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On stress-state optimization in steel-concrete composite structures

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Abstract. The plastic resistance of a concrete-filled column commonly is given as a sum of the components and taking into account the effect of confinement. The stress state in a composite column is determined by taking into account the non-linear relationship of modulus of elasticity and Poisson's ratio on the stress level in the concrete core. The effect of confinement occurs at a high stress level when structural steel acts in tension and concrete in lateral compression. The stress state of a composite beam is determined taking into account non-linear dependence on the position of neutral axis. In order to improve the stress state of a composite element and increase the safety of the construction the appropriate strength of steel and concrete has to be applied. The safety of high-stressed composite structures can be achieved by using high-performance concrete (HPC). In this study stress analysis of the composite column and beam is performed with the purpose of obtaining the maximum load-bearing capacity and enhance the safety of the structure by using components with the appropriate strength and by taking into account the composite action. The effect of HPC on the stress state and load carrying capacity of composite elements is analysed.

1. Introduction

Composite steel-concrete elements, formed as hollow steel sections filled with concrete have advantages in different architectural and structural solutions. It is shown [1] that a composite column with a corresponding content of reinforcement can provide at least 90-minutes of fire-resistance rating. The composite structure characterizes with higher ductility than concrete elements and advantages of steel may be successfully used in connections of composite structures [2]. In addition to these advantages, the steel tubes surrounding the concrete columns eliminate permanent form-work, which reduces construction time. Furthermore, steel tubes not only assist in the carrying axial load, but also provide confinement to the concrete and increase load carrying capacity of a column.

Eurocode 4 [3] gives a simplified design method for composite structures, which is applicable for practical purposes. The method is based on assumption that ultimate strength of material is attained simultaneously in all parts of the section. According to [3] to determine the resistance of a section against bending moment, full plastic stress distribution in the section has been assumed.

Investigations regarding the interaction between steel shell and concrete core have been performed by some authors [4–7]. In general, it is determined that for the service load no bond exists and that the core and shell act as two independent parts. In the early stages of loading, Poisson's ratio for concrete is lower than that for steel and the steel tube has no restraining effect on the concrete core. When lateral expansion of the concrete becomes greater than that of the steel a radial pressure develops at the steel-concrete interface. At this stage, the concrete core is stressed triaxially and the steel tube biaxially. It is shown in investigations [8] that concrete compaction affects the properties of the core and may also influence the interaction between the steel tube and concrete core.

Investigations with high strength concrete [9–11] show that the original contribution lies mainly in the use of concrete filling, especially ultra-high-performance concrete (UHPC), in slender composite elements for bridges rather than massive elements more commonly associated with concrete structures.

In this study stress analysis of the composite column and beam is performed with the purpose of obtaining the maximum load-bearing capacity and enhance the safety of the structure by using components with the appropriate strength and by taking into account the composite action. The effect of UHPC on the stress state and load carrying capacity of composite elements is analysed.

2. Analytical models for stress analysis

2.1. Composite steel-concrete elements in axial compression

The behaviour of short straight concrete-filled steel column under axial loading is discussed. Disregarding the local effects at the ends, the stress state in the cross-section of the middle part of the column is analysed. A reinforced concrete core with radius R_0 is included in the steel tube with thickness t (figure 1). The reinforcement consists of symmetrically arranged longitudinal steel bars and circumferential reinforcing wire located at radius R_s from axis z.

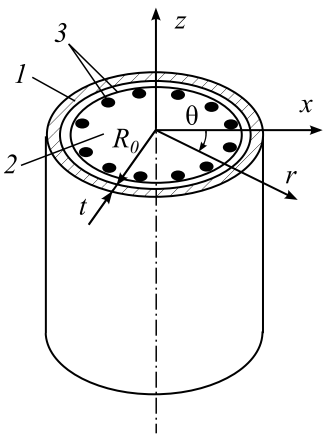

Figure 1. Concrete-filled hollow steel column with reinforcement, where 1 – steel column, 2 – concrete, 3 – reinforcing steel.

