

Mathematical model for studying cyclist kinematics in vehicle-bicycle frontal collisions

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Abstract. For the development of effective vehicle related safety solutions to improve cyclist protection, kinematic predictions are essential. The objective of the paper was the elaboration of a simple mathematical model for predicting cyclist kinematics, with the advantage of yielding simple results for relatively complicated impact situations. Thus, the use of elaborated math software is not required and the calculation time is shortened. The paper presents a modelling framework to determine cyclist kinematic behaviour for the situations in which a M1 category vehicle frontally hits the rear part of a bicycle. After the primary impact between the vehicle front bumper and the bicycle, the cyclist hits the vehicle's bonnet, the windscreen or both the vehicle's bonnet and the windscreen in short succession. The head-windshield impact is often the most severe impact, causing serious and potentially lethal injuries. The cyclist is represented by a rigid segment and the equations of motion for the cyclist after the primary impact are obtained by applying Newton's second law of motion. The impact time for the contact between the vehicle and the cyclist is yielded afterwards by formulating and intersecting the trajectories for two points positioned on the cyclist's head/body and the vehicle's windscreen/bonnet while assuming that the cyclist's equations of motion after the primary impact remain the same. Postimpact kinematics for the secondary impact are yielded by applying linear and angular momentum conservation laws.

1. Introduction

Bicycling is an economical, healthy and environmentally friendly mode of transportation. In 2010, 7,4% of EU cyclists used the bicycle as a main mode of transport [1]. In Europe, as the number of bicycles and their usage in urban traffic is predicted to grow based on the increased policy interest in promoting cycling [2], new concerns arise regarding cyclist safety. Cyclists are non-motorized road users which present a high degree of vulnerability in traffic and a low degree of observability during night time. Accident statistics show that although driving safety significantly improved in the last few decades, cycling safety did not improve at the same rate [3]. Thus, additional efforts in accident analysis have to be made in order to understand cyclist kinematics (velocity, projection distance) and dynamics (interacting forces, motion and equilibrium) following a motor vehicle based collision.

Cyclist kinematics are characterized by a high degree of variability. One challenge here is the lack of current vehicle-cyclist impact formulations, given by the general assumption that cyclist and pedestrian kinematics, dynamics and injury causation mechanisms present a high degree of similarity. Therefore, safety standards and measures regarding the design of vehicles frontal profiles for pedestrian impacts are considered to cover a part of the cyclist impact typologies [4-10].



While the primary impact between the bumper and the bicycle/pedestrian leg differs in terms of inertia and geometry, a relatively high degree of similarity can be indeed established for the impact between the human head and the vehicle's bonnet, windshield or roof. However, the Wrap Around Distance (WAD) measured along the vehicle profile from the ground to the cyclist/pedestrian head impact location is higher for cyclists than for pedestrians [11], which overall increases the vertical coordinate of the point of impact, modifies the angle of impact and consequently leads to more severe head impacts.

A second challenge is the increased modelling complexity of vehicle-bicycle collisions in comparison to vehicle-pedestrian collisions, caused by the addition of the bicycle and by the interactions which appear between the bicycle and the cyclist.

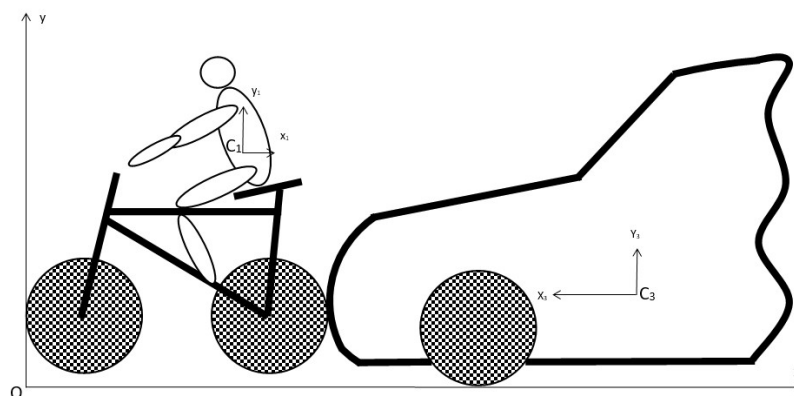
The study presented here shows a simple two-dimensional approach for front-rear vehicle-bicycle collisions, divided into three stages: 1) *Mathematical modelling of the primary vehicle-bicycle impact*
2) *Determining post-primary impact cyclist kinematics and the time of the secondary impact*
3) *Mathematical modelling of the secondary cyclist-vehicle impact*

