

# Modelling and Simulation Analysis of Rolling Motion of Spherical Robot

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**Abstract.** This paper presents the findings of modelling, control and analysis of the spherical rolling robot based on pendulum driven within the simulation environment. The spherical robot is modelled using Lagrange function based on the equation of rolling motion. PD-type Fuzzy logic controller (FLC) was designed to control the position of the spherical robot where 25 rules were constructed to control the rolling motion of spherical robot. It was then integrated with the model developed in Simulink-Matlab environment. The output scaling factor (output gain) of the FLC was heuristically tuned to improve the system performance. The simulation results show that the FLC managed to eliminate the overshoot response and demonstrated better performance with 29.67% increasing in settling time to reach 0.01% of steady state error.

## 1. Introduction

Spherical robot is one type of mobile robot that shows significant advantages over others due to its geometrical shape. It is spherical in shape with driving mechanism located inside the spherical robot. The robot is naturally roll to locate itself and capable to recover from collisions with obstacle. So, the robot can be thrown and dropped without risk to be overturned like other types of mobile robot [1]. This allowed the spherical robot to manoeuvre comfortably in any environments to perform any desired tasks.

This unique features in mobility and safety enable it to be used in many applications such as education, entertainment, security, navigation, monitoring and exploration [2]–[7]. This increases the interest of researchers to explore and develop this area. The number of different mechanical structures has been developing in recent years. Several basic principles have been used to generate the motion of spherical robot. The early version of spherical robot actuated using wheel inside the sphere. The wheel is driven by a motor and can be turned to cause change in direction [8]. Almost similar concept is also been used in [9] where the small remote car is used to provide force to propel the robot. These kinds of mechanisms are easier to be modelled and controlled as long as no accurate tracking control is required. A simple equation of motion has been proposed using linear dynamic model of the system to model the system. Using simple model make the control design become easier but the simple model is unable to capture the nonlinearity and complexity of the system. Therefore, it makes the methods to be unsuitable to be used to model a spherical robot.

Spherical robot also can be actuated by changing the center of the mass of the system. The concept of imbalance mass is used by moving the mechanism like pendulum [10]–[17]. This moving mass will cause unbalance to the system and make the spherical robot to roll. The dynamic model of the pendulum



and kinematic model of rolling disk is used to model the system. The complex modelling process can also be simplified with decoupled approach when dealing with more than one degree of freedom [13].

Besides, rolling motion of spherical robot can also be achieved using three independent inertia disc [18]. For this type of design, the system is model based on its geometrical framework and adopting attitude dynamic governed by Poincare equation of motion. Based on this model, a nonlinear state feedback control law is used to achieve asymptotic stabilization of position and reduced attitude. This nonlinear feedback controller is defined from linear state feedback control law that has been transformed using body coordinate velocities.

The kinematic and dynamic model of the spherical robot can also be model using Euler angle and Lagrange equation. Most of the modelling methods discussed before linearized the model to make the controller design easier. LQR with full state feedback controller [15], simple feedback controller [19], gaits control based on motion equation [17] is designed to tackle the stability or tracking the trajectory of spherical robot. But, the linearization cause the elimination of nonlinear part of the system and may cause some behaviour of the system that are related to the unmodelled dynamics, parameter variations, disturbance and uncertainty being neglected. This will affect the design controller.

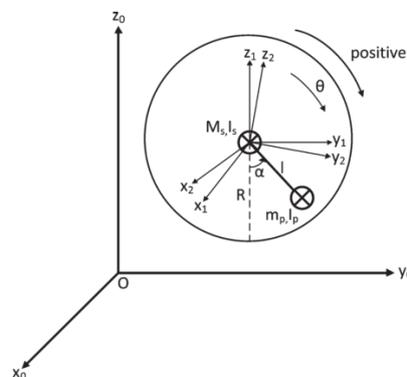
To overcome this problem, the intelligent controller is introduced and seem to be effective in other mobile robot [20], [21]. In [22], adaptive neuron fuzzy controller where PID controller was combined with fuzzy controller and sliding mode control theory based learning algorithm is introduced to improve the robustness of the system and ensure faster convergence than traditional learning technique.

