Double- diffusive convection in an anisotropic and inclined porous layer with cooperating horizontal gradients of temperature and concentration

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Abstract. Double-diffusive natural convection in porous layer saturated with a binary fluid is presented in this study. The layer is anisotropic with respect to both its permeability and diffusivity and is inclined with respect to the horizontal axis. The vertical walls of porous cavity are subjected to uniform temperature and concentration where the other surfaces are assumed to be adiabatic and impermeable. The governing equations are solved by the finite volume method. The study focuses on the effects of anisotropy in permeability and the inclination angle of the flow structures on heat and mass transfer. The results are presented and analyzed in terms of streamlines, isotherms, isoconcentrations lines and average Nusselt and Sherwood numbers. They indicate the existence of three regimes. A diffusive regime, a transition regime and an asymptotic regime.They are in good agreement with those found in the literature.

1. Introduction

The heat and mass transfer phenomena within porous configurations are particularly important to study because of the large number of industrial or environmental applications in which they are present. In most cases, natural convection of heat and/or solutal source plays a decisive role in the transfers involved.

Cheng and Chang [1] studied the natural convection heat transfer from impermeable horizontal surfaces in a saturated porous medium. Bejan and Khair [2] examined the natural convection boundary layer flow driven by both temperature and concentration gradients. Most studies [3-5] of thermosolutal convection induced in isotropic or anisotropic porous medium saturated by a fluid consider a single layer of porous medium in the case of rectangular cavities. Among these studies, it can be mentioned the numerical investigations of the simplified problem of square porous cavity where the vertical walls are held at constant temperatures and concentrations and the horizontal surfaces are considered adiabatic and impermeable. The scale analysis permits to treat this problem in the two extreme cases of heat-driven and solule-driven natural convection.

A special focus has been made by several authors on the case where a plane porous layer is inclined to the horizontal and bounded by impermeable isothermal walls. A temperature difference between the walls may lead to an unstable stratification that has a different from that of a horizontal layer, Darcy–Bénard [6]. In fact, the inclination to the horizontal results in a basic stationary state where the fluid is not at rest, but circulates along a single cell of infinite width. The basic velocity field is parallel, bidirectional, and with a vanishing mass flow rate. However, most of the porous materials are anisotropic due to preferential orientation for either enhancing or for deteriorating heat transfer. Hadadi et al. [7] studied numerically thermosolutal convection within inclined porous collector vertically. The effect of the inclination angle is taken into account and analyzed. The results are presented in terms of streamlines, isotherms, and isoconcentrations and are mainly analyzed in terms of the average heat and mass transfers. Dimensional analysis is applied to predict analytically the

evolution of transfer and is compared to numerical results obtained from simulations of double-diffusive natural convection in such bi-layered porous medium for the different situations.

The problem of double-diffusive convection in an inclined horizontal bi-layered porous cavity was also considered by Hadadi and Bennacer [8]. The results were also presented and analyzed in terms of streamlines, isotherms, isoconcentrations lines and average Nusselt and Sherwood numbers. Numerical and a scale analysis are used to characterize the effect of the permeability ratio on the heat and mass transfer in vertical bi-layered porous cavity. Singh and al. [9] have studied the natural convection heat transfer in inclined porous square cavities. They concluded that the larger inclination angle may be optimal for the energy efficient processes involving inclined enclosures due to larger heat flow circulations with enhanced thermal mixing.

The aim of the present study is to emphasize on the natural convection and heat and mass transfer in inclined porous layer with reference to horizontal. The saturated porous medium is anisotropic in permeability. The vertical walls of the cavity are subjected to uniform temperature and concentration. The study focuses on the effects of anisotropy in permeability and the inclination angle of the flow structures on heat and mass transfer. The results are presented in terms of streamlines, isotherms, and isoconcentrations and are mainly analyzed in terms of the average heat and mass transfers.

2. Problem formulation

The physical model is presented in Figure 1. It is represented by an inclined porous layer saturated by a binary fluid. The inclination angle of the enclosure with respect to the horizontal plane is denoted by α. The porous medium is considered homogeneous and anisotropic in permeability and thermal conductivity. The vertical walls of the porous cavity are subjected to uniform conditions of temperature and concentration, whereas, the horizontal walls are assumed to be adiabatic and impermeable. A general Brinkman-Forchheimer extended Darcy model is used to account for the flow in the porous medium.