In analysis, it is assumed an in-plane membrane stressed state in steel tube and σ_z^a, σ_θ^a, ε_z^a, ε_θ^a are stresses and strains in the axial and circumferential direction, respectively. In the cylindrical concrete core, axially loaded by uniformly distributed pressure, because the concrete core is stressed triaxially for plasticized concrete, it is assumed that there exists the relation $\sigma_r^c = \sigma_\theta^c$ [12]. Here and below, the indices a, c and s refer to the steel tube, concrete and reinforcement, respectively.

The cross-section area of longitudinal reinforcement is A_z^s, but in the circumferential direction A_θ^s, placed with the given step. In the model, it is taken that reinforcement is spread throughout the cross-sectional area, and as a result anisotropic concrete core is formed. On the basis of compatibility conditions, four equations can be written in the following form:

$$\frac{1}{E^a}\sigma_z^a - \frac{\nu^a}{E^a}\sigma_\theta^a = \frac{1}{E_z^c}\sigma_z^c - \frac{\nu_{zr}^c}{E_r^c}\sigma_r^c - \frac{\nu_{z\theta}^c}{E_\theta^c}\sigma_\theta^c \tag{1}$$

$$\frac{\nu^a}{E^a}\sigma_z^a - \frac{1}{E^a}\sigma_\theta^a = \frac{\nu_{zr}^c}{E_z^c}\sigma_z^c - \frac{1}{E_r^c}\sigma_r^c + \frac{\nu_{z\theta}^c}{E_\theta^c}\sigma_\theta^c \tag{2}$$

$$\frac{1}{E_z^s}\sigma_z^s = \frac{1}{E_z^c}\sigma_z^c - \frac{\nu_{zr}^c}{E_r^c}\sigma_r^c - \frac{\nu_{z\theta}^c}{E_\theta^c}\sigma_\theta^c \tag{3}$$

$$\frac{1}{E_\theta^s}\sigma_\theta^s = \frac{1}{E_\theta^c}\sigma_\theta^c - \frac{\nu_{r\theta}^c}{E_r^c}\sigma_r^c - \frac{\nu_{z\theta}^c}{E_z^c}\sigma_z^c \tag{4}$$

The equilibrium equations in the direction z and θ, respectively, are as follows:

$$\frac{A_\theta^s}{R_s}\sigma_\theta^s + \frac{t}{R_0}\sigma_\theta^a + \sigma_\theta^c = 0 \tag{5}$$

$$A^c\sigma_z^c + A^a\sigma_z^a + A^s\sigma_z^s + q\left(A^a + A^c\right) = 0 \tag{6}$$

Here A^c, A^a, and A^s are the cross-sectional areas of the concrete, structural steel, and reinforcing steel, E^a is modulus of elasticity of steel tube, E_z^c, E_r^c, and E_θ^c are moduli of elasticity of the reinforced anisotropic concrete core in the directions z, r and θ, respectively, but E_z^s and E_θ^s – moduli of elasticity of the reinforcement in z and θ directions, q is a uniformly distributed axial load applied to the entire section. Eqs (1)–(4) have been written by taking into account the transverse deformation. Here, ν^a – Poisson's ratio of the steel tube, ν_{ij}^c are Poisson's ratios of the reinforced concrete ($i, j = r, z, \theta$; $i \neq j$).

2.2. *Composite steel-concrete elements in bending*

The load carrying capacity of composite elements in bending can be characterised by plastic deformation of structural steel in tension, by cracking of the concrete in compression or by both. At the same time, stresses in reinforcement can be less or more of the yield limit of steel. The stress analysis of composite elements in bending has been performed with certain assumptions: 1) the hypothesis of flat sections has been used; 2) the concrete lying in the tension zone of the section is neglected; 3) the conditions of deformation continuity of structural steel and concrete fulfil in the compression zone; 4) relationship for stress distribution in the compression zone of concrete has been assumed in following non-linear form:

$$\sigma_z^c = f_{cd}\left[1 - \left(1 - \frac{\varepsilon_z^c}{\varepsilon_{c2}}\right)^{n_c}\right] \tag{7}$$

where σ_z^c and ε_z^c are stresses and strains at distance z from neutral axis, respectively; f_{cd} is the compressive strength of concrete; n_c is a characteristic of the stress diagram form and according to [13] $n_c = 2$; $\varepsilon_{c2} = 2.0‰$, i.e. the strain at reaching the maximum strength ($\sigma_z^c = f_{cd}$). The cross section of reinforced composite beam and the stress-strain distribution diagrams are shown in figure 2.

