The Kinematic Analysis of Flat Leverage Mechanism of the Third Class

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Abstract. It is necessary to make link mechanisms calculation to the strength at designing of flat link mechanisms of high class after definition of block diagrams and link linear sizes i.e. it is rationally to choose their forms and to determine the section sizes. The algorithm of the definition of dimension of link mechanism lengths of high classes (MHC) and their metric parameters at successive approach is offered in this work. It this paper educational and research software named GIM is presented. This software has been developed with the aim of approaching the difficulties students usually encounter when facing up to kinematic analysis of mechanisms. A deep understanding of the kinematic analysis is necessary to go a step further into design and synthesis of mechanisms. In order to support and complement the theoretical lectures, GIM software is used during the practical exercises, serving as an educational complementary tool reinforcing the knowledge acquired by the students.

1. Introduction

In the teaching of subjects related to Machine Theory, supporting and complementing theoretical lectures with a simulation and analysis software, helps the students to understand deeply and visually the theoretical bases of the Mechanisms Science [1]. The software has been developed focusing, not only on educational purposes but also on research in the field of computational kinematics and mechanism design applications [2-3]. The software presented in this article also has potential to be used by students of Bachelor or Master Degrees in Mechanical Engineering and other subjects related to Robotics, Mechanism Design, etc. GIM has been developed in a modular structure [4-5]. After defining the kinematic structure of a linkage in the Geometry module, the user can perform the motion simulation in the Motion module. GIM is mainly oriented to the field of kinematic analysis, motion simulation and dimensional synthesis of planar mechanisms. In any case, it also includes other modules for workspace and singularity evaluation [6-7] and static analysis of mechanical structures.

2. Materials and Methods

The mechanisms of the third class, thanks to their structure, can provide difficult laws of the movement of the points and the driven members. For example, by means of such mechanism it is possible to reproduce set movements of two driven members (output) at one driving (entrance) member [1]. The questions of synthesis and kinematic analysis of mechanisms of the third class are insufficiently studied therefore these mechanisms received limited application in the national economy. In this subsection the technique of the kinematic analysis of the transmission mechanism of the third

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class with usage of vector calculation on the example of the mechanism of the third class with a sliding bar 5 and driving member 1 is considered (see Fig. 1).

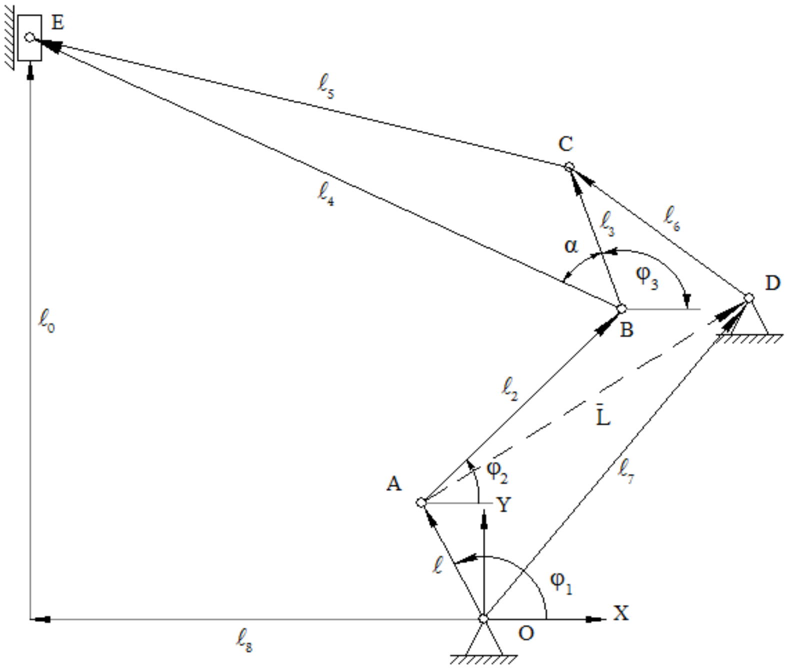

Figure 1. Kinematic scheme of flat lever mechanism of the third class.