In this paper, a fuzzy logic controller is designed with 25 fuzzy rules to control the position of rolling spherical robot to follow the reference point. Fuzzy logic controller is design based on the human's heuristic knowledge on the system. This control method is significant in reducing error result from the complex mathematical modelling as the control is expressed in a natural language. With fuzzy logic controller system manage to improve the transient response of the system. The paper is organized as mathematical model based on pendulum driven in next section, and design of fuzzy logic controller in section 3. The simulation result and discussion will be presented in section 4 and finally, the conclusion in section 5.

## 2. Model of spherical robot

### 2.1 Kinematic modelling

Kinematic model of the system is derived based on equation of motion of the rolling wheel or disk. In the model ling process, the spherical robot is assumed to roll without slipping, no friction force acting between the spherical robot and horizontal plane. Besides that, the geometric center of the shell and mass of the robot are assumed to be eccentric. This assumptions help to simplify the modelling process. Similar modelling based on pendulum driven had been discussed briefly in [13], [22].



**Figure 1.** Modelling of the Spherical robot with pendulum driven. [22]

Figure 1 represent the motion of spherical robot in 2D-plane with  $\theta$  and  $\alpha$  as angle of rotation of spherical robot and pendulum around the x-axis respectively.  $X_0, Y_0, Z_0$  is the reference frame fixed to the motion ground while  $X_1, Y_1, Z_1$  and  $X_2, Y_2, Z_2$  are the moving frames attached to the center of the spherical robot with radius of R. The kinematic system of the model can be represent with it angular velocity,  $\omega_s$  and linear velocity,  $v_s$  of the spherical robot, the angular velocity,  $\omega_p$ , and linear velocity,  $v_p$  of the mass of the pendulum. Kinematics equations can be written in vector form as below:

$$\omega_s = -\dot{\theta}i \quad (1)$$

$$V_s = R\dot{\theta}j \quad (2)$$

$$\omega_p = (\dot{\alpha} - \dot{\theta})i \quad (3)$$

$$V_p = (-R\dot{\theta} + (\dot{\alpha} - \dot{\theta})l\cos(\alpha - \theta))j + ((\dot{\alpha} - \dot{\theta})l\sin(\alpha - \theta))k \quad (4)$$

## 2.2 Dynamic model

The dynamic model of the rolling spherical robot can be derived using Lagrangian function of L. L is the subtraction of kinetic energy of linear and rotation system with potential energy of the system.

Equation of L is described as below:

$$L = K - P$$

$$\begin{aligned} &= \frac{1}{2} M_s \|V_s\|^2 + \frac{1}{2} I_s \|\omega_s\|^2 + \frac{1}{2} m_p \|V_p\|^2 + \frac{1}{2} I_p \|\omega_p\|^2 - m_p g r_p = z \\ &= \frac{1}{2} M_s (R\dot{\theta})^2 + \frac{1}{2} I_s (-\dot{\theta})^2 + \frac{1}{2} I_p (\dot{\alpha} - \dot{\theta})^2 + \frac{1}{2} m_p ((-R\dot{\theta} + (\dot{\alpha} - \dot{\theta})l\cos(\alpha - \theta))^2 \\ &+ ((\dot{\alpha} - \dot{\theta})l\sin(\alpha - \theta))^2) - m_p g l\cos(\alpha - \theta) \end{aligned} \quad (5)$$

$M_s$  and  $m_p$  representing the masses of the sphere and pendulum. While, mass of moment of inertia,  $I_s$  and  $I_p$  of sphere and pendulum is derived as equations 6 and 7.

$$I_s = \frac{2}{3} M_s R^2 \quad (6)$$

$$I_p = \frac{1}{12} m_p l^2 + m_p \left(\frac{l}{2}\right)^2 \quad (7)$$

The system also is assumed to have viscous friction acting between the sphere and the surface where the equation can be written in an energy dissipation function of velocities of the system,  $\dot{q}_i$  and the damping constant,  $\xi$ ,

$$S = \frac{1}{2} \xi \dot{q}_i^2 = \frac{1}{2} \xi (\dot{\theta}^2 + \dot{\alpha}^2) \quad (8)$$

Then, the Euler Lagrange system is written as follow:

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} + \frac{\partial S}{\partial \dot{q}_i} = Q_i \quad (9)$$

Where  $q_1$  and  $q_2$  are  $\theta$  and  $\alpha$  respectively are the generalized coordinates.  $Q_1 = Q_2 = \tau$  represent the input torque to rotate the pendulum that is rotated through an input torque and a reaction torque about the shaft occurs in the opposite direction. Then, the final rolling motion of spherical robot is written as below:

$$M_{11} \ddot{\theta} + M_{12} \ddot{\alpha} + C_{11} + G_{11} = \tau \quad (10)$$

$$M_{22} \ddot{\alpha} + M_{21} \ddot{\theta} + C_{21} + G_{21} = \tau \quad (11)$$

