IOP Conf. Series: Materials Science and Engineering **251** (2017) 012045 doi:10.1088/1757-899X/251/1/012045

Designing a supply chain of ready-mix concrete using Voronoi diagrams

E Kozniewski[1], M Orlowski[1] and Z Orlowski[2]

[1] Department of Geoinformation and Spatial Economy,
Bialystok University of Technology
15-351 Białystok, ul. Wiejska 45E, Poland
[2] Department of Geomechanics, Building and Geotechnics,
AGH University of Science and Technology,
30-059 Kraków, al. Mickiewicza 30, Poland

E-mail: m.orlowski@pb.edu.pl

Abstract. Voronoi diagrams are used to solve scientific and practical problems in many fields. In this paper Voronoi diagrams have been applied to logistic problems in construction, more specifically in the design of the ready-mix concrete supply chain. Apart from the Voronoi diagram, the so-called time-distance circle (circle of range), which in metric space terminology is simply a sphere, appears useful. It was introduced to solve the problem of supplying concrete-related goods.

1. Introduction

Voronoi diagrams are used in many fields ranging from Photogrammetry for (semi) automatic urban model reconstruction [6, 7, 15, 16, 18, 21] reconstruction of roads from satellite imagery [3], in Cartography [11], analysis of morphology of granular materials, in Medicine for representation, reconstruction and visualization of human organs [4], topographical analysis, Computer Graphics in the field of object recognition and analysis [17] as well as a geographical study of school attendance areas [1, 2]. This concept is introduced on the basis of distance, in a specific metric space [9]. The concept of the Voronoi diagram may be helpful in many problems where we are dealing with distance optimization. In this paper Voronoi diagrams have been applied to logistic problems in construction, more specifically in the design of the ready-mix concrete supply chain. Apart from the Voronoi diagram, the so-called time-distance circle (circle of range), which in metric space terminology is simply a sphere, appears useful. It was introduced to solve the problem of supplying concrete-related goods.

2. Voronoi Diagram

A *metric space* $(\boldsymbol{M},d)$ is a set M in which each pair a, b of its elements is assigned a real non-negative number $d(a, b)$ satisfying the following postulates:

$$d(a,b) = 0 \Leftrightarrow a=b \text{ (identity axiom)}, \quad \text{(i)}$$
$$d(a,b) = d(b,a) \text{ (symmetry)}, \quad \text{(s)}$$
$$d(a,b) + d(b,c) \geq d(a,c) \text{ (triangle inequality)}. \quad \text{(t)}$$

Function *d(a, b),* which satisfies the postulates (i), (s), (t), is called *a metric (distance)* of space $\boldsymbol{M}$. In metric space ($\boldsymbol{M}$,d), the (*closed*) *sphere* centered at a with radius r>0 is defined as a set of points $x\in\boldsymbol{M}$ contained in:

$$\boldsymbol{B}(a, r) = \{x\in\boldsymbol{M}: d(a,x) \le r\}. \quad \text{(b)}$$

The open sphere $\boldsymbol{B}(a, r)$ is a set of points whose distance from a is less than r.
Let us assume that $\boldsymbol{S}=\{a_1,a_2,\ldots,a_n\}$ is a set of n points in metric space ($\boldsymbol{M}$,d). Each point is assigned a Voronoi region $\boldsymbol{VD}(a_i)$,

$$\boldsymbol{VD}(a_i) = \{x\in\boldsymbol{M}: \forall_{i\neq j,\ i,j=1,2,\ldots,n}\ d(a_i,x)\le d(a_j,x)\}. \quad \text{(vd1)}$$

containing the points of space for which point a_i is nearest among all the points of $\boldsymbol{S}$.
Hence, for each pair of points (a_i, a_j) we divide the space into two regions

$$\boldsymbol{V}(a_i, a_j)=\{x: d(a_i,x)\le d(a_j,x)\},\ \boldsymbol{V}(a_j, a_i)=\{x: d(a_j,x)\le d(a_i,x)\}. \quad \text{(vd2)}$$

More specifically, the Voronoi region will be understood as a set [5]

$$\boldsymbol{VD}(a_i)=\bigcap_{i\neq j,j=1,2,\ldots,n}V(a_i, a_j), \text{ for } 1,2,\ldots,n. \quad \text{(vd3)}$$

