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Nonlinear differential system applied of a mechanical plan model of the automotives used for the nonlinear stability analysis

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Abstract. This paper proposes a plan mechanical model for the vehicles with two axles, taking into account the lateral deflection of the tire. For this mechanical model are determined two mathematical models under the nonlinear differential equations systems form without taking into account the action of the driver and taking into account. The analysis of driver-vehicle system consists in the mathematical description of vehicle dynamics, coupled with the possibilities and limits of the human factor. Description seeks to emphasize the significant influence of the driver in handling and stability analyzes of vehicles and vehicle-driver system stability until the advent of skidding. These mathematical models are seen as very useful tools to analyzing the vehicles stability. The paper analyzes the influence of some parameters of the vehicle on its behavior in terms of stability of dynamic systems.

1. Introduction

In this paper we are proposing a mathematical model for automotives moving, a model who includes some of the position parameters and moving parameters.

This mathematical model can be used into a preliminary stability analyzes by dynamic systems theory or by appling the classical mechanic theory ([4], [7], [8]).

Dynamic Systems Theory aims to analyze the stability of a system both in the field of stability and in the field of unstable movements or even chaotic movements. Chaotic movements do not occur for any parameter values, but only for certain values thereof. Hence, it is possible to choose parameter values other than those for which it is possible chaotic motion. In this way, artificial dynamic systems, designed and built, will have stable and predictable movements.

For this reasons is necessary to know the conditions that produce chaotic movements to avoid them.

2. The mathematical model of automotives moving

The paper takes into consideration the plan of a two-axle vehicle, for which the rolling and pitch oscillations are neglected.

The mechanical model is shown in Figure 1, and the notations are detailed below:

- C1, C2, C3, C4 - the center of contact area between tire and road;
- F_{x1}, F_{x2}, F_{x3}, F_{x4} - the longitudinal forces of contact area between tires and road;
- F_{y1}, F_{y2}, F_{y3}, F_{y4} - the lateral forces of contact area between tire and road;

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- δ_1, δ_2 - the cornering angle for front axle;

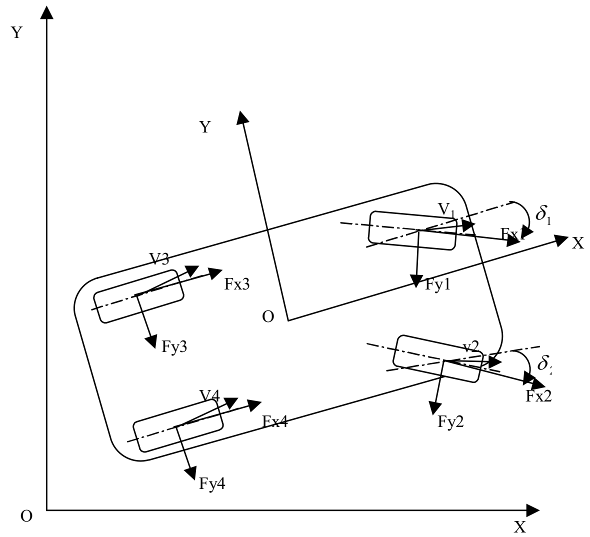

Fig. 1. The plan model of vehicle

The next references systems are taking into account:

a reference system is attaching to the suspended mass of the vehicle; this system has the origin in center mass point of this, and Ox, Oy axle making the plan of principal direction of displacement; the Oz axle is perpendicular to Ox, Oy axles;

a fixed $O_1x_1y_1z_1$ coordinate system presented in Fig. 1.

The next degree of freedom is taking into account:

x, define the displacement along longitudinal axle;

y, define the displacement along transversal axle;

u, define the displacement along vertical axle;

θ_1, define the circular displacement of suspended mass around Ox axle;

θ_2, define the circular displacement of suspended mass around Oy axle;

φ, define the circular displacement of suspended mass around Oz axle.

