

Fatigue Analysis of Rotating Parts. A Case Study for a Belt Driven Pulley.

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Abstract. The present study is focused on the life estimation of a rotating part as a component of an engine assembly namely the pulley of the coolant pump. The goal of the paper is to develop a model, supported by numerical analysis, capable to predict the lifetime of the part. Starting from functional drawing, CAD Model and technical specifications of the part a numerical model was developed. MATLAB code was used to develop a tool to apply the load over the selected area. The numerical analysis was performed in two steps. The first simulation concerned the inertia relief due to rotational motion about the shaft (of the pump). Results from this simulation were saved and the stress – strain state used as initial conditions for the analysis with the load applied. The lifetime of a good part was estimated. A defect was created in order to investigate the influence over the working requirements. It was found that there is little influence with respect to the prescribed lifetime.

1. Introduction

In the present study, we assessed lifetime of pulley by numerical methods. The pulley is a component part of a belt drive. The drawing process was used to manufacture the pulley. The pulley works under both the fatigue compressive loading due to belt tension and the torque due to rotation. Therefore, to assure the safety of the belt drive system is very important to investigate the lifetime and optimization of the pulley [1]. According to [2], optimization of this part has been widely carried out through the stress analysis based approach. Common applications in the automotive industry include crankshafts, water pumps, air-conditioner compressors and power steering pumps. Although the shape and the usage of pulleys are very simple, evaluating the design of a pulley is difficult because of the loading conditions and installation environment. According to [1] Euler, first investigated the belt-drive mechanics and established the relationship between belt tension and coefficient of friction and Reynolds, developed creep theory to explain speed loss in power transmission belts. Furthermore, several researchers have studied the mechanics of V-belts, for example, [3] who described the relationship between ribs shear deformation and transmitted force. The applied stress distribution of the pulley under high-tension and torque was obtained by using FEA (Finite Element Analysis). The main features of fatigue analysis are listed below:

- Develop the three-dimensional model of the pulley, the FE (Finite Element) model;
- Perform critical stress/strain analysis under multiple loading;
- Process the maximum principal stress and strain history obtained by analysis with rotational angle;
- Input the probabilistic S–N (P–S–N) curve into an analytical tool;
- Compute the lifetime of the pulley at a specification level.



Pulleys are one of the main components used in a belt conveyor, supporting the loads generated from the belt tension according to its wrapping angle [7]. Few available reports [7,8] have shown that the majority of failure of pulleys are due to fatigue or overload failures, whereas few other investigators [9] have attributed a typical failure of a pulley to material and manufacturing process fault analyzing after detailed microstructural and finite element analysis. According to SS-EN 13306 [10], failure is the termination of the ability of an item (i.e., any part, component, device, subsystem, functional unit, component, or system that can be individually considered) to perform a required function. In other words, failure can be referred to as an event or process of component deterioration.

2. Model description and technical specifications

Starting from the CAD model of the part (Figure 1) a numerical model was developed. As specified in the functional drawing the bolts used to assembly the pulley on the pump's rotor were represented in the model. Figure 2 presents the numerical model that will be further used.



Figure 1. Pulley model (as generated)

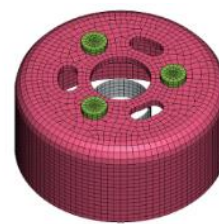


Figure 2. Numerical model

The part must resist 2.10^7 alternated stresses under a static load of 1016 N applied on an angle sector of 97° and 28mm width (Figure 3).

A custom MATLAB code was developed as a tool to apply the load over the selected area. The load is applied for each circumferential sector using a SIN function. The tool can also transform to load and apply it on the required area of the part (Figure 4).

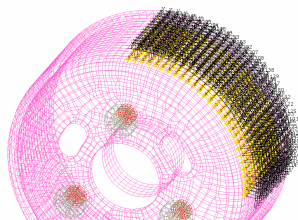


Figure 3. Area selected for the applied load

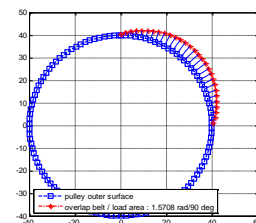


Figure 4. Load definition

For each node on the selected area a user defined coordinate system was created normal to the surface. The load defined by the *SIN* function and scaled accordingly was applied on the node along local Ox axis.