2. Methods

In order to simplify the model and reduce the number of unknown variables, the following hypotheses were formulated:

- 1) The primary vehicle-bicycle impact:
 - a. during the primary impact, the cyclist weight force is applied only to the bicycle's saddle
 - b. handlebar and pedal forces are neglected
 - c. vehicle velocity losses are neglected
- 2) Post-primary vehicle-bicycle impact:
 - a. between the primary and secondary impact, no other contacts occur
 - b. after the primary impact, gravity is the only force acting on the cyclist
 - c. bicycle and vehicle post impact vertical velocities are neglected
 - d. air resistance has a negligible effect upon the cyclist
- 3) The secondary cyclist-vehicle impact:
 - a. the contact occurs for a very short time
 - b. the impact does not produce a moment around the vehicle's center of mass

The cyclist is considered a rigid body. Following the primary impact between the vehicle and the bicycle (fig 1.a.), the cyclist will be subjected to an air projection towards the vehicle's frontal profile. Depending on the height of the bicycle's saddle and on the vehicle's impact velocity and frontal profile geometry, the cyclist hits the vehicle's bonnet (fig 1.b.), the windscreen (fig 1.c.) or both the vehicle's bonnet and the windscreen in short succession. The objective of this paper is to study cyclist kinematic parameters for the first two cases.



a. Primary impact schematised

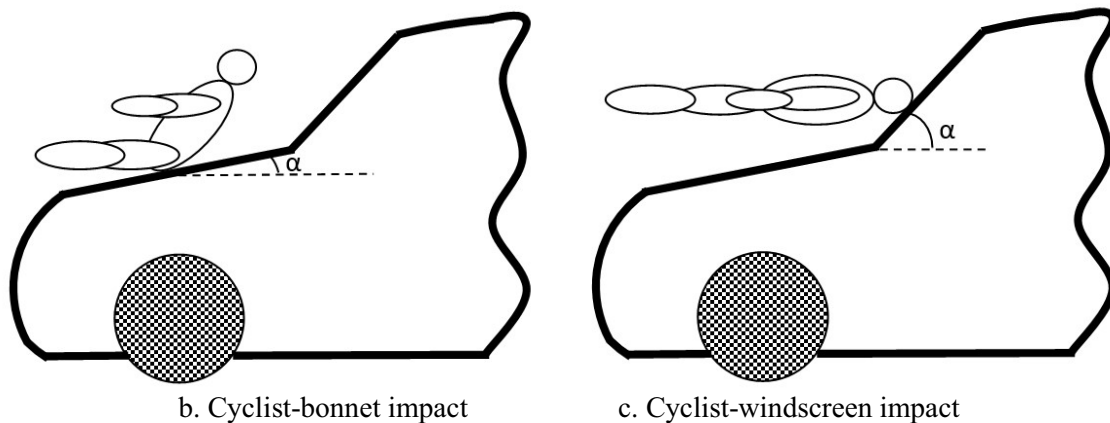


Fig. 1 (a, b, c) Primary impact and possible secondary impact configurations

The global origin is positioned with the Ox axis at ground level and Oy axis vertically upwards, tangential to the bicycle's front wheel. The cyclist coordinate system origin is the centre of mass of the cyclist (C_1) during the instant of the primary impact between the bicycle and the vehicle ($t_0=0$). The origin of the vehicle's coordinate system is at its centre of mass (C_3) with the positive Ox axis direction aligned with the vehicle's travel direction and the positive Oy axis direction vertically upwards.