On the basis of equilibrium conditions and taking into account the hypothesis of flat sections, the position of neutral axis can be determined by using non-linear equation:

$$\begin{gathered} x^2\left[\frac{f_{cd}b}{\left(1 + E_u^c/E^c\right)\varepsilon_{c2}} - 2t\left(E^a + E_t^a\right)\right] + \\ x\left[E^a\left(-bt + 2ht - 2t^2\right) + E_t^a\left(bt + 4ht + 2t^2\right) + E^zA^z\right] + \\ E^a\left(hbt + 2ht^2\right) - E_t^a\left(hbt + 2h^2t + 2ht^2\right) \\ -E^sA^sd = 0 \end{gathered} \tag{8}$$

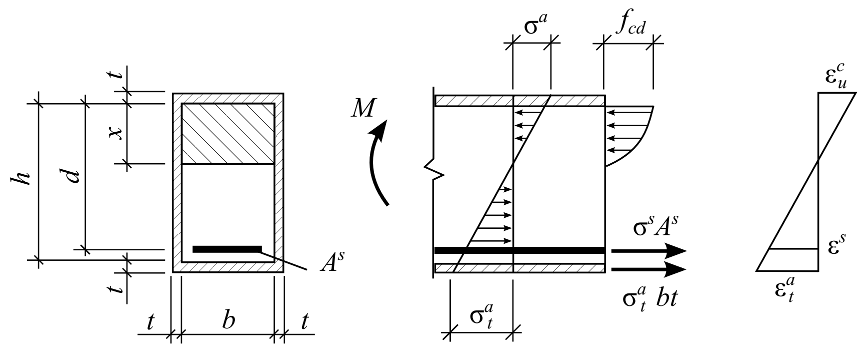

Figure 2. Stress and strain distribution in composite beam.

where E_u^c and E^c are tangent and initial modulus of the concrete, respectively, E_t^a is modulus of constructional steel in tension, ε_{c2} is ultimate deformation of concrete in compression. The ultimate bending moment M_u is determined by using the condition of equality of the moments in section of the beam:

$$\begin{aligned} M_u = b\int_0^x f_{cd}\left[1-\left(1-\frac{\varepsilon_z^c}{\varepsilon_{c2}}\right)^{n_c}\right] zdz + \sigma^a btx + \\ +\frac{2}{3}\left[\sigma^a btx\,(x+t)+\sigma_t^a t\,(h-x)\,(h+t-x)\right]+ \\ +\sigma^s A^s\,(d-x)+\sigma_t^a bt\,(h-x) \end{aligned} \tag{9}$$

Here b, h and d are width, depth and effective depth of concrete section, respectively.

3. Numerical solution and analysis

3.1. Strength analysis of composite column

In order to perform the stress analysis of a composite column the system (1) – (6) of six linear equations was solved. In matrix form this system can be written as:

$$[\mathrm{A}]\,\mathbf{X} = \mathbf{B} \tag{10}$$

The unknown components of the vector $\mathbf{X}$ are stresses in the steel tube, concrete, longitudinal and circumferential reinforcement, i.e.:

$$\mathbf{X} = [\sigma_z^a, \sigma_\theta^a, \sigma_z^c, \sigma_\theta^c, \sigma_z^s, \sigma_\theta^s]^{\mathrm{T}} \tag{11}$$