3. Fatigue Analysis

In the design of machine elements [4], three fundamental equations are used:

$$\begin{aligned}\sigma_{c,t} &= \frac{P}{A} \\ \sigma_b &= \frac{M_b \cdot y}{I} \\ \tau &= \frac{M_t \cdot R}{J}\end{aligned}\tag{1}$$

They are referring to the stress for compression/tension cases, bending or torsion. The notations used in set of equations (1) are: P - axial load; A - cross sectional area; M_b - bending moment; y - coordinates of a point on the cross section at which the stress is to be determined; I - area moment of inertia of the section; M_t - applied torque; R - radius of the section; J - torsion constant of the section.

3.1. S-N curve

The fatigue or endurance limit of a material is defined as the maximum amplitude of completely reversed stress that the standard specimen can sustain for an unlimited number of cycles without fatigue failure. The results of these tests are plotted by the means of the S-N curve. The S-N curve is the graphical representation of the stress (S_f) versus the number of stress cycles (N_f) before the fatigue failure on a log-log chart.

3.2. Endurance limit

When the laboratory data regarding the endurance limit of the materials are not available, the following procedure is adopted.

Two separate notations are used regarding the endurance limit (S'_e) and (S_e) where:

- S'_e is the stress of a rotating beam specimen subjected to reversed bending stress;
- S_e is the stress of a particular mechanical component subjected to reversed bending stress.

There is an approximate relationship between the endurance limit and the ultimate tensile strength (S_{ut}) of the material (e.g. for steels):

$$S'_e = 0.5 \cdot S_{ut} \quad (2)$$

The relationship between (S'_e) and (S_e) is as follows:

$$S_e = k_a \cdot k_b \cdot k_c \cdot k_d \cdot S'_e \quad (3)$$

where:

- k_a is surface finish factor that for machined of cold – drawn parts is given by the equation $k_a = a \cdot (S_{ut})^b$ where $a = 4.51$ and $b = -0.265$.
- k_b is the size factor which for larger the machine part, greater is the probability that a flaw exist somewhere in the volume and in this case of a value of 0.85 was assigned.
- k_c is the reliability factor that the greater the likelihood that a part will survive, the more is the reliability thus a value of 0.659 was assigned.
- k_d is the factor to account for stress concentration

4. Results. Structural analysis

4.1. Inertia Relief

The numerical simulation was performed in two steps with the first step focused on the inertia relief due to rotational motion about the shaft (of the pump). Results are presented in Figure 5.

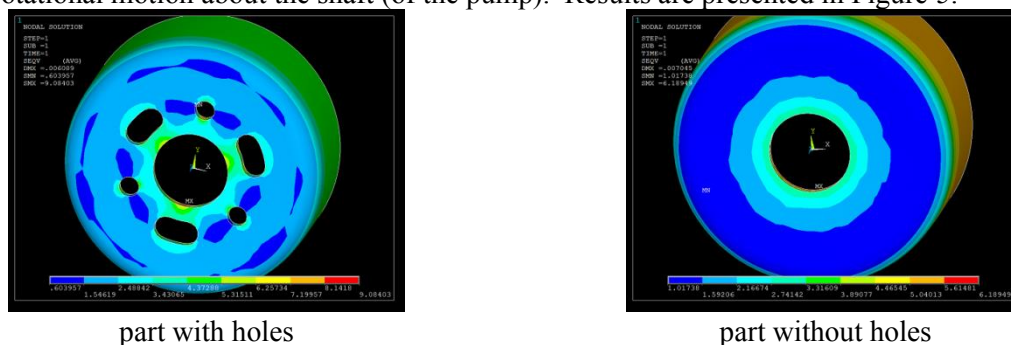


Figure 5. Inertia relief. Stress analysis

The stress recorded on the bolt location has a value of about 7 MPa while the stress on the flange has a value of about 3 MPa. Results from this simulation were saved and the stress – strain state of the assembly will be further used as initial conditions for the analysis with the load applied. Stress concentration is defined as the localization of high stresses due to the irregularities present in the component and the abrupt changes of the cross section. The modifying factor k_d to account for the effect of stress concentration is defined as:

$$k_d = \frac{1}{k_f} \quad (4)$$

where k_f is the stress concentration factor [4].