For this mechanism it is possible to work out three following vector equations of isolation:

$$\left.\begin{aligned} \bar{l}_2+\bar{l}_3-\bar{l}_6 &= \bar{L} \\ \bar{l}_3+\bar{l}_5-\bar{l}_4 &= 0 \\ \bar{l}_2+\bar{l}_4-\bar{l}_0 &= \bar{l}_8-\bar{l}_1 \end{aligned}\right\} \tag{1}$$

where $\bar{L}=\bar{l}_7-\bar{l}_1$ the known vector-function of the generalized coordinate φ_1.
We choose any two from required vectors in the system Eq. (1) and we spread out them on orthogonal vectors $\bar{L}$ and $\left(\bar{k}\times\bar{L}\right)$. Let it be a vector $\bar{l}_2$ and $\bar{l}_3$:

$$\left.\begin{aligned} \bar{l}_2 &= p_2\bar{L}+q_2\left(\bar{k}\times\bar{L}\right), \\ \bar{l}_3 &= p_3\bar{L}+q_3\left(\bar{k}\times\bar{L}\right), \end{aligned}\right\} \tag{2}$$

$\vec{i}$, $\vec{j}$, $\vec{k}$ cross heading of system of coordinates $Oxyz$; P_2, q_2 and P_3, q_3 are unknown decomposition coordinates [2].

The others vectors on the basis of equality Eq. (1) can be expressed through chosen. The vector $\bar{l}_4$ is defined by turning movement vector $\bar{l}_3$ on the corner α between them, and the other vector $\bar{l}_6$, $\bar{l}_5$ and $\bar{l}_0$ are defined from equalities Eq. (1). Substituting in the received expressions Eq. (2), we have:

$$\left.\begin{aligned}
\bar{l}_4 &= \frac{l_4}{l_3}\left[p\bar{L} + q\left(\bar{k}\times\bar{L}\right)\right] \\
\bar{l}_5 &= \left(\frac{l_4}{l_3}p - p_3\right)\bar{L} + \left(\frac{l_4}{l_3}q - q_3\right)\left(\bar{k}\times\bar{L}\right) \\
\bar{l}_6 &= \left(p_2 + p_3 - 1\right)\bar{L} + \left(q_2 + q_3\right)\left(\bar{k}\times\bar{L}\right) \\
\bar{l}_0 &= \bar{l}_1 - \bar{l}_8 + \left(p_2 + \frac{l_4}{l_3}p\right)\bar{L} + \left(q_2 + \frac{l_4}{l_3}q\right)\left(\bar{k}\times\bar{L}\right)
\end{aligned}\right\} \tag{3}$$

where: $p = p_3\cos\alpha - q_3\sin\alpha$, $q = p_3\sin\alpha + q_3\cos\alpha$.

In system of the equations Eq. (3) parameters P_2, q_2, P_3, q_3 are unknown [3]. These parameters can be determined, having squared both parts of the equations Eq. (2) and the third equation of the system Eq. (2) and scalarly multiplying the last equation on $\bar{l}_8$:

$$\left.\begin{aligned}
&\left(p_2^2 + q_2^2\right)L^2 = l_2^2 \\
&\left(p_3^2 + q_3^2\right)L^2 = l_3^2 \\
&\left[\left(p_3 - p_2 - 1\right)^2 + \left(q_2 + q_3\right)^2\right]L^2 = l_6^2 \\
&\left(p_2 + \frac{l_4}{l_3}\right)\left(l_8, \bar{L}\right) + \left(q_2 + \frac{l_4}{l_3 q}\right)\left[\bar{l}_8, \bar{k}, \bar{L}\right] = l_8^2 - \left(l_8, l_1\right)
\end{aligned}\right\} \tag{4}$$

The nonlinear system of the equations generally has not the only solution and number of these solutions is equal to number of assemblies of the mechanism. The system Eq. (4) can be simplified, having reduced to two equations relatively q_2 and q_3. For this purpose from the second and the last system parameters Eq. (4) p_3 и p_2 we express through q_3 and q_2:

$$p_3 = \pm\left[\frac{l_3^2}{L^2} - q_3^2\right]^{\frac{1}{2}} \tag{5}$$

$$p_2 = a_{11}p_3 + a_{12}q_3 + a_{13}q_2 + a_{14} \tag{6}$$

where:

$$\left.\begin{aligned}
a_{11} &= -\frac{l_4}{l_3}\left(\cos\alpha - a_{13}\sin\alpha\right) \\
a_{12} &= \frac{l_4}{l_3}\left(\sin\alpha - a_{13}\cos\alpha\right) \\
a_{13} &= -\frac{\left[l_8, \bar{k}, \bar{L}\right]}{\left(\bar{l}_8, \bar{L}\right)} \\
a_{14} &= \frac{l_8^2 - \left(\bar{l}_8, \bar{l}_1\right)}{\left(\bar{l}_8, \bar{L}\right)}
\end{aligned}\right\} \tag{7}$$