Vector of constants $\mathbf{B}$ and the matrix of system [A] are determined from the stress–strain relationships and equilibrium equations. They have the following form:

$$\mathbf{B} = [0, 0, -q\,(A^a + A^s)\,, 0, 0, 0]^{\mathrm{T}} \tag{12}$$

$$\mathrm{A} = \begin{bmatrix} \frac{1}{E^a} & -\frac{\nu^a}{E^a} & -\frac{1}{E_z^c} & \frac{\nu_{zr}^c}{E_r^c} + \frac{\nu_{z\theta}^c}{E_\theta^c} & 0 & 0 \\ \frac{\nu^a}{E^a} & -\frac{1}{E^a} & -\frac{\nu_{zr}^c}{E_z^c} & \frac{1}{E_r^c} - \frac{\nu_{r\theta}^c}{E_\theta^c} & 0 & 0 \\ A^a & 0 & A^c & 0 & A_z^s & 0 \\ 0 & 0 & \frac{1}{E_z^c} & -\left(\frac{\nu_{zr}^c}{E_r^c} + \frac{\nu_{z\theta}^c}{E_\theta^c}\right) & -\frac{1}{E_z^s} & 0 \\ 0 & \frac{h}{R_0} & 0 & 1 & 0 & \frac{A_\theta^s}{R_s} \\ 0 & 0 & -\frac{\nu_{z\theta}^c}{E_z^c} & -\frac{\nu_{r\theta}^c}{E_r^c} + \frac{1}{E_\theta^c} & 0 & -\frac{1}{E_\theta^s} \end{bmatrix} \tag{13}$$

In the analysis, the following geometrical parameters were chosen: diameter of circular hollow section $d = 40.6$ cm, thickness of the steel tube section $t = 0.88$ cm, reinforcement ratio $\mu = 4\%$ of the concrete section, distance from central axis to the reinforcement bars $R_s = 15.5$ cm, cross-sectional area of longitudinal reinforcement $A_z^s = 49$ cm^2 and secondary reinforcement $A_\theta^s = 0.1$ cm^2 (per cm of column length). According to [3], the modulus of elasticity for the reinforcement is $E^s = 200000$ MPa, for structural steel – $E^a = 210000$ MPa and Poissons ratio $\nu^a = 0.3$.

Despite the heterogeneous nature of concrete, it is assumed as homogeneous material and the mechanical behaviour is expressed in the form of a non-linear stress–strain relationship. However, with increasing of compressive strength, the modulus of elasticity increases. It is shown in [9] that the ultimate strain, i.e. the strain at maximum stress, increases with increasing of compressive strength.

In order to determine mechanical characteristics of concrete under compression stresses, prisms were tested in uni-axial compression. Development of axial strain ε_a and lateral strain ε_l fixed in the test is shown in figure 3. In the first phase of loading, the stress-strain relation is almost linear. Due to the heterogeneity of concrete, a uniform uni-axial stress applied to a concrete specimen cause locally non-uniform stress state. At a stress, corresponding approximately to 50% of the peak stress, the stress-strain relation exhibits a visible non-linear response. Approximately at 85% of the peak stress, the response is marked non-linear.

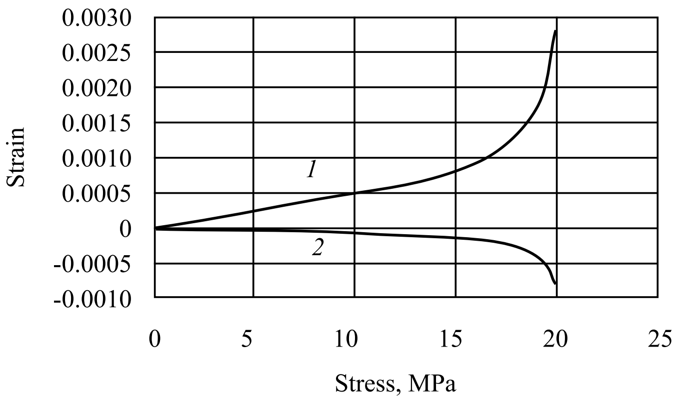

Figure 3. Stress–strain relation for concrete prisms in uni-axial compression: 1 – ε_a; 2 – ε_l.

At the initial stage of loading, Poisson's ratio of the concrete core is lower than that of steel. Therefore, the steel tube expands faster in the radial direction as compared with the concrete core; hence, the steel tube does not restrain the concrete core. As a result, centric loaded column is affected by the difference between the values of Poissons ratio of the steel tube ν^a and the concrete core ν^c.