Based on results of the inertia relief analysis, the stress concentration factor can be evaluated as follows [4]:

$$k_{f,b} \approx \frac{7.0}{4.5} = 1.55$$

$$k_{f,f} = \frac{5.0}{3.5} = 1.45 \quad (5)$$

where:

- - $k_{f,b}$ is the stress concentration factor for the bolt holes;
- - $k_{f,f}$ is the stress concentration factor for the flange;

4.2. Analysis of an undamaged part

An repeated load cycle with an amplitude of 1 was defined for the load. The resulting (computed) values of the applied load are presented in Figure 6.

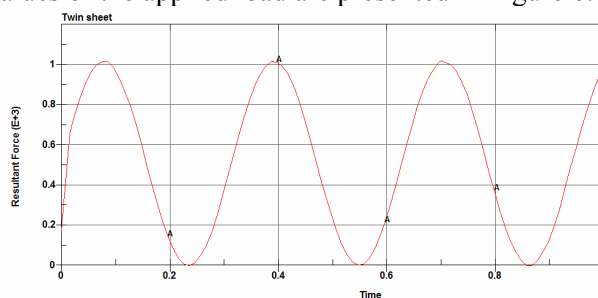


Figure 6. Resultant applied load (as computed)

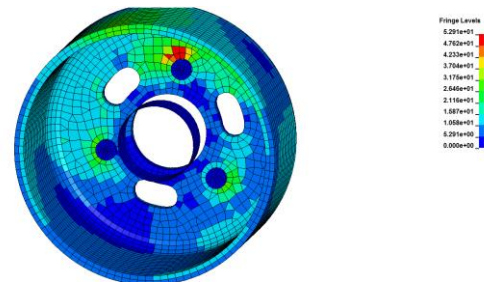


Figure 7. Stress state

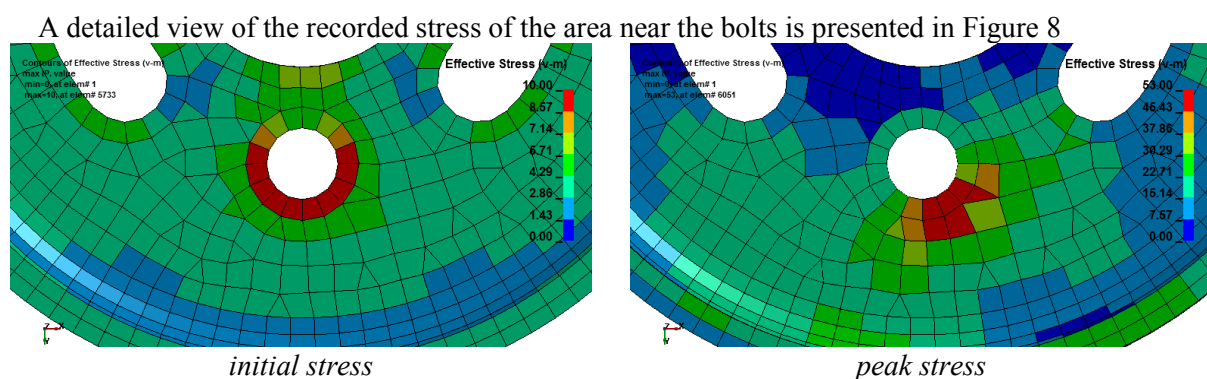


Figure 8. Stress history at the bolt location.

A detailed view of the recorded stress on the flange of the pulley is presented in Figure 9.

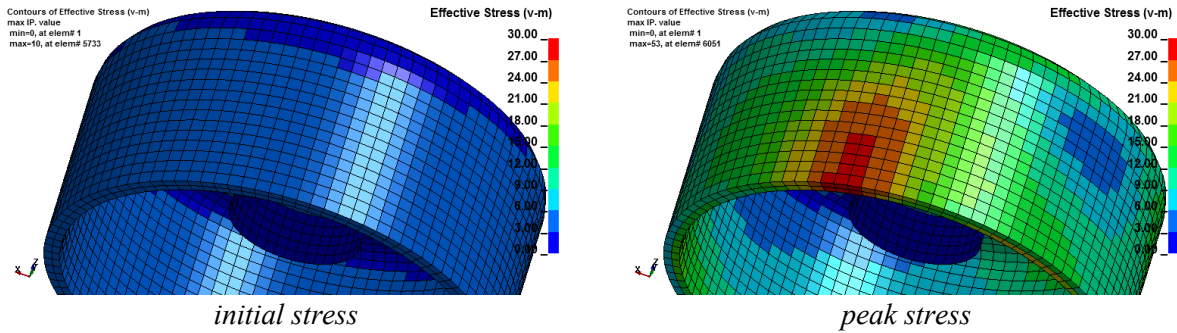


Figure 9. Stress history on the flange.