The points lying on the boundary of the Voronoi regions form the Voronoi diagram. If (M, d) is a Euclidean plane as a set $\boldsymbol{S}$, we can take a set of sides of a plane polygon and apply the above definitions to the points of this polygon. Then we obtain the Voronoi diagram for the polygon. The boundary elements of the Voronoi regions, apart from line segments are parabolae. Figure 1a illustrates a set of regions for the ordinary Voronoi diagram on the Euclidean plane [5] (Fig. 1a) Voronoi diagram for the polygon inscribed in the curve indicating the optimum division of the river into regions. These regions form rinsing aggregate acquisition points located closest to the straight line sections that approximate the coast line [12, 13] (Fig. 1b).

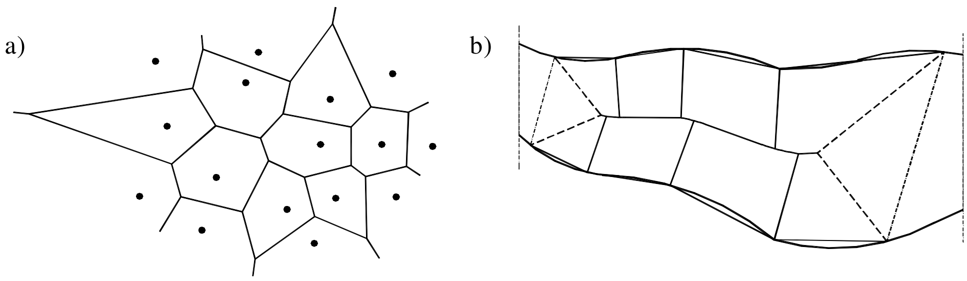

Figure 1. Exemplary Voronoi diagrams: a) classic on the Euclidean plane; b) for the sections of the river border, the polygon indicating the optimal division of the river section into the regions for the extraction of the rinsing aggregate [13].

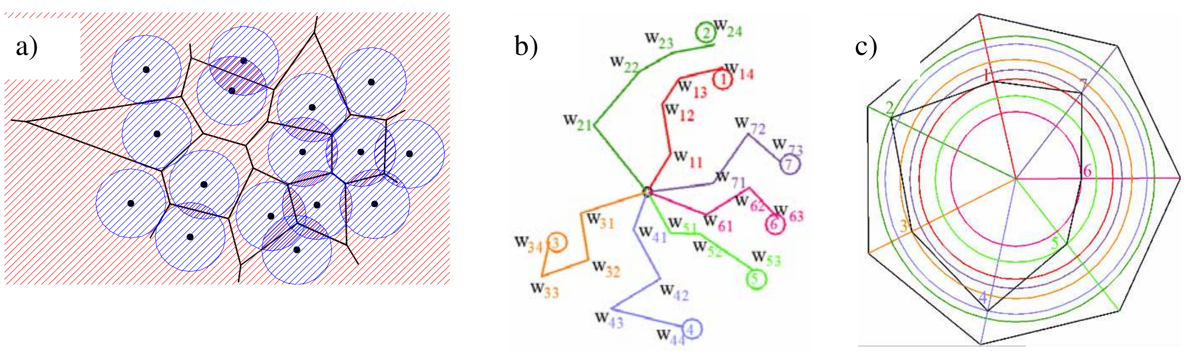

Figure 2. Voronoi diagram and a circle of range; a) the range (sum of all circles hatched in blue) of a set of points in the Euclidean plane, an area highlighted in red is out of range of a set of points; b) a

graph tree interpretation of a circle of range, where the lengths of graph ridges express the distances [km] of the vertices: 1,2,3,4,5,6,7 from the centre; c) a radar graph, where the radii of appropriate Euclidean circles denote the distances [km] of the vertices: 1,2,3,4,5,6,7.