The initial tangent modulus of concrete and modulus at a given stress level have been used in the analysis. Based on experimental results, Poisson's ratio of concrete in accordance to the stress level, starting with $\nu^c = 0.15$, was used. The moduli of elasticity of a reinforced anisotropic concrete core E_z^c, E_r^c, and E_θ^c as well as Poissons ratios were determined on the basis of the reinforcement theory [14] taking into account the content of reinforcement μ and mechanical properties of constituents.

The stresses in components of the structure and applied load N for conventional concrete and UHPC are given in table 1. In the case of conventional concrete, stresses in the steel section, concrete and reinforcement exceed the strength limit. By using UHPC as column filling, the stress level in components is satisfactory and load carrying capacity is maintained.

Table 1. Stress distribution (MPa) in constituents of centric-loaded composite column with diameter thickness ratio $D/t = 44$.

Strength of concrete, MPa	Axial force N, kN	σ_z^a	σ_θ^a	σ_z^c	σ_θ^c	σ_z^s	σ_θ^s
30	3200	-332	-4.3	-47	-0.1	-316	94
180	3200	-255	-2.0	-58	-0.2	-243	73

According to [3], the load-bearing capacity of the concrete core is increased due to limitation of transverse strains. This effect is shown in figure 4. The transverse compression of the concrete σ_r^c at high stress level leads to three-dimensional effects, which increase the column resistance.

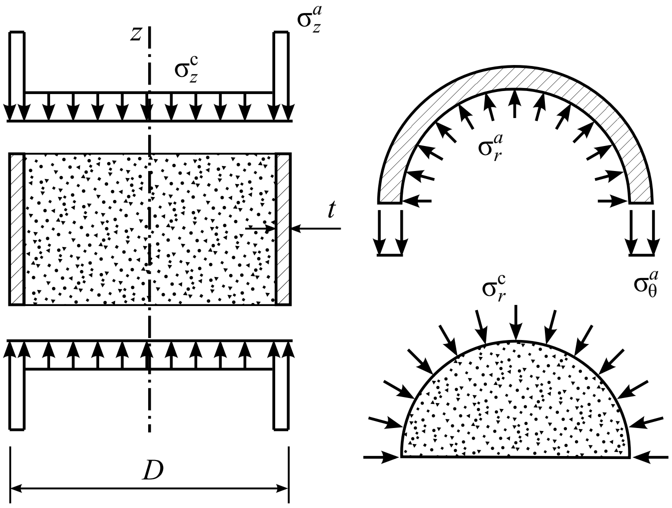

Figure 4. Stresses in composite column.

Note that when axial stress in concrete is $\sigma_z^c \cong 0.85 f_{ck}$, Poisson's ratio of the concrete increases and becomes higher than ratio of the steel. It is shown that in the region before fracture, Poisson's ratio is approximately 0.3 or more. In the calculation of stresses for design load this ratio was adopted equal to 0.3 [15]. Due to this feature, circular tensile stresses σ_θ^c in the concrete core of the column change to compressed stresses.

The tube diameter-to-thickness ratio, concrete compressive strengths and steel yield strengths are the main mechanical characteristics influencing the load carrying capacity of columns [16]. Taking into account the design strength of steel and concrete as well as limit ratios of the circular hollow sections the analysis of load bearing capacity of the composite column is performed. It is shown in figure 5 that the first load limiting factors are concrete design strength f_{cd} and diameter thickness ratio D/t. Using concrete with strength class C35/45 and steel with grade Fe235 the load bearing capacity of the composite column increases by 20% in comparison with concrete of class C30/37. For a thin wall hollow section ($D/t = 90$) instead of section with $D/t = 46$ the steel economy can be 50%.

