \sigma_u^4 / N$ are the expectation and variance of the original ED method for SS, and $u_1(n)$ is the residual noise after subspace filtering.

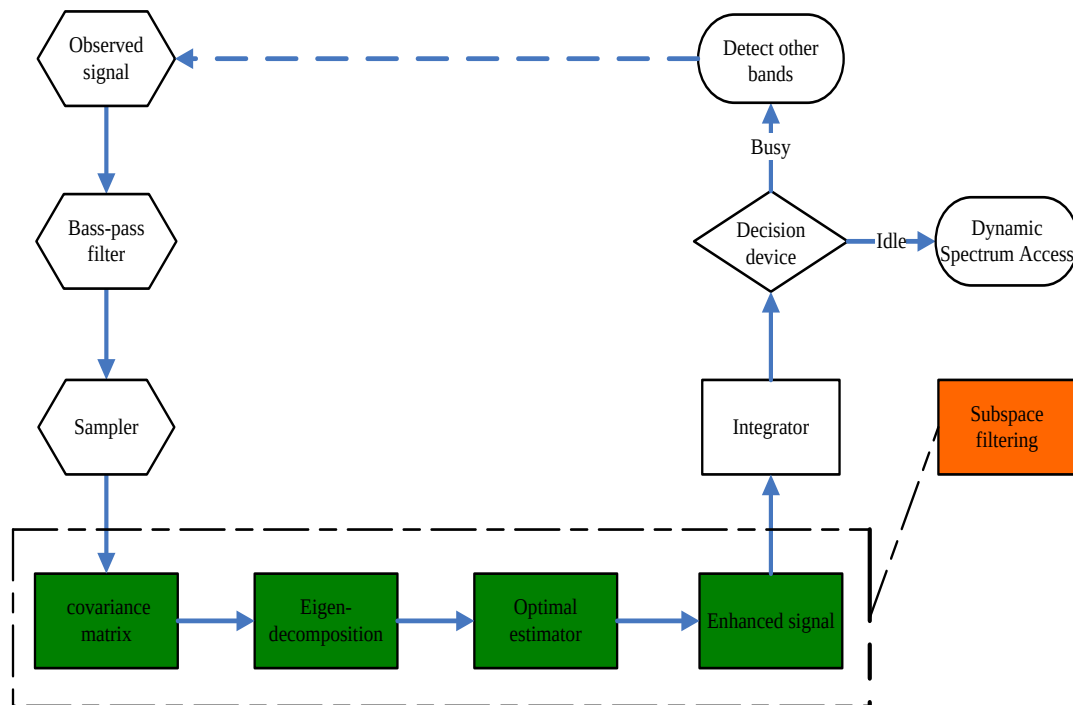


Figure.1. the flowchart of subspace based spectrum sensing

As Figure.1 shown, the general procedures for subspace based spectrum sensing are listed. The green pane denotes the subspace filtering, which corresponds to Eq.5-11. The decision device works dependent on Eq.15-16 or Eq.17-19. Note that when decision device considers current radio band to be busy, a CR will start to monitor and sense other bands for opportunistic occupation within a certain amount of time.

4. Simulation and analysis

In this section, Simulations are made to check the proposed method. The simulation band is conducted at Very high frequency (VHF), where the carrier frequency is set as the sampling frequency of SU is $1.0 \times 10^8 \text{ Hz}$; the sampling frequency f_s of SU is $3.0 \times 10^8 \text{ Hz}$ and the corresponding sampling number N is 100. Considering the sensing time and detection performance simultaneously, $N = 100$. In the simulation, the OFDM signal is firstly generated and then white Gaussian noise with mean zero and variance one is added into the OFDM signal. After the sampling and energy normalization of the noise-only case and the signal-plus-noise case, subspace based filtering is applied to them, respectively. Ultimately, we calculate the detection and false alarm probability by the corresponding energy comparisons with the presupposed threshold.