Transport of ready-mix concrete requires exceptional time discipline. Therefore, when planning the transport of ready-mix concrete to the construction site one should take into account not so much the distance, but the required time. Then, we are interested in the "distance in time" between the place of production and the construction site, taking into account the mode of transport (means of transport, route, time of day, load-bearing capacity of roads…) [14]. This problem is particularly acute in congested cities, where pouring of concrete is often done late in the evening and at night. Delivery of concrete does not need to be done in minimal time, but it should take place within a certain time allowed; otherwise the project does not make sense. Therefore, thorough examination of possible routes and delivery times remains crucial. Thus, we are ultimately dealing with 'distance in time'. For such a time distance, beyond the Voronoi diagrams, we introduce circles of range (Fig. 2) – formally spheres in a metric space (b).

3. Graph of a tree structure and radar chart

In consideration of the location of ready-mix concrete production plants we use precisely the time distance, which is a function defined in a form of a graph and dependent on many parameters: traffic, time of day, road conditions, load-bearing capacity of the road. To observe mathematical accuracy, for the sake of simplification, we will talk about time-distance, or about the time needed to cover a certain distance. An interpretation of the circle of range (or a sphere in a metric space) in the space with such a "distance" was shown in Fig. 2 from two perspectives. The first in the form of a graph tree, where the root of branches [8, 10] is the center of the circle, while the vertices of the graph are all the points where the time distance from the center is less than the projected size of the " time radius". The second – in the form of the so-called radar chart, or a polygon shown in the Euclidean plane. The distances of the vertices of the polygon (1, 2, 3, 4, 5, 6, 7) from the center indicate the distances [km] which can be covered in time which equals the 'time radius'. One could say that the circle of time is a polygon in the Euclidean space.

4. Supply chain of ready-mix concrete

Supply chain of ready-mix concrete is a combination of the companies involved in the production of ready-mix concrete, its transport and laying the mixture in the construction element.

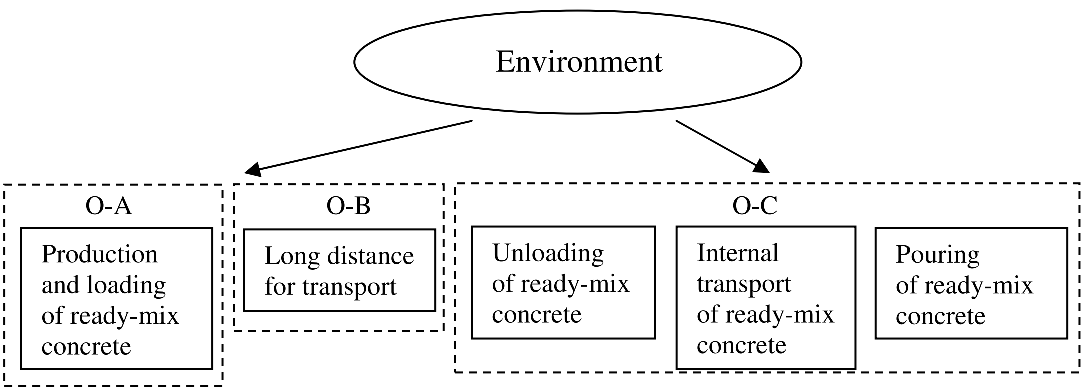

Figure 3. The logistic and production structure of the concrete works [19].

Supply chain of ready-mix concrete is usually a serial structure (Fig. 3). It involves the following nodes:

- The O-A node: preparation, production and loading of concrete mix onto trucks and the required time t_{zal},
- The O-B node: long-distance freight of ready-mix concrete and the required time t_{trd},
- The O-C node: unloading, handling and pouring of concrete mix and the required time t_{roz}.

Supply chain of ready-mix concrete should meet the following technological and organizational conditions [19, 20]: a) harmonizing the capacity of individual nodes of the supply chain; b) achieving high quality of the processes in the individual nodes; c) ensuring the continuity of the process of pouring concrete at the specific location, and thus ensuring the continuity of ready-mix concrete supply; d) observing the so-called "timing condition" (as a special technological requirement). The elapsed time from the completion of the ready-mix concrete to the completion of laying in the concreted piece must not exceed the permissible time (t_{dop}) – time of concrete binding:

$$t_{zal} + t_{trd} + t_{roz} < t_{dop}.$$

4.1. Distance-limited diagrams – case study

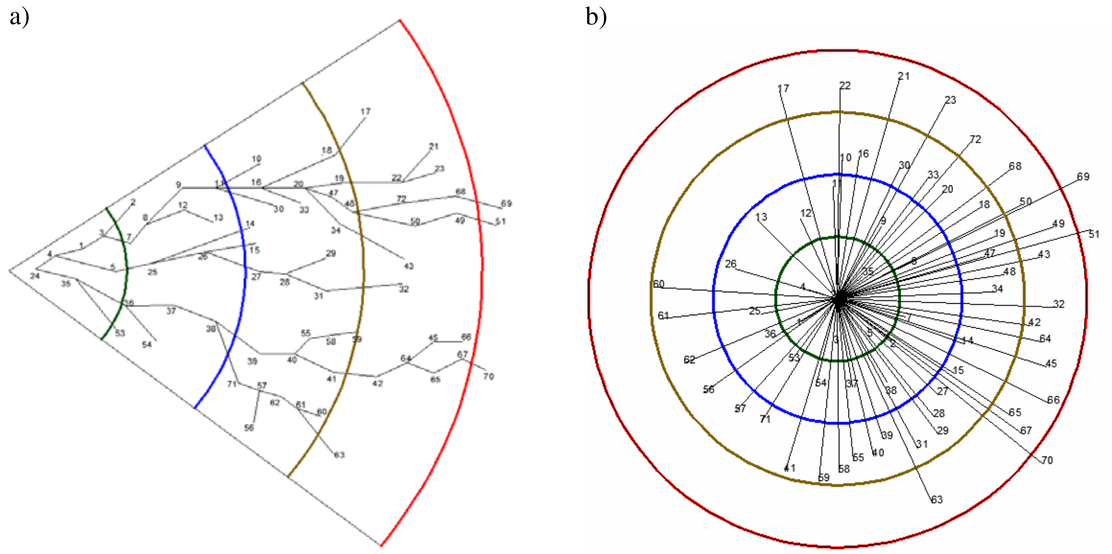

Figure 4. Operating range of ready-mix concrete plant F (Elewatorska Street in Bialystok) determined for: a) complex graph-tree; b) radar chart for transport times: 5 min (green), 10 min (blue), 15 min (orang), 20 min (red) between 9^{00} and 14^{30}.

4.1.1. Using the concepts introduced and defined in chapter 1 based on the Voronoi diagrams concerning the distance and the circles of range, the authors have determined the zones of operating range of concrete plants located in Bialystok. When determining the operating range zones, a distance-time criterion is used to determine the distance which a transit mixer can cover within a given time, in given ambient conditions. In the case of large urban areas, the primary factor characterizing environmental conditions is the time interval (time of day) in which the transport takes place.

4.1.2. Figure 4 shows the range areas in two forms: as a graph tree and a radar plot. These graphs concern the concrete plant F located at Elewatorska Street in Bialystok in the time interval $9^{00} \div 14^{30}$.

4.1.3. Figure 4a shows the graph in a form of a tree with its root at the point of the ready-mix concrete plant mentioned above. The numbers on the "tree branches" correspond to the vertex numbers of the base graph. Each vertex is assigned a specific point on the map. Using this graph representation, we are able to determine the "fastest" (the shortest) path to any vertex on the graph.

4.1.4. Figure 4b shows a graph with a central point being the counterpart of the discussed ready-mix concrete plant with radii pointing to the vertices on the graph. Numbers at the ends of the radii correspond to the numbered vertices on the graph. Using this graph representation one can quickly find the journey times for any vertex on the graph.