Where,

$$M_{11} = M_s R^2 + m_p R^2 + m_p l^2 + I_s + I_p + 2m_p R l \cos(\alpha - \theta)$$

$$M_{12} = M_{21} = -m_p l^2 - I_p - m_p R l \cos(\alpha - \theta)$$

$$\begin{aligned}
 M_{22} &= m_p l^2 + I_p \\
 C_{11} &= m_p R l \sin(\alpha - \theta) (\dot{\alpha} - \dot{\theta})^2 + \xi \dot{\theta} \\
 C_{21} &= \xi \dot{\alpha} \\
 G_{11} &= -G_{21} = -m_p g l \sin(\alpha - \theta)
 \end{aligned}$$

The equations 10 and 11 are used to model the simulink block diagram of spherical robot for rolling motion as in figure 3. The pendulum is actuated using dc motor. The simple model of DC motor is modelled as in equations 12 and 13 where  $\theta_m, k_e, k_t, b, R_m, V_m, L$  and  $J$  are the rotation of motor, moment of inertia of the motor, electromotive force constant, motor torque constant, motor's viscous friction constant, electrical resistance, voltage armature, electrical inductance and moment of inertia of the motor respectively. The system then was represented in Simulink block diagram for simulation.

$$\ddot{\theta}_m = \frac{1}{J} (k_t i - b \dot{\theta}_m) \quad (12)$$

$$\frac{di}{dt} = \frac{1}{L} (-R_m i + V_m - k_e \dot{\theta}_m) \quad (13)$$

### 3. Fuzzy logic control (FLC)

Fuzzy logic control is a controller that used the human language and knowledge as an element to control the system. This controller do not used an exact value to determine the input to the plant. Fuzzy logic controller consists of 4 main components; fuzzification, rule based, inference mechanism and defuzzification of the output FLC.

The first step in designing the FLC is to select the input-output parameters of the fuzzy controller. In order to control the position of the system PD type of fuzzy logic is designed. This type of FLC requires error ( $e$ ) and rate of change of error ( $\dot{e}$ ) as the input to the FLC, while producing the voltage of the DC motor as the output to the actuator. The voltage of DC motor will vary the angle of rotation of the pendulum that causes the rotation of the spherical robot.

The inputs and output of the FLC are represented as membership functions as Big Negative (BN), Small Negative (SN), Zero (Z), Small Positive (SP), and Big Positive (BP) using triangular membership function with range [-1 1]. The membership function of inputs and output of the FLC are plotted in figures 2,3 and 4.

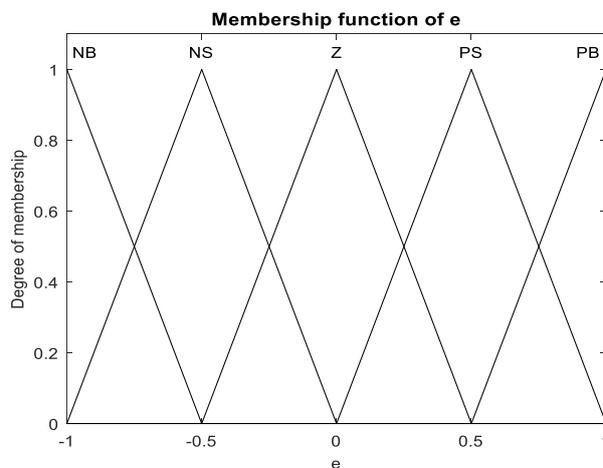


Figure 2. Membership function of error,  $e$ .