3. Mathematical modelling of the primary vehicle-bicycle impact

During the primary impact, the resultant impact force F_e is applied on the bicycle's rear wheel with its corresponding force pair acting upon the vehicle's bumper, and a pair of resultant contact forces F_b appear and interact with the bicycle's saddle and the cyclist's prop point (fig. 2).

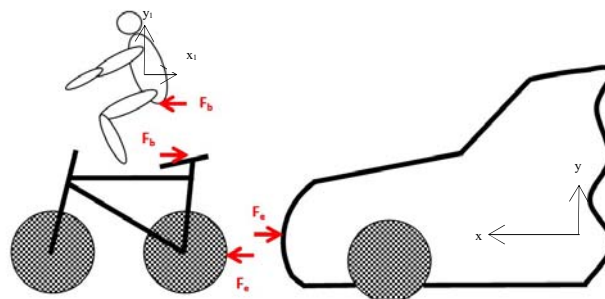


Fig. 2. Pairs of forces which appear during the primary impact

Upon the cyclist's center of mass (C_1) acts its own weight force G_1 while in its prop point (A) acts the bicycle's saddle afferent normal force N_A (fig 3).

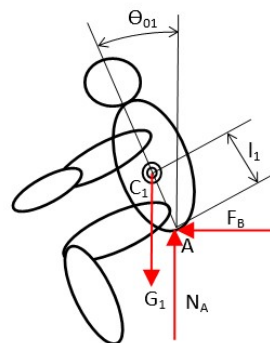


Fig. 3. Forces which act upon the cyclist during the primary impact

Considering the cyclist, the bicycle and the vehicle as rigid segments and applying the second principle of mechanics during the primary impact, the following system of equations results, where the 1, 2 and 3 indices stand for the cyclist, the bicycle and the vehicle respectively:

- For the cyclist

$$m_1 \ddot{x}_1 = -F_b \quad (1)$$

$$m_1 \ddot{y}_1 = -G_1 + N_A \quad (2)$$

- For the bicycle

$$m_2 \ddot{x}_2 = F_b - F_e \quad (3)$$

- For the vehicle

$$m_3 \ddot{x}_3 = F_e \quad (4)$$

Where \ddot{x}_1 , \ddot{x}_2 and \ddot{x}_3 are the center of mass acceleration components on the Ox axis, \ddot{y}_1 is the cyclist's center of mass acceleration component on the Oy axis and m_1 , m_2 , and m_3 are physical body masses.

From the condition of moments equilibrium taken about the cyclist's center of mass C_1 point, the following equation is yielded:

$$J_1 \ddot{\theta}_1 = -F_b l_1 \cos \theta_{01} + N_A l_1 \sin \theta_{01} \quad (5)$$

All the equations can be written in the form:

$$\begin{bmatrix} m_1 & & & \\ & m_1 & & \\ & & J_1 & \\ & & & m_2 \\ & 0 & & & m_3 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{y}_1 \\ \ddot{\theta}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{bmatrix} = \begin{bmatrix} -F_b \\ -G_1 + N_A \\ -F_b l_1 \cos \theta_{01} + N_A l_1 \sin \theta_{01} \\ F_b - F_e \\ m_3 \ddot{x}_3 = F_e \end{bmatrix} \quad (6)$$

Where θ_{01} is the initial angular displacement of the cyclist, $\ddot{\theta}_{01}$ is the cyclist's angular acceleration, l_1 is the distance from the cyclist's center of mass to its prop point A on the saddle of the bicycle and J_1 is the cyclist's moment of inertia.