Using the data plotted in Figure 7 and Figure 8 the following input parameters can be evaluated:

- the maximum stress: σ_{max} (**53 MPa** recorded **near the bolt hole** during loading / **27 MPa** recorded **on the flange** during loading).
- the minimum stress: σ_{min} (**7 MPa** recorded **near the bolt hole** during inertia relief / **3.5 MPa** recorded **on the flange** during inertia relief).

Therefore:

- the mean stress [4] (table 3):

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} \quad (6)$$

- the stress amplitude [4] (table 3):

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} \quad (7)$$

- yield stress [5]: $S_{yt} = 210 \text{ MPa}$ (material **DC 04** DIN EN 10130:2006)
- ultimate tensile stress [5]: $S_{ut} = 350 \text{ MPa}$ (material: **DC 04** DIN EN 10130:2006)
- surface finish factor [6, 4]: $k_a = 4.51 \cdot (350)^{-0.265} = 0.995$;
- size factor [6, 4]: $k_b = 1.0$;
- reliability factor [4]: $k_c = 0.659$;
- stress concentration factor [4]: k_d ;
- endurance limit stress [4]

$$S'_e = 0.5 \cdot 350 = 175 \text{ MPa} \quad (8)$$

- endurance limit stress [4]

$$S_e = k_a \cdot k_b \cdot k_c \cdot k_d \cdot S'_e \quad (9)$$

The S-N curve (Figure 10) will be used to determine the number of working cycles before failure.

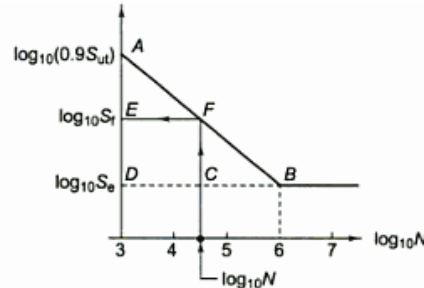


Figure 10. S-N curve [4]

The coordinates of failure point F can be determined using the equation of the \overline{AB} line and the stress amplitude σ_a and the mean stress σ_m .

$$S_f = \frac{\sigma_a \cdot (0.9 \cdot S_{ut})}{(0.9 \cdot S_{ut}) - \sigma_m} \quad (10)$$

The abscissa points can be determined:

- $p_1 = \log_{10}(0.9 \cdot S_{ut})$;
- $p_2 = \log_{10}(S_e)$;
- $p_3 = \log_{10}(S_f)$;

Table 1 presents the results of this study.

Table 1. Results

Parameter	Bolt location	Flange location
S_{yt}	210.000	210.000
S_{ut}	350.000	350.000
σ_{max}	53	27
σ_{min}	7	3.5
σ_m	30	15.25
σ_a	23	11.75
k_f	1.550	1.450
k_d	0.645	0.690
S'_e	175.000	175.000
S_e	60.397	64.562
S_f	25.421	12.348
$p_1 = \log_{10}(0.9 \cdot S_{ut})$	2.4983	2.4983
$p_2 = \log_{10}(S_e)$	1.7810	1.8100
$p_3 = \log_{10}(S_f)$	1s.4051	1.0916
n_f	7.5717	9.1310
$N_f = 10^{n_f}$	3.73E+07	1.35E+09

Fatigue diagrams are presented in Figure 11 and Figure 12.