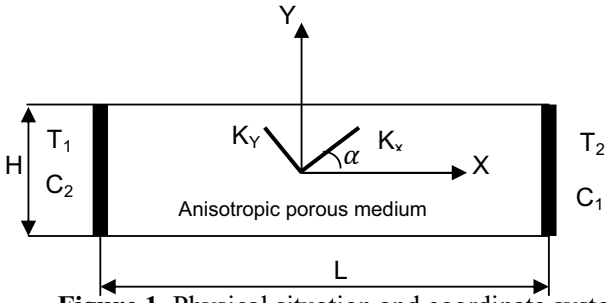


Figure 1. Physical situation and coordinate system.

Under the usual Boussinesq approximation, the governing equations for steady, laminar, two-dimensional boundary layer flow under above assumptions can be written as:

$$\vec{\nabla} . \vec{V} = 0 \tag{1}$$

$$\rho_f \left[\frac{1}{\varphi}\frac{\partial \vec{V}}{\partial t} + \frac{1}{\varphi^2}(\vec{V}\vec{\nabla})\vec{V}\right] = -\vec{\nabla}P - \mu\frac{\vec{V}}{\bar{\bar{K}}} - \frac{C_f}{\bar{\bar{K}}^{1/2}}|\vec{V}|\vec{V} + \mu_{eff}\vec{\nabla}^2\vec{V} + \rho_f\vec{g} \tag{2}$$

$$\sigma\frac{\partial T}{\partial t} + \vec{V}\vec{\nabla}\,T = \vec{\nabla}.\left(\bar{\bar{\lambda}}\vec{\nabla}.T\right) \tag{3}$$

$$\varphi\frac{\partial c}{\partial t} + \vec{V}\vec{\nabla}C = \vec{\nabla}.\left(D_{eq}\vec{\nabla}C\right) \tag{4}$$

where ρ_f is the fluid density(kg/m^3), g is the gravitational acceleration (m/s^2), μ_{eff} apparent dynamic viscosity for Brinkman's model(kg/m.s) ,D the equivalent mass diffusivity(m^2/s) and σ ratio of heat capacities.

$\bar{\bar{K}}$ and $\bar{\bar{\lambda}}$ are the second order flow permeability and thermal diffusivity tensors defined, respectively, by

$$\bar{\bar{K}} = \begin{bmatrix} K_1 cos\alpha^2 + K_2 sin\alpha^2 & (K_1 - K_2) sin\alpha \cos\alpha \\ (K_1 - K_2) sin\theta \cos\theta & K_2 cos\alpha^2 + K_1 sin\alpha^2 \end{bmatrix} \quad (5)$$

$$K = \frac{K_x}{K_y} \quad (6)$$

Substituting K (equation (6)) in equation (5) and by applying inverse, the permeability tensor can be reduced to

$$\bar{\bar{K}}^{-1} = \frac{1}{K_1} \begin{bmatrix} cos\alpha^2 + K sin\alpha^2 & (1-K) sin\alpha \cos\alpha \\ (1-K) sin\alpha \cos\alpha & K cos\alpha^2 + sin\alpha^2 \end{bmatrix} \quad (7)$$

$$\bar{\bar{\lambda}} = \begin{bmatrix} \lambda & 0 \\ 0 & 1 \end{bmatrix} \quad (8)$$

$$\lambda = \frac{\lambda_x}{\lambda_y} \quad (9)$$

Substituting λ (equation (9)') in equation (8)', the thermal conductivity tensor can be reduced to

$$\bar{\bar{\lambda}}^{-1} = \begin{bmatrix} \lambda & 0 \\ 0 & 1 \end{bmatrix} \quad (10)$$

The dimensionless governing equations are written as:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (11)$$

$$\frac{1}{\varphi}\frac{\partial U}{\partial t} + \frac{1}{\varphi^2}\left(U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y}\right) = -\frac{\partial P}{\partial X} + \frac{Pr}{\varphi}\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial X^2}\right) - \frac{Pr}{Da}[(U cos\alpha^2 + K sin\alpha^2) +$$

$$V((1-K) sin\alpha \cos\alpha)] - \frac{C_f}{\sqrt{Da}}\left[U\left(cos\alpha^2 + \sqrt{K} sin\alpha^2\right) + V\left(\left(1-\sqrt{K}\right) sin\alpha \cos\alpha\right)\right] \times \sqrt{U^2 + V^2}$$