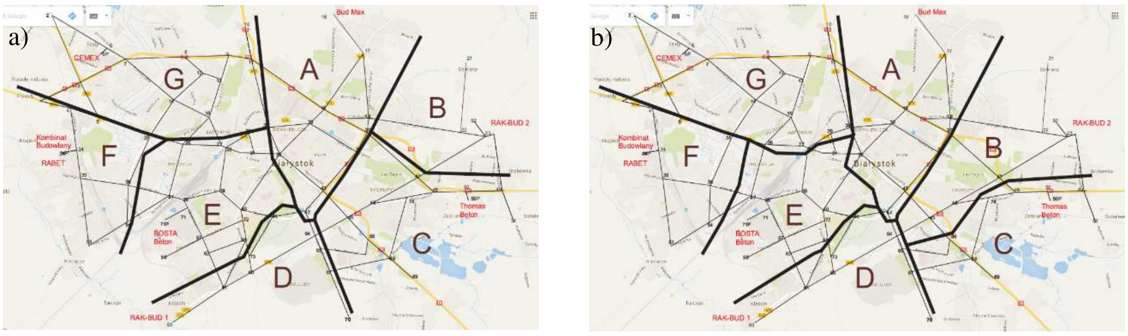

Figure 5. The Voronoi Diagram, referring to the location of seven ready-mix concrete production plants in Białystok, prepared in the "metric" with the time- distance specified on the directed graph (distance in time, measured between: a) 14.30-16.30 hours; b) between16,30-22,00 hours) . The graph is marked with numbered vertices (the number in a rectangle drawn with a bar line) and segments connecting these vertices, i.e. the edges of the graph.

4.1.5. Figure 5 shows the Voronoi diagram referring to the location of seven concrete production plants in Bialystok. Letters A ÷ G indicate the areas in which the time of delivery of concrete by the ready-mix concrete plant (located in the area) is the shortest. A – the area served by the ready-mix concrete plant BUDMAX; B – the area served by the ready-mix concrete plant RAK BUD 2, C – the area served by the ready-mix concrete plant Thomas Beton; D – the area served by the ready-mix concrete plant RAK BUD 1; E –the area served by the ready-mix concrete plant BOSTA Beton; F –the area served by the ready-mix concrete plant Kombinat Budowlany i and RABET, G – the area served by the ready-mix concrete plant CEMEX.

IOP Conf. Series: Materials Science and Engineering **251** (2017) 012045 doi:10.1088/1757-899X/251/1/012045

Figure 6. The operating area of the ready-mix concrete plant RABET located in Elewatorska Street in Bia³ystok, designated for transport times: 5 min (green), 10 min (blue), 15 min (orange), 20 min (red), between $9^{00}\div14^{30}$ hours, determined for the composed graph.

4.1.6. Figure 6 shows an example of the operating areas of the ready-mix concrete plant located in Elewatorska Street in Bialystok, designated for transport times: 5 min (green), 10 min (blue), 15 min (orang), 20 min (red), between 900÷1430 determined for the composed graph.

5. Conclusions

1. The presented analysis of the supply chain in the field of concrete mix transport is also a proposal for concrete creation of concrete operating areas for the managers of production and supply of concrete in any city.
2. The proposed method of selecting concrete transport route, using metric space theory and Voronoi diagrams, is a modern tool for optimizing road connections (time) from the concrete plant to the installation site.
3. The operating areas of a concrete batching plant illustrate the possibility of providing potential demand for concrete mix in the city at various time intervals.
4. In the analyzed example, the operating ranges show that the current distribution of the concrete mix production nodes fully covers the demand in the city area.
5. The analysis showed that all concrete plants were able to transport concrete without delaying admixtures within the city within 20 minutes (long distance) at any time of the day.
6. The procedures used in this work, geometric concepts, time intervals (time of day), methodology of measuring, recording and compiling travel times can serve as a reference for similar types of analysis.
7. An illustration of the route of transport of concrete mix (in the example shown) indicates the high frequency of use of both city bypasses passing through the tunnel and viaduct. This is confirmed by the use of two metrics: the railway (viaducts as two major nodes) and the metrics "river" ("river" as bypass).

References

[1] Ahmadi Nejad Masouleh F 2010 *Constructing Weighted Voronoi Diagrams Using Computer Programs*. https://www.google.pl/search?newwindow=1&site=&source=hp&q=Constructing+Weighted+Voronoi+Diagrams+Using+Computer+Programs.+&oq=Constructing+Weighted+Voronoi+Diagrams+Using+Computer+Programs.+&gs_l=hp.3...2656.2656.0.4752.2.2.0.0.0.0.70.134.2.2.0....0...1c.1.64.hp..0.0.0.0.qyJ1HUTMY4s (Access: 21-01-2017)

[2] Ahmadi Nejad Masouleh F 2006 *A Geographical Study of School Attendance Areas Using Multiplicatively Weighted Voronoi Diagrams*. A Case of Rasht City Iran. Geographical Review of Japan, 79, 700-714 http://giswin.geo.tsukuba.ac.jp/sis/thesis/Fatemeh_AHAMADI_NEJAD_MASOULEH.pdf (Access: 21-01-2017)