If we neglect the roll and pitch oscillations of the vehicle, forming a system that models the movement plane of the vehicle, and which follow the variables y, v_y, φ, ω. It is also considered a constant speed v_x = const. A system that modeling the vehicle plan moving are obtained, and this system has y, v_y, φ, ω variables and the velocity v_x is considerate constant ([11], [12]). We also intend to study the stability of the vehicle under the influence of disturbances that will be considered as a change in the steering angle of the wheels.

This change may be a periodic perturbation $\delta_p(t)$ applied to the steering axle tires, having the form:

$$\delta_p(t) = Q\cos(\omega_p t) \quad (1)$$

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with Q, ω_d – amplitude and frequency of the disturbance. The origin of such disturbances can be steering or periodic deformations of the street.

In addition to this disruption may occur steering angle changes by the action of the driver, if he perceives a lateral deviation of the vehicle from the desired trajectory. The driver tries to control vehicle steering angle $\delta_s(t)$, as yet to reduce the deviation from the theoretical line of travel. This action of the driver can be modeled mathematically, inserting a feedback loop k, a response time Tr and the driver's front vision L, the driver's command can be described by a differential equation taking into account the state component y(t) as follows ([5], [6]):

$$\dot{\delta}_s = -\left[\frac{k}{T_r}\left(y + \frac{L}{v_x}\dot{y}\right) + \frac{\delta_s}{T_r}\right]. \tag{2}$$

The above will be supplemented by consideration of external perturbations and by taking into account the action of the driver [6] on the steering angle of the wheels.

The analysis of driver-vehicle system consists in the mathematical description of vehicle dynamics, coupled with the possibilities and limits of the human factor. Description seeks to emphasize the significant influence of the driver in handling and stability analyzes of motor vehicles and vehicle-driver system stability until the advent of skidding.

Given the assumptions stated above but considering the action of the driver, it is obtained the next system:

$$\dot{v}_y = \frac{1}{m}\cdot\left\{(c_{11}\cdot\left(\delta_s + Q\cos(\omega_d t) - arctg\frac{v_y + a\cdot\omega}{v_x - \frac{e}{2}\cdot\omega}\right)\right.$$

$$-c_{13}\cdot\left(\delta_s + Q\cos(\omega_d t) - arctg\frac{v_y + a\cdot\omega}{v_x - \frac{e}{2}\cdot\omega}\right)^3)\cdot\cos(\delta_s + Q\cos(\omega_d t))$$

$$+(c_{21}\cdot\left(\delta_s + Q\cos(\omega_d t) - arctg\frac{v_y + a\cdot\omega}{v_x + \frac{e}{2}\cdot\omega}\right) - c_{23}\cdot\left(\delta_s + Q\cos(\omega_d t) - arctg\frac{v_y + a\cdot\omega}{v_x + \frac{e}{2}\cdot\omega}\right)^3)$$

$$\cdot\cos(\delta_s + Q\cos(\omega_d t) + \left(F_{x1} + F_{x2}\right)\cdot\sin\left(\delta_s + Q\cos(\omega_d t)\right)$$

$$+\left(c_{31}\cdot\left(arctg\frac{v_y-b\cdot\omega}{v_x-\frac{e}{2}\cdot\omega}\right)-c_{33}\cdot\left(arctg\frac{v_y-b\cdot\omega}{v_x-\frac{e}{2}\cdot\omega}\right)^3\right)$$

$$\left.+\left(c_{41}\cdot\left(arctg\frac{v_y-b\cdot\omega}{v_x+\frac{e}{2}\cdot\omega}\right)-c_{43}\cdot\left(arctg\frac{v_y-b\cdot\omega}{v_x+\frac{e}{2}\cdot\omega}\right)^3\right)\right\}-\omega\cdot v_x$$