$$+ PrRa(T + NC) \quad (12)$$

$$\frac{1}{\varphi}\frac{\partial U}{\partial \tau} + \frac{1}{\varphi^2}\left(U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y}\right) = -\frac{\partial P}{\partial X} + \frac{Pr}{\varphi}\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) - \frac{Pr}{Da}[U((1-K) sin\alpha \cos\alpha) +$$

$$V(K cos\alpha^2 + sin\alpha^2)] - \frac{C_f}{\sqrt{Da}}\left[U\left(\left(1-\sqrt{K}\right) sin\alpha \cos\alpha\right) + V\left(\sqrt{K}\, cos\alpha^2 + sin\alpha^2\right)\right] \times \sqrt{U^2 + V^2}$$

$$+ PrRa(T + NC) \quad (13)$$

$$\frac{\partial T}{\partial t} + \left(U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y}\right) = \frac{1}{Pr}\bar{\bar{\lambda}}^{-1}\left(\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2}\right) \quad (14)$$

$$\frac{\partial C}{\partial t} + \frac{1}{\varphi}\left(U\frac{\partial C}{\partial X} + V\frac{\partial C}{\partial Y}\right) = \frac{1}{Le}\left(\frac{\partial^2 C}{\partial X^2} + \frac{\partial^2 C}{\partial Y^2}\right) \quad (15)$$

The non-dimensional boundary conditions can be expressed as follows:

$\mathrm{U} = \mathrm{V} = 0$, $\mathrm{T} = 0$ et $\mathrm{C} = 0$ at $\mathrm{t} = 0$

$\frac{\partial \mathrm{T}}{\partial \mathrm{x}} = 0$, $\frac{\partial \mathrm{C}}{\partial \mathrm{x}} = 0$, $\mathrm{U} = \mathrm{V} = 0$ at $\mathrm{Y} = 0, \forall\, \mathrm{X}$

$\frac{\partial \mathrm{T}}{\partial \mathrm{x}} = 0$, $\frac{\partial \mathrm{C}}{\partial \mathrm{x}} = 0$, $\mathrm{U} = \mathrm{V} = 0$ at $\mathrm{Y} = 1$, $\forall\, \mathrm{X}$

$\mathrm{T} = 1$, $\mathrm{C} = 0$, $\mathrm{U} = \mathrm{V} = 0$ at $\mathrm{X} = 0$, $\forall\, \mathrm{Y}$

$\mathrm{T} = 0$, $\mathrm{C} = 1$, $\mathrm{U} = \mathrm{V} = 0$ at $\mathrm{X} = 4$, $\forall\, \mathrm{Y}$

Where φ is the porosity, C_f : Forchheimer coefficient, U et V : dimensioneless horizontal and vertical velocities, X and Y are dimensioneless horizontal and vertical coordinates. T, C, P and t: are respectively the dimensioneless, temperature, concentration, pressure and time.

The dimensionless variables are defined as: Darcy number *(Da),* Prandtl number *(Pr)*, Rayleigh number *(Ra)*, Lewis number *(Le)* and Buoyancy ratio number *(N).*

The average Nusselt and Sherwood numbers are defined by

$$Nu = \int_0^1 \left(\frac{\partial T}{\partial X}\right)_{X=0} dY \tag{16}$$

$$Sh = \int_0^1 \left(\frac{\partial C}{\partial X}\right)_{X=0} dY \tag{17}$$

3. Numerical method

The governing equations along with the boundary conditions are solved numerically employing finite volume method. For pressure and velocity coupling, the SIMPLER algorithm is used. The system of algebraic equations is solved iteratively by means of the Thomas algorithm. The iteration process is continued until the following convergence criterion is achieved.

$$\left(\left|\frac{\phi_{i,j}^{t+\Delta t} - \phi_{i,j}^{t}}{\phi_{i,j}^{t+\Delta t}}\right|\right) \leq 10^{-4} \tag{18}$$

where ϕ is a dependent variable U, V, T et C . The indices i, j indicate a mesh point and Δt being the increment of time.

For the accuracy of the numerical results, the present study is compared with the previous study Bennacer et al. [10] and Ni and Beckermann [11] for horizontal rectangular porous cavity as shown in table 1. It is observed that the present results are in good agreement with that of [10] and [11]. This favorable comparison leads confidence in the numerical results to be reported in the next section. The relative differences observed between the results being less than about 1.5%.

Table 1. Comparison the Nusselt number of present results with those of Bennacer et al. [8] and Ni and Beckerman [9].
Da =10^{-7}, A = 1, Ra = 10^3, N= 0, α =0°.