The cyclist's velocity is equal to the bicycle's velocity during the instant of the primary impact between the bicycle and the vehicle. Considering that the bicycle's and vehicle's movement in the aftermath of the primary impact is solely on the Ox axis and neglecting vertical velocities which may appear depending on the difference between the bicycle's and vehicle's centers of mass vertical coordinates ($\dot{z}_2 = 0$; $\dot{z}_3 = 0$), the following equation is yielded:

$$\dot{x}_2 = \dot{x}_A \quad (7)$$

A good approximation that can avoid solving the equation (1)-(4) is to consider that the vehicle transfers entirely its velocity to the point A of the cyclist:

$$\dot{x}_2 = \dot{x}_A = \dot{x}_3 \quad (8)$$

In this case it can be considered that the vehicle's velocity at impact is the initial velocity (if there is no braking).

During the plastic impact of the bicycle, the kinematic conditions are:

$$x_1 = x_2 + l_1 \sin \theta_1 \quad (9)$$

$$y_1 = y_2 + l_1 \cos \theta_1 \quad (10)$$

and after differentiation:

$$\dot{x}_1 = \dot{x}_2 + l_1 \dot{\theta}_1 \cos \theta_1 \quad (11)$$

$$\dot{y}_1 = -l_1 \dot{\theta}_1 \sin \theta_1 \quad (12)$$

and

$$\ddot{x}_1 = \ddot{x}_2 + l_1 \ddot{\theta}_1 \cos \theta_1 - l_1 \dot{\theta}_1^2 \sin \theta_1 \quad (13)$$

$$\ddot{y}_1 = -l_1 \ddot{\theta}_1 \sin \theta_1 - l_1 \dot{\theta}_1^2 \cos \theta_1 \quad (14)$$

$$\begin{bmatrix} \ddot{x}_1 \\ \ddot{y}_1 \\ \ddot{\theta}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{bmatrix} = \begin{bmatrix} l_1 \cos \theta_1 & 1 & 0 \\ -l_1 \sin \theta_1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{bmatrix} + \begin{bmatrix} -l_1 \sin \theta_1 \\ -l_1 \cos \theta_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \dot{\theta}_1^2 \quad (15)$$

By substituting (15) in (7) and premultiplying the equations with

$$\begin{bmatrix} l_1 \cos \theta_1 & 1 & 0 \\ -l_1 \sin \theta_1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^T$$

The following equations are yielded:

$$\begin{bmatrix} m_1 l_1^2 \cos^2 \theta_1 + m_2 l_1^2 \sin^2 \theta_1 + J_1 & m_1 l_1 \cos \theta_1 & 0 \\ m_1 l_1 \cos \theta_1 & m_1 + m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{bmatrix} + \begin{bmatrix} -(m_1 + m_2) l_1^2 \sin \theta_1 \cos \theta_1 \\ -m_1 l_1 \sin \theta_1 \\ 0 \end{bmatrix} \dot{\theta}_1^2 = \begin{Bmatrix} G_1 l_1 \sin \theta_1 - 2F_b l_1 \cos \theta_1 \\ -F_e \\ F_e \end{Bmatrix} \quad (16)$$

It is possible to obtain the differential motion equations of the system. There are 3 differential equations with 3 unknowns x_2 , θ_1 and x_3 .

4. Determining post-primary impact cyclist kinematics and the time of the secondary impact

Following the primary impact, the cyclist is launched towards the vehicle's frontal profile on a parabolic trajectory. This yields the following equations of motion for the cyclist's center of mass C_1 :

$$x_1 = x_{01} + \dot{x}_1 t \quad (17)$$

$$y_1 = y_{01} - \frac{gt^2}{2} \quad (18)$$

Where t_1 is the time of the secondary impact between the cyclist's head/body with the vehicle's windscreen/bonnet and g is the acceleration of gravity.

The secondary impact takes place when a point P positioned on the cyclist's head/body reaches a point Q positioned on the vehicle's windscreen/bonnet (fig 4).