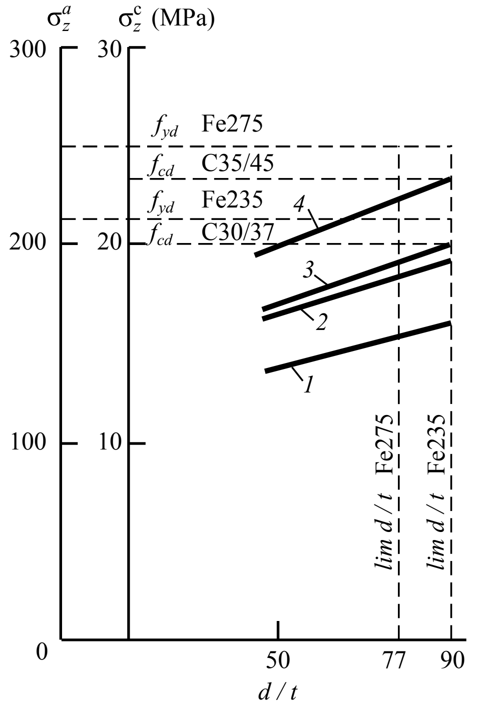

Figure 5. Variation of stresses in structural steel and concrete with ratio D/t: 1 – σ_z^a, 2 – σ_z^c for load $N = 4000$ kN; 3 – σ_z^a, 4 – σ_z^c for $N = 5000$ kN.

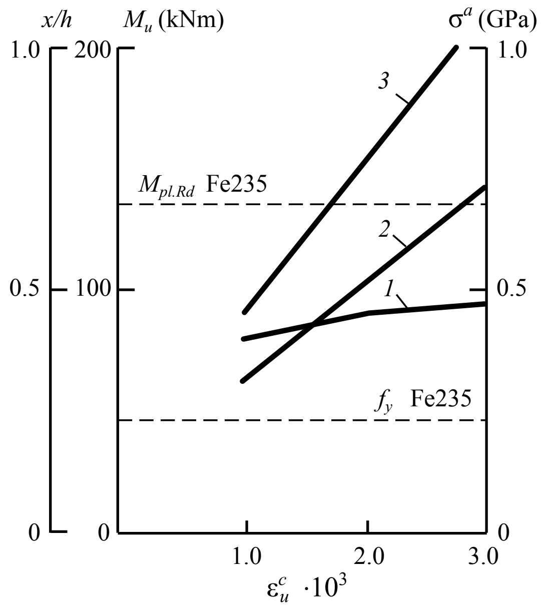

Figure 6. Influence of ultimate concrete deformation in compression on position of neutral axis, stresses in structural steel and ultimate bending moment: 1 – x/h, 2 – σ^a, 3 – M_u.

3.2. Strength analysis of composite beam

Results of stress analysis of the composite beam are shown in Fig 6. By using a non-linear approach the position of neutral axis of reinforced concrete element depends on ultimate deformation of concrete in compression ε_u^c. When $\varepsilon_u^c < 1 \times 10^{-3}$, the stress level in structural

Table 2. Comparison of composite beam with conventional concrete and UHPC.

Ultimate concrete deformation, $\varepsilon_u^c < 1 \times 10^{-3}$	Compressive strength of concrete, MPa			
	30		180	
	Relative compressive depth, x/h	Ultimate moment M_u, kNm	Relative compressive depth, x/h	Ultimate moment M_u, kNm
1	0.42	92.7	0.27	186.8
2	0.47	155.4	0.34	271.8
3	0.49	248.0	0.39	383.4

steel in tension zone is equal to the yield limit f_y (steel grade Fe235). The ultimate bending moment M_u in this case is 40% less than plastic moment $M_{pl.Rd}$ determined according to [3]. A comparison of composite beam load bearing capacity in the case of conventional concrete and UHPC depending on ultimate compressive strain of concrete and compressive depth is given in table 2.

Conclusions

1. The stress state in a composite column is determined, taking into account the dependence of the modulus of elasticity and Poisson's ratio on the stress level in the concrete. The main load limiting factors are concrete design strength and diameter thickness ratio. By using higher concrete strength class the load bearing capacity of the composite column increases and steel economy can be reached by using thin hollow sections.

2. In the case of a non-linear approach, the position of neutral axis of composite beam depends on the ultimate deformation of concrete in compression ε_u^c. When $\varepsilon_u^c < 1 \times 10^{-3}$ the ultimate bending moment is 40% less than plastic moment $M_{pl,Rd}$ determined according to Eurocode 4 and stress in structural steel in tension zone are equal to yield limit;

3. The optimization of working conditions and cross section area of a composite structure as well as the prevention of a failure in the case of small thickness of structural steel and fire can be realized by using appropriate strength of concrete and steel. By using UHPC as steel element filling the increase of load carrying capacity can be significant.

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