$K = K_y/K_x$	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}
Bennacer et al [10]	1.00	1.29	4.17	13.48	37.56	80.62
Ni and Beckermann [11]	1.01	1.30	4.17	13.41	37.37	80.34
Present study	1.04	1.26	4.19	13.09	37.31	80.33

4. Results and discussion

In the non-dimensional governing equations the Forchheimer constant (C_f) and porosity are taken respectively 0.55 and 0.8 according to Ergun relation [12]. The Prandtl number is taken to be 0.71 throughout the computations.

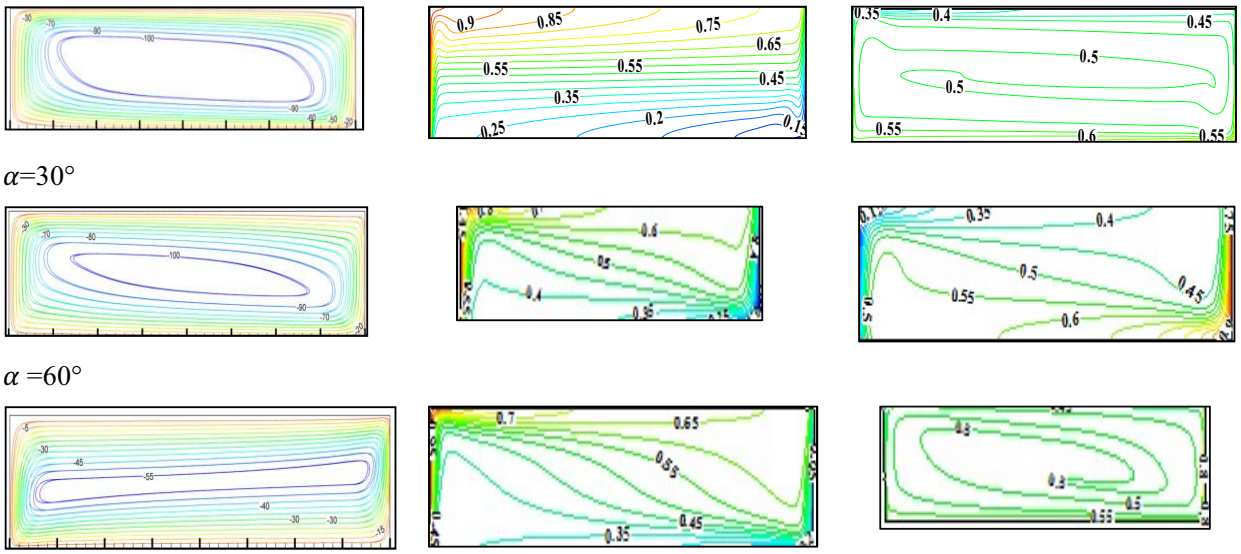
In 'figure 2' the effects of the inclination are illustrated for K= 10 and N = 10. The results are presented in terms of streamlines (on the left), isotherms (at the center) and isoconcentration (on the right) contours for different values of α. When the inclination decreased to α = 0° the pattern of streamlines of Figure 2 indicates that a large portion of the fluid in the center of the cavity is now stagnant due to the blocking effect of the vertical stratification of the density field in this area and the flow circulation is restricted to thin boundary layers of almost constant thickness, along the vertical walls. The isotherms are distorted and span in the diagonal direction of the entire cavity, the thermal boundary layer tends to be formed, located respectively on the lower and upper parts of the lateral walls. It is noted that the deformation of isoconcentrations increases as the tilt angle increases.

Figure 3 shows the average Nusselt and Sherwood numbers for different inclination of principal axes α with different permeability ratio K for Da = 10^{-4}, Ra=10^6 and k =10. The effects of *K* on the transfers are noticeable. The heat and mass transfers increase with the increase of α, passes through a peak, and then begins to decrease. The peak in Nusselt and Sherwood number occurs at about 30 ° for

different values of K. The increase of permeability in vertical direction with the variation of α from 0° to 90° leads to decrease in energy and mass transport, resulting in decrease of velocity.

The effect of the (buoyancy ratio) N on heat and mass transfer Nusselt (Nu) and Sherwood (Sh) are illustrated in 'figure 4' for α=30° and Da =10^{-4}. For a fixed value of N the heat and mass transfers increase with the increase of When K >10, the Nusselt and Sherwood numbers are found to be independent of K. Indeed, the resistance to the flow and then transfers decreases. For a fixed K and in a diffusive regime, one observes an increase of Nusselt and Sherwood with an increase of N, however it remains moderate. However over this regime, the transfers intensify. In fact, the buoyancy forces that induce the fluid motion are assumed to be cooperative. Nusselt reaches a maximum for K=10^2. However, for Sherwood the behavior is reversed, the maximum mass transfer occurs for K=10.