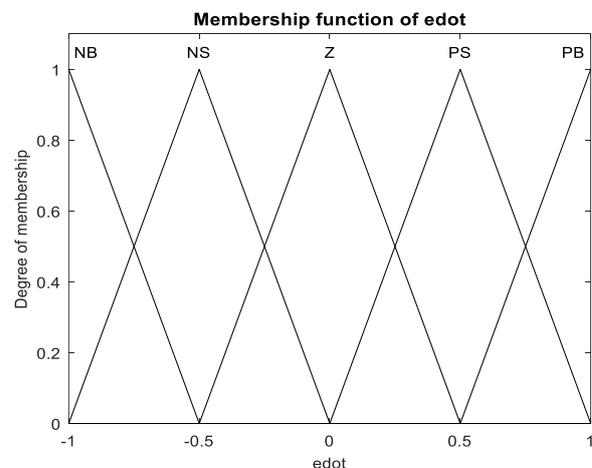
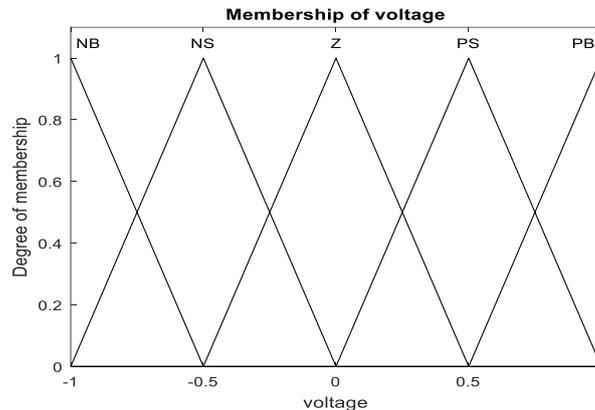


Figure 3. Membership function of rate change of error,  $\dot{e}$



**Figure 4.** Input membership function of input  $e$ ,  $\dot{e}$ , and output voltage.

The 25 rules of membership function are constructing based on the experiences and the understandings of the system. Here, the understanding is gained through its mathematical model that describes its behaviour and experiences of the person. table 1 show the fuzzy rules used in this controller.

**Table 1.** Rule base of PD type Fuzzy logic controller for position control

Voltage		Change of error( $\dot{e}$ )				
		BN	SN	Z	SP	BP
Error ( $e$ )	BN	BN	BN	SN	SN	Z
	SN	SN	SN	SN	Z	SP
	Z	SN	SN	Z	SP	SP
	SP	SN	Z	SP	SP	BP
	BP	Z	SP	SP	SP	BP

The general form of linguistic rule listed as below:

**“If premise, Then consequent.”**

As this fuzzy controller takes two inputs; error and rate of change of error, the two inputs are combined using AND. The rule can be understood as **“If** error is big negative (BN) **and** change of error is big negative (BN), **Then**, the voltage supply to DC motor is also big negative (BN)”. This rule quantifies the situation where the simulated position of spherical robot is bigger than the desired output and it is moving fast forward (fast changing in error). So, it means the robot move further away from the desired position. Then, the negative voltage is supplied to the DC motor so that it moves on the opposite direction to the desired position.

#### 4. Simulation result and discussion

For the simulation, parameters of the spherical robot have been assumed as  $I_s = 0.6\text{m}$ ,  $I_p = 0.2\text{m}$ ,  $M_s = 1\text{m}$ ,  $m_p = 0.06\text{kg}$ ,  $R = 0.15\text{m}$ ,  $l = 0.145\text{m}$  and  $\zeta = 1$ . While the parameters of dc motor are assumed as  $k_e = 1$ ,  $k_t = 1$ ,  $b = 1$ ,  $R_m = 1\text{ohm}$ ,  $L = 1\text{H}$  and  $J = 1\text{kg.m}^2$ .

The model was first simulated without controller to evaluate the output response of the spherical robot to the input signal. The result to this step input test is shown in figure 5. The transient response of the spherical robot without the controller shows that 6.6% overshoot, rise time,  $t_s$  at 4.54s, peak time,  $t_p$  at 5.51s and settling time,  $t_s$  at 11.29s.

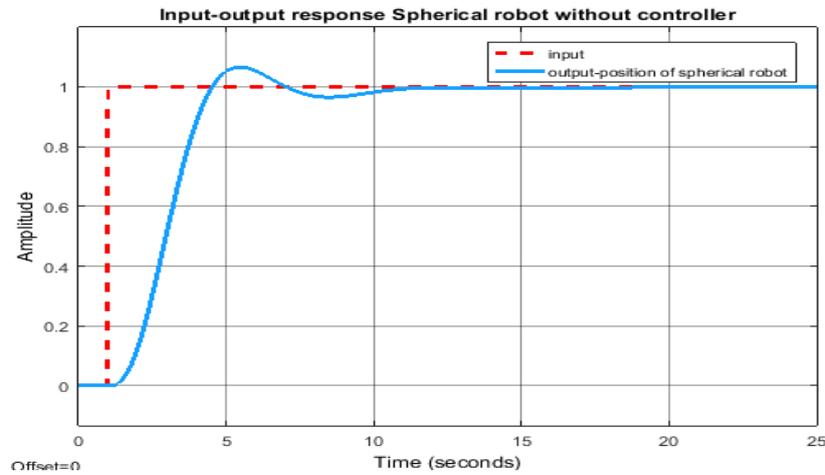


Figure 5. Step input test of spherical robot plant without controller.