[3] Akel S 2005 *Dense DTM Generalisation Aided by Roads Extracted from LIDAR Data.ISPRS III/3, III/4, V/3 Workshop "Laser scaninng"*, Enschede, the Netherlands, September 12-14, 2005

[4] Barequet G, Goodrich M T, Levi-Steiner A and Steiner D 2003 *Straight-skeleton Based Contour Interpolation*. Symposium on Discrete Algorithms, Baltimore, Maryland, 119-127

[5] de Berg M, van Kreveld M, Overmars M and Schwarzkopf O 2008 *Computational Geometry: Algorithms and Applications*. Springer-Verlag Berlin Heidelberg, 147 – 162

[6] Brenner C 2001 *City Models – Automation in Research and Practice. Photogrametric Week '01*. D. Fritsch & R. Spiller, Eds. Wichmann, Wien

[7] Dikaiakou M, Efthymiou A, Chrysanthou Y 2003 Modelling the Walled City of Nicosia. The 4th International Symposium on Virtual Reality, Archaelogy and Intelligent Cultural Heritage

[8] Deo N 2004 Teoria grafów i jej zastosowanie w technice i informatyce (*Graph Theory with Applications to Engineering and Computer Science*) Państwowe Wydawnictwo Naukowe, Warszawa

[9] Deza M M and Deza E 2009 *Encyclopedia of Distances*. Springer-Verlag Berlin Heidelberg

[10] Gross J L and Yellen J 2005 *Graph Theory and Its Applications.* Chapman and Hall/CRC

[11] Haunert J H and Sester M 2004 *Using the Straight Skeleton for Generalisation and Multiple Representation Environment*. ICA Workshop on 'Generalisation and Multiple Representation', 20-21 August 2004, Leicester

[12] KoŸniewski E 2006 *Zastosowanie geometrii dachów do wyznaczania linii rozdziału mas ziemnych* (*Application of Roof Geometry to Determine the Separation Lines of Earth Masses*). Zeszyty Naukowe Politechniki Gdańskiej. Budownictwo Lądowe. Tom III (LIX), Nr 602, 215-222

[13] KoŸniewski E 2007 *Geometria dachów. Teoria i zastosowanie* (*Geometry of Roofs. Theory and Application*). Wydawnictwo Politechniki Białostockiej, Białystok.

[14] KoŸniewski E and Orłowski M 2011 *Obszary efektywnego zasięgu betonowni* (*Optimum range areas of ready-mixed concrete plants*). Budownictwo i Inżynieria Œrodowiska Vol. 2 no. 4, 531-540

[15] Laycock R G and Day A M 2003 *Automatically Generating Large Urban Environments Based on the Footprints Data of Buildings*. The 8th Solid Modelling Symposium, Seattle, USA

[16] Laycock R G and Day A M 2003 *Automatically Generating Roof Models from Building Footprints*. The 11th International Conference in Central Europe on Computer Graphics, Visualisation and Computer Vision '2003, Plzen-Bory, Czech Republic, February 3-7, 2003.

[17] Lawlor O S 2004 *Impostor for Parallel Interactive Computer Graphics*. Dissertation for the degree of Doctor of Philosophy in Computer Science in the Graduate College of the University of Illinois at Urbana-Champaign

[18] Novaes A G N 2010 *Solving Continuous Urban Districting Problems with Voronoi Diagrams*. Laboratory of Transportation and Logistics, Department of Civil Engineering, Federal University of Santa Catarina, Brazil

[19] Orłowski Z and Orłowski M 2014 *Zarządzanie ryzykiem łańcucha dostaw betonu towarowego* (*Managing the Supply Chain Risk of Ready-Mix Concrete*). Logistyka nr 6, 8178 – 8186
[20] Sobotka A 2010 *Logistyka przedsiębiorstw i przedsięwzięć budowlanych* (*Logistics of Enterprises and Construction Projects*). Wyd. AGH, Kraków
[21] Swift C, Leinemann K and Schaefer G 2003 *Real Times Systems for Urban Modelling*. Proc. 17th European Simulation Multiconference, Nottingham, UK, 411-416