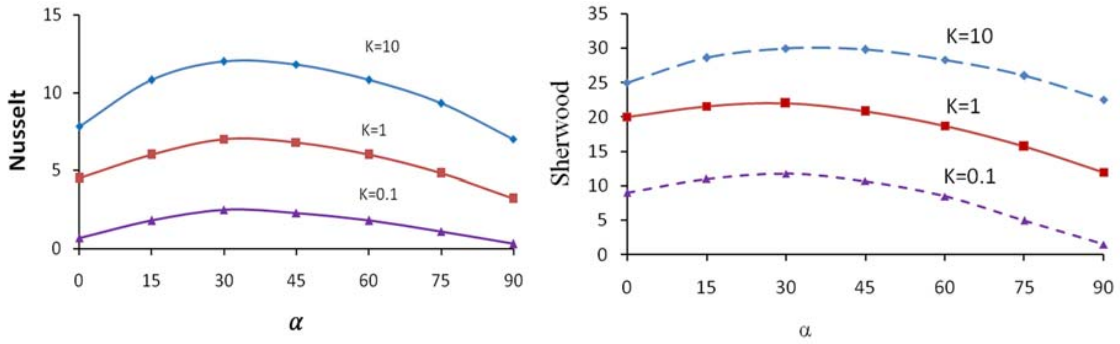
α=0°



α=30°

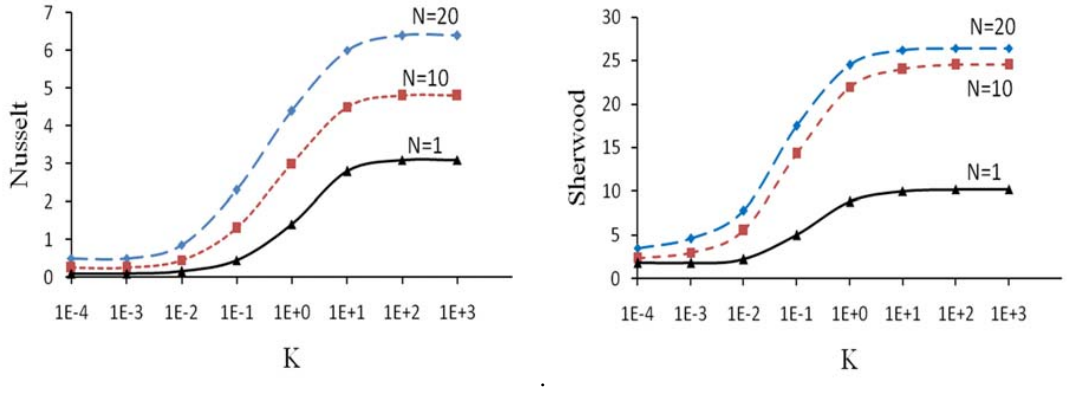
α =60°

Figure 2. Streamlines (left), isotherms (center) and isoconcentration (right) for Ra = 10^6, Da = 10^{-4} , λ=10 and K=10 for different tilted angle α.



(a) (b)

Figure 3. Effect of α on (a) average Nusselt and (b) Sherwood for various values of K for N = 10, Ra = 10^6 and Da = 10^{-4}, λ=10



(a) (b)

Figure 4. Effect of K on (a) average Nusselt and (b) Sherwood for various values of N for α=0°, Ra=10^6 and Da=10^{-4}, λ=10.

5. Conclusions

A study is made of natural convection heat and mass transfer in a horizontal porous cavity. The porous medium is assumed to be both thermally and hydrodynamically anisotropic with the principal axes of anisotropic permeability inclined with respect to the gravity force. The major results obtained in the present investigation can be summarized as follows.

For large permeability ratios (K > 1) as the inclination angle varies from α=0° to 90°,the heat and mass transport is maximum for α = 30°. For small permeability ratios (K < 1) as the anisotropic orientation angle varies from α=0° to 90°, the convective strength of the flow field gradually diminishes.

The numerical results indicate the existence of three regimes, namely, a diffusive regime for low values of K, a transition regime where Nusselt and Sherwood numbers increase with an increase of K and an asymptotic regime where Nusselt and Sherwood become independent of K.

6. References

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