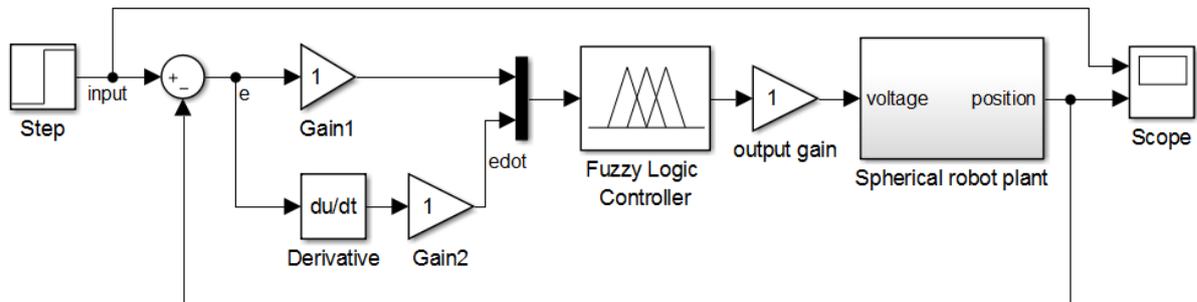


Figure 6. Simulink diagram of fuzzy logic controller of spherical robot.

Once the PD-type fuzzy logic controller is implemented in the simulation as in figure 6, the result shows an improvement in the transient response where the controller manages to eliminate the overshoot problem. To improve the output response, it is necessary to tune the output gain of the FLC,  $g$ . When the value of output gain FLC,  $g$  is increased gradually to 6, the settling time is reduced from 11.29s to 7.94s as shown in figure 7. Yet, increasing the value of  $g$  managed to accelerate the output response but the distortion build in the process can cause the problem to the system. So, the best output gain used to get faster response and least distortion before reach the reference point is 4.

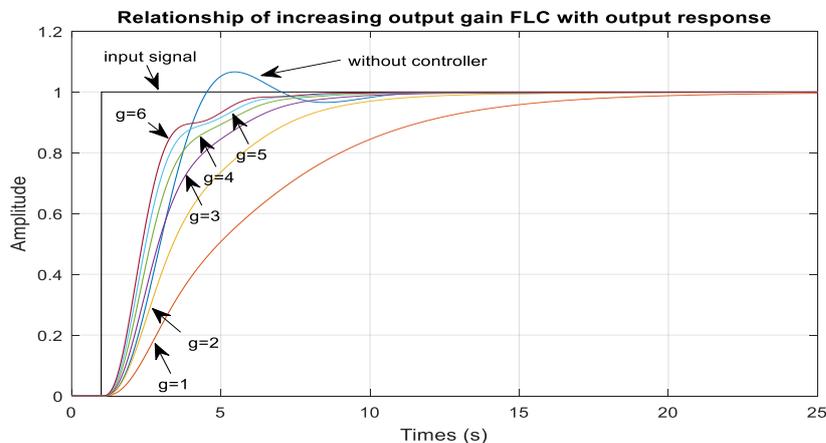


Figure 7. Output response's graph with different values of FLC's output gain,  $g$ .

Besides, several fuzzy rules are also being designed, tuned and simulated but, the result shows that changing the several rules like BN to SN or vice versa does not give significant impact to the performance of the spherical robot. While, amplifying the input gain of  $e$  and  $\dot{e}$  also does not improved the performance of the rolling motion.

## 5. Conclusion

In this study, modelling and Fuzzy Logic controller has been designed and investigated. The controller was focused to control the response of the system to follow the reference input. The modelling of the spherical rolling robot was based on single DOF pendulum driven. The response of the system without controller was compared with the PD-type controller to analyse the effect of fuzzy logic controller. The effect of increasing output gain of controller was analysed. The response manages to reach the reference point faster as the gain increasing, but, the system will suffer to the oscillation before reaching the reference point if the output gain is too high.

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