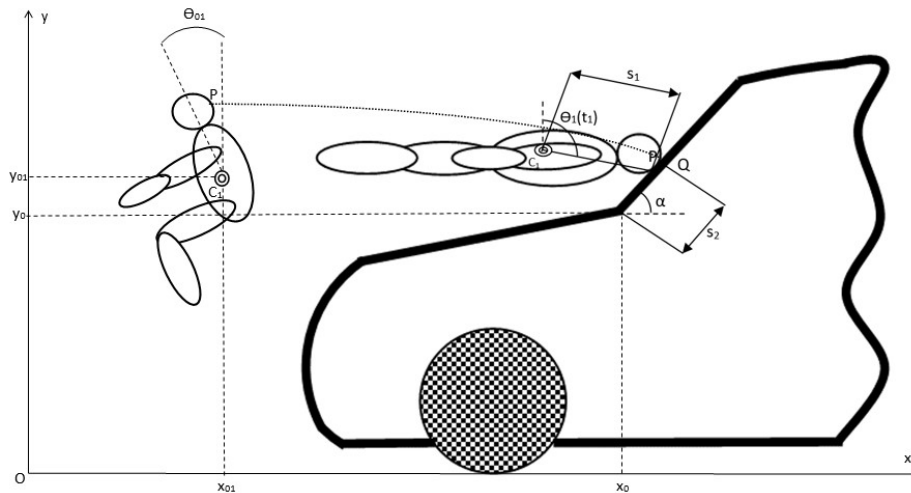


Fig 4. – Cyclist post-primary impact kinematics and secondary impact configuration

The secondary impact time can be determined by intersecting the trajectories of points P and Q. For point P the following equations of motion result:

$$x_P = x_1 + s_1 \cos \theta \quad (19)$$

$$y_P = y_1 + s_1 \sin \theta \quad (20)$$

$$\theta = \theta_{01} - \omega_0 t \quad (21)$$

Where s_1 is the distance from the cyclist's center of mass C_1 to the point P.

Considering the vehicle's bonnet or windscreen as a line of slope $\tan(\alpha)$, the following equations of motion for point Q result:

$$x_Q = x_0 + s_2 \cos \alpha \quad (22)$$

$$y_Q = y_0 + s_2 \sin \alpha \quad (23)$$

Where s_2 is the distance from the vehicle's center of mass C_3 to the point Q and α is the horizontal distance from center of mass C_3 to the starting point of the windscreen/bonnet slope.

From (22) and (23) the following equation is yielded:

$$\frac{x_Q - x_0}{y_Q - y_0} = \frac{\cos \alpha}{\sin \alpha} = \cot \alpha \quad (24)$$

The secondary impact time can be determined by applying the following conditions:

$$x_P = x_Q \quad (25)$$

$$y_P = y_Q \quad (26)$$

From (24), (25) and (26), the following equation results:

$$(x_P - x_0) = (y_P - y_0) \cot \alpha \quad (27)$$

Equation (27) can be rewritten as a transcendent equation:

$$(x_{01} + \dot{x}_1 t + s_1 \cos \theta_1 - x_0) = \left(y_{01} - \frac{gt^2}{2} + s_1 \sin \theta_1 - y_0 \right) \cot \alpha \quad (28)$$

Where θ_1 is the initial angular displacement of the cyclist at the time of the secondary impact t_1 .

Equation (28) doesn't have close-form solutions, it is solved approximately using the bisection method, yielding the time of the secondary impact t_1 .

5. Mathematical modelling of the secondary cyclist-vehicle impact

Considering the secondary impact is produced at t_1 between the cyclist's head and the windscreen as it is the case for collisions with moderate or high vehicle impact velocities, cyclist postimpact kinematic parameters can be determined by applying the laws of conservation of linear and angular momentum (29), (30), (31) and by formulating the equations for the relative motion of the cyclist in relation to the vehicle (32), (33):