In Figure.2, a comparison is made between SSF and the other spectrum sensing schemes. Note that SSF (1) represents the sensing scheme in Eq.15-16, where the remainder energy after subspace filtering is compared with 0 to determine whether PU exists. SSF (2) denotes the spectrum hole sensing with Eq.17-19, where the residual noise after subspace filtering is assume to be Gaussian distribution. In addition, 3-MV describes a cooperative sensing scheme named Majority Voting (MV) and 3 indicate the number of secondary user (SU). From Figure.2, SSF (1) and SSF (2) have a distinct advantage over ED. Compared with 3-MV, SSF (1) has an approximate performance and SSF (2) is superior to it. The superiority of SSF on performance derives from less noise in the signal after subspace filtering. Consequently, the proposed method is competent in monitoring and sensing spectrum holes.

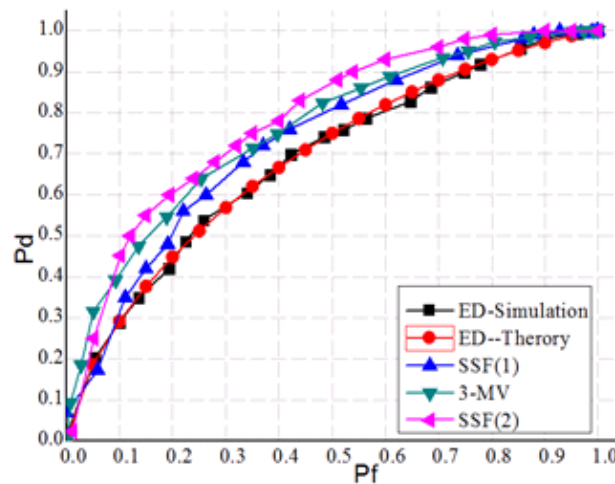


Figure 2. ROC curve comparison in AWGN channel (SNR=10dB).

As a supplement, Figure.3 gives the detection probability for a given false alarm probability ($P_f = 0.1$) when $SNR = -15\text{ dB}$. From the histogram, the performance of SSF still outperforms ED and SSF (2) has the best performance in contrast with ED and SSF (1), which corresponds to the simulation made in Figure.2.

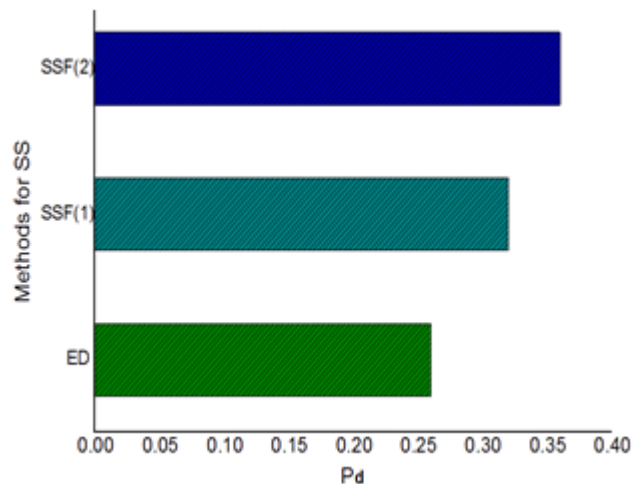


Figure 3. The detection probability for a given false alarm probability.

To sum up, the simulation results in Figure.2-3 fits our prediction and the theoretical derivation in Section 3. After subspace based filtering, the decision has less influence from environmental noise. Hence, the decision is more accurate than before.

5. Conclusions

In this paper, a subspace filtering based spectrum sensing scheme is proposed to monitor and sense spectrum holes as accurate as possible. The proposed method starts with subspace filtering for the received signal at the receiver. Then the energy of remainder signal is considered as the criterion to determine whether the radio frequency band is idle. The simulation based on MATLAB platform has been verified the proposed method.

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