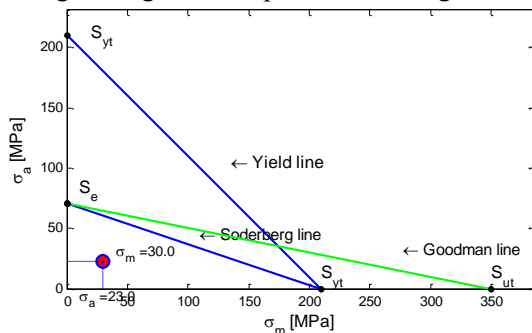


Figure 11. Fatigue diagram for bolt location

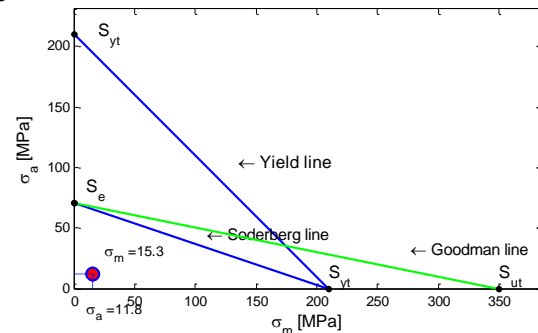
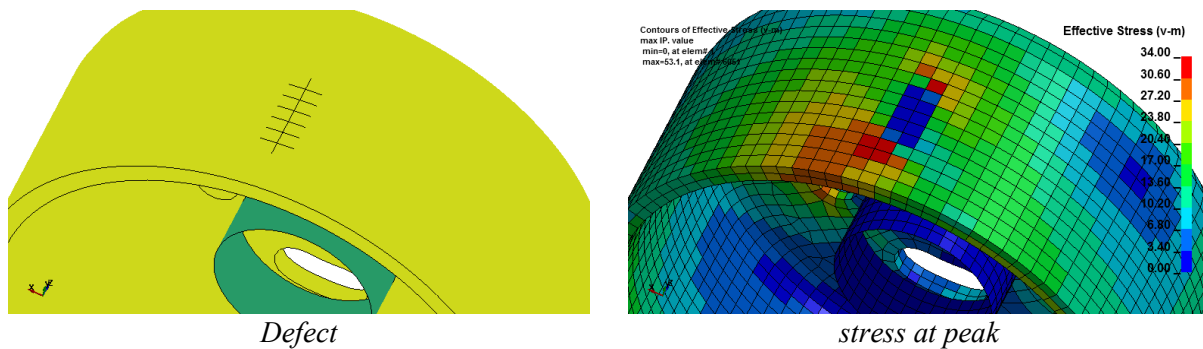


Figure 12. Fatigue diagram for the flange

4.3. Analysis of a damaged part

A simple damage was defined on the flange of the part. The damage simulates a crack with a length of 20 mm. In order to produce the damage the elements were disconnected. There were not used any specialized card to define the crack.

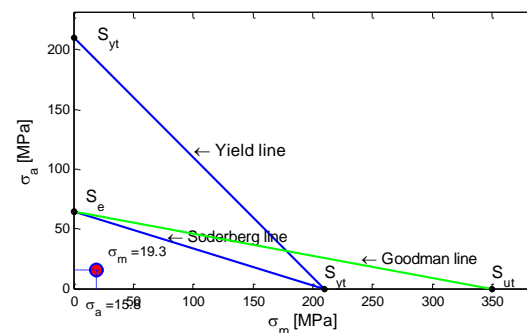
Figure 12 presents both the defect and the stress state at peak load on the flange of the part.

**Figure 13.** Part with discontinuities

It can be noticed that there is a discontinuity in the stress on the area with the defect. Considering the above mentioned elements the life time can be estimated by the values listed in Table 2.

Table 2. Results

Parameter	Flange location
S_{yt}	210.000
S_{ut}	350.000
σ_{max}	35
σ_{min}	3.5
σ_m	19.25
σ_a	15.75
k_f	1.450
k_d	0.690
S'_e	175.000
S_e	64.562
S_f	16.775
$p_1 = \log_{10}(0.9 \cdot S_{ut})$	2.4983
$p_2 = \log_{10}(S_e)$	1.8100
$p_3 = \log_{10}(S_f)$	1.2247
n_f	8.551
$N_f = 10^{n_f}$	3.56E+08

**Figure 14.** Fatigue diagram for the damaged flange

5. Conclusions

The paper presents a simple methodology useful to investigate the lifetime. Numerical analysis was used to obtain input data for the mathematical model. The analysis show that considering the requirements the part complies with the prescribed working conditions.

For the selected material **DC 04** the estimated number of working cycles is 3.73E+07 above the required limit of 2.1e7. There were analyzed points located on the top of the part (bolt fixation) and points on the flange (circumference).

A second model uses a simple method to define a defect of the part. The defect is of the type of a crack and is located on the flange. The numerical analysis showed, as expected, that there is discontinuity in the stress. Considering the previously used model the life estimation analysis was also performed.

Results show that this type of defect does not affect the functionality of the part. This was some additional rejection criteria can be defined that can account for the existence of defects on the part.

References

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