$$\dot{\omega}_z=\frac{1}{J_{zz}}\cdot\{\ -\frac{e}{2}\cdot\left(F_{x1}-F_{x2}\right)\cdot\cos(\delta_s+Q\cos(\omega_p t))-a\cdot(F_{x1}+F_{x2})\cdot\sin(\delta_s+Q\cos(\omega_p t))$$

$$+\frac{e}{2}\cdot\left(-F_{x3}+F_{x4}\right)-(c_{11}\cdot\left(\delta_s+Q\cos(\omega_p t)-arctg\frac{v_y+a\cdot\omega}{v_x-\frac{e}{2}\cdot\omega}\right)$$

$$-c_{13}\cdot\left(\delta_s+Q\cos(\omega_p t)-arctg\frac{v_y+a\cdot\omega}{v_x-\frac{e}{2}\cdot\omega}\right)^3)\cdot\left(-\frac{e}{2}\cdot\sin(\delta_s+Q\cos(\omega_p t))+a\cdot\cos(\delta_s+Q\cos(\omega_p t))\right)$$

$$+b\cdot\left(c_{31}\cdot\left(arctg\frac{v_y-b\cdot\omega}{v_x-\frac{e}{2}\cdot\omega}\right)-c_{33}\cdot\left(arctg\frac{v_y-b\cdot\omega}{v_x-\frac{e}{2}\cdot\omega}\right)^3\right)+$$

$$b\cdot\left(c_{41}\cdot\left(arctg\frac{v_y-b\cdot\omega}{v_x+\frac{e}{2}\cdot\omega}\right)-c_{43}\cdot\left(arctg\frac{v_y-b\cdot\omega}{v_x+\frac{e}{2}\cdot\omega}\right)^3\right)\}$$

$$\dot{y}=v_y\cos\varphi+v_x\sin\varphi$$

$$\dot{\varphi}=\omega \tag{3}$$

$$\dot{\delta}_s=-\left[\frac{k}{T_r}\left(y+\frac{L}{v_x}\cdot\left(v_y\cos\varphi+v_x\sin\varphi\right)\right)+\frac{\delta_s}{T_r}\right]$$

Previous equations forming a system of linear differential equations of the general form $\dot{X}=f(X,t,\gamma)$, $X\in R^5$ where: $X=(v_y,\omega,y,\varphi,\delta_s)^T$.

The parameters m, I_Z, ., were grouped under the γ symbol.

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3. Analysis of the influence of physical parameters on the stability of the system

The numerical parameters used in this mathematical analysis are:

For the case of a car with a load of 1 people: v=25 km/h, a= 1113 mm, b = 1457 mm, m=1358kg, Iz= 2611Kgm2;

For the case of a car with a load of 5 people: v=25 km/h, a = 1234 mm, b = 1336 mm, m=1667kg, Iz=2802 Kgm2;

Further, the numerical solutions of the proposed mathematical model are presented, in the case of a movement after a sinusoidal trajectory and the influence of the variation of the Iz parameter on the dynamic behavior of the vehicle-driver system is analyzed.

System solutions represent: S<0>=t , S<1>=vy, S<2>= ω , S<3>=y, S<4>= φ , S<5>= δ_s ;

The case of single-passenger vehicle loading:

System solutions are:

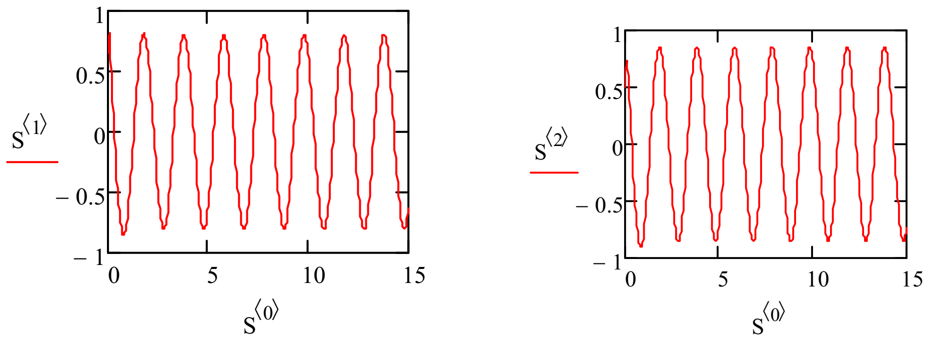
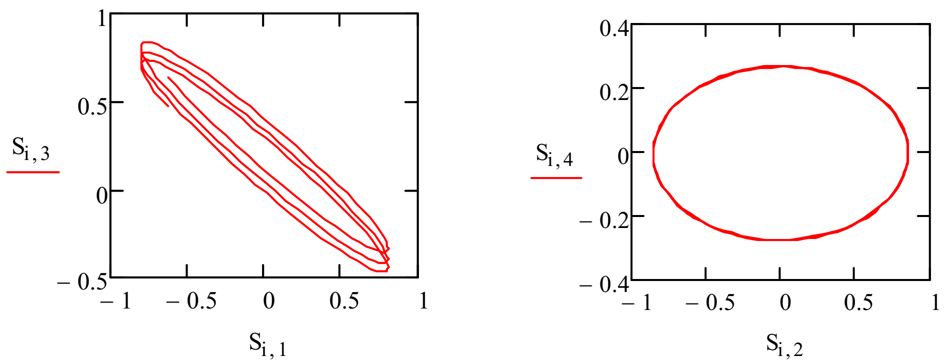


Diagram in the phase space:



Wheel lateral forces:

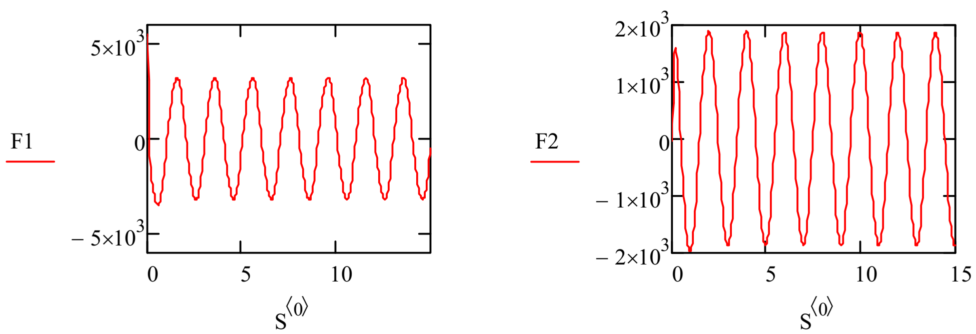


Fig. 2. The case of single-passenger vehicle loading

The movement is periodically stable, the amplitude of the lateral speed is 0.8, the amplitude of the angular velocity is 0.75 rad / sec and the wheel force amplitude is approximately 3500 (in case of F1) and 2000 (in case of F2).

The case of loading the vehicle with five passengers:

System solutions are:

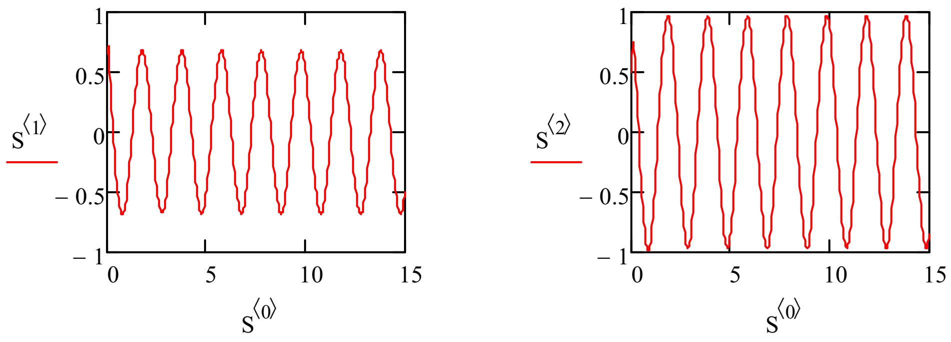
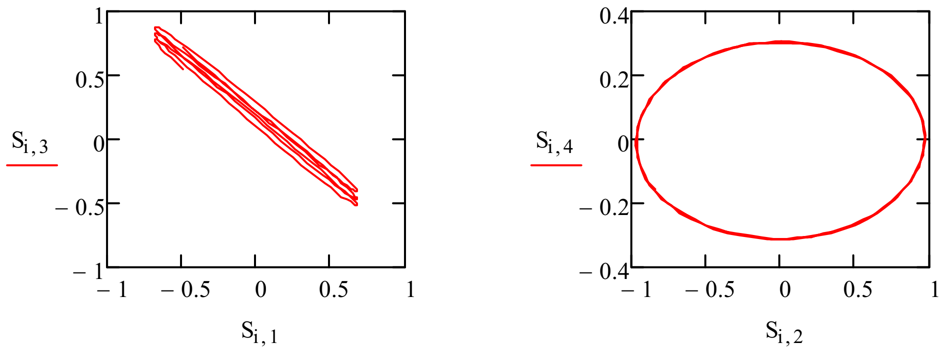


Diagram in the phase space:



Wheel lateral forces:

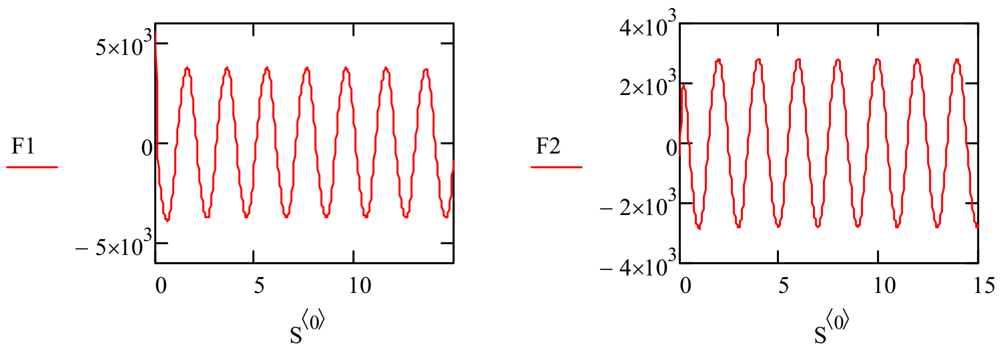


Fig. 3. The case of five-passengers vehicle loading

The movement is periodically stable, the amplitude of the lateral speed is 0.7, the amplitude of the angular velocity is 0.98 rad / sec and the wheel force amplitude is approximately 3800 (in case of F1) and 2800 (in case of F2).

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4. Conclusions

For the proposed mathematical model and for the types of motion analyzed, variation of the moment of inertia (made by changing the load of the vehicle) changes both the movement character and the motion parameters, both by the amplitude of the angular velocity and the lateral velocity, and more particularly by the amplitude of the lateral force on the wheel.

Analyzing the previous results, we find the following: a variation of the inertia moment of approximately 20% leads to a variation of the lateral speed amplitude by 12%, a variation of the angular velocity of 26% and a variation of lateral forces up to 28%, reaching values comparable to the force of adhesion, which causes the vehicle to skid.

The variation of momentum of inertia produces a change in system behavior, from a periodic movement with 3 subarmons to a quasi-periodic movement with 5 subarmons.

These conclusions refer to the simplified plan layout where the center of the table is placed in the plane of the tread and on the symmetry axis of the car. If computational possibilities allow the analysis of some car models, including other physical and geometric parameters (unsuspended masses and suspended mass, rigidity of tires and suspension), the moment of inertia can influence the character of the movement in a considerable way.

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