$$m_1 \dot{x}'_1 = m_1 \dot{x}_1 + m_3 \dot{x}_3 \quad (29)$$

$$m_1 \dot{y}'_1 + m_3 \dot{y}_3 = m_1 \dot{y}_1 \quad (30)$$

$$J_1 \dot{\theta}'_1 + m_3 s_1 \dot{x}'_1 \cos \theta'_1 - m_3 s_1 \dot{y}'_1 \sin \theta'_1 = J_1 \dot{\theta}_1 + m_3 s_1 \dot{x}_1 \cos \theta_1 - m_3 s_1 \dot{y}_1 \sin \theta_1 \quad (31)$$

$$\dot{x}'_1 + \dot{\theta}'_1 s_1 \cos \theta'_1 - \dot{x}_3 = 0 \quad (32)$$

$$\dot{y}'_1 - \dot{\theta}'_1 s_1 \sin \theta'_1 - \dot{y}_3 = 0 \quad (33)$$

Where \dot{x}_1, \dot{y}_1 are cyclist preimpact velocity components at t_1 , \dot{x}_3, \dot{y}_3 are vehicle preimpact velocity components at t_1 , \dot{x}'_1, \dot{y}'_1 are cyclist postimpact velocity components, \dot{x}_3, \dot{y}_3 are vehicle postimpact velocity components, $\dot{\theta}_1$ is the cyclist's preimpact angular velocity at t_1 , $\dot{\theta}'_1$ is the cyclist postimpact angular velocity, θ_1 is the cyclist's preimpact angular displacement at t_1 and θ'_1 is the cyclist postimpact angular displacement.

The system of equations (29)-(33) is solved using numerical integration, yielding cyclist and vehicle postimpact kinematic parameters.

The modelling for the secondary impact described can also be applied to the contact between the cyclist's body and the vehicle's bonnet when this is the case, however an additional modelling phase would be required to yield the parameters of the tertiary impact between the cyclist's head and the vehicle's windscreen or roof.

6. Input data for the mathematical model

Input data for the model consists of the following constants and initial parameters: physical body masses m_1, m_2, m_3 , resulting force acting upon the primary impact F_e , cyclist moment of inertia J_1 , windshield/ bonnet angle α , initial cyclist angular displacement θ_{01} , cyclist position at primary impact x_{01}, y_{01}, s_1 , vehicle position at primary impact x_0, y_0, s_2 and vehicle, bicycle and cyclist initial velocities $\dot{x}_1, \dot{x}_2, \dot{x}_3$.

Example: Considering a vehicle having a velocity of 72 km/h. The cyclist has a velocity of 36 km/h. In the short time of the crash the vehicle's velocity practically doesn't change. The cyclist receives a momentum that changes the velocity of his center of mass, to 54 km/h. At the same time the cyclist receives a moment of momentum which causes a spin, and the velocity for the contact point between the cyclist and the bicycle becomes 72 km/h. The angular velocity of the cyclist, which receives a rotation motion around his mass center, becomes 50 rad/s. The time between the moment in which the bicycle is collided by the vehicle and the moment of the contact between cyclist and the vehicle (windscreen) is, for example, 0,04 s and the rotation angle of the cyclist in this time is approx. 2 rad. With this rotation (approximately 114°), the cyclist's head reaches and collides the windscreen at its base (as it is the case in the most situations for this accident typology).

7. Conclusions

A simple mathematical model for predicting the impact and cyclist kinematics after impact that provides insights into the crash dynamics was elaborated. The rigid body approach yields simple results for relatively complicated impact situations, does not require the use of elaborated math software and the calculation time is significantly shortened. The main advantage of this approach is the necessary time and cost to obtain some conclusions concerning an impact. Cyclist kinematic parameters are a result of the primary impact between the vehicle and the bicycle and are directly influenced by inertia, geometry of the vehicle and cyclist's riding position at impact.

The mathematical model developed has the following limits:

- 1) is a simple two-dimensional model with concentrated masses;
- 2) the cyclist is represented by a rigid body and therefore deformations cannot be calculated
- 3) the model takes into account only one collision between the cyclist and the vehicle's frontal profile.

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