Substituting parameters p_2 and p_3 in the first and third equations of system Eq. (4), we have system of two equations relatively q_2 and q_3:

$$\left.\begin{aligned}&\left\{\pm\left[\frac{l_3^2}{L^2}-q_3^2\right]^{\frac{1}{2}}\cdot a_{11}+a_{12}q_3+a_{13}q_2+a_{14}\right\}^2+q_2^2=\frac{l_3^2}{L^2}\\&\left\{\pm\left[\frac{l_3^2}{L^2}-q_3^2\right]^{\frac{1}{2}}\cdot(1+a_{11})+a_{12}q_3+a_{13}q_2+a_{14}-1\right\}^2+(q_2+q_3)^2=\frac{l_6}{L^2}\end{aligned}\right\}\quad(8)$$

Here pluses and minuses before the radical correspond to different assemblies of the mechanism [4]. From solutions of the system Eq. (8) taking into account Eq. (6) and Eq. (7) we determine unknown parameters P_2, q_2, P_3, q_3. As initial approximations of projection of the corresponding vectors functions $\bar{L}$ and $\left(\bar{k}\times\bar{L}\right)$ for a certain position of the mechanism are set by synthesis conditions. Provisions of other links of the mechanism are from scalar products of cross heading of system of coordinates $Oxyz$ and the found vectors:

$$\left.\begin{aligned}&tg\varphi_2=\frac{\bar{l}_2\bar{j}}{\bar{l}_2\bar{i}}=\frac{p_2L_y+q_2L_x}{p_2L_x+q_2L_y}\\&tg\varphi_3=\frac{\bar{l}_3\bar{j}}{\bar{l}_3\bar{i}}=\frac{p_3L_y+q_3L_x}{p_3L_x-q_3L_y}\\&tg\varphi_4=\frac{\bar{l}_4\bar{j}}{\bar{l}_4\bar{i}}=\frac{pL_y+qL_x}{pL_x+q_3L_y}\\&tg\varphi_6=\frac{\bar{l}_6\bar{j}}{\bar{l}_6\bar{i}}=\frac{(p_2+p_3-1)L_y+(q_2+q_3)L_x}{(p_2+p_3-1)L_x-(q_2+q_3)L_y}\\&l_0=l_{1y}-l_{8y}+\left(p_2+\frac{l_4}{l_3}p\right)\cdot L_x+\left(q_2+\frac{l_4}{l_3}q\right)\cdot L_y\end{aligned}\right\}\quad(9)$$

where $L_x=l_{7x}-l_1\cos\varphi_1$, $L_y=l_{7y}-l_1\sin\varphi_1$.

We pass to definition of analogs of speeds and speedups. Differentiating Eq. (1) on φ_1 taking into account $\varphi_3'=\varphi_4'=\varphi_5'$, we receive:

$$\left.\begin{aligned}&\bar{l}_2\,\varphi_2'+\bar{l}_3\,\varphi_3'-\bar{l}_6\,\varphi_6'=-\bar{l}_1\\&\bar{l}_2\,\varphi_2'+\bar{l}_4\,\varphi_3'-\bar{i}\,l_0'=-\bar{l}_1\end{aligned}\right\}\quad(10)$$

Analogs of speeds of links can be defined from the solution of linear system Eq. (10):

$$\left.\begin{aligned}&\varphi_2'=\frac{1}{\Delta}\{(\bar{l}_1,\bar{j})[\bar{l}_3,\bar{k},\bar{l}_6]-(\bar{l}_4,\bar{j})[\bar{l}_1,\bar{k},\bar{l}_6]\}\\&\varphi_3'=-\frac{1}{\Delta}\{(\bar{l}_1,\bar{j})[\bar{l}_2,\bar{k},\bar{l}_6]-(\bar{l}_2,\bar{j})[\bar{l}_1,\bar{k},\bar{l}_6]\}\\&\varphi_6'=\frac{1}{l_6^2}[(\bar{l}_2,\bar{l}_6)\varphi_2'+(\bar{l}_3,\bar{l}_6)\varphi_3'+(\bar{l}_1,\bar{l}_6)]\\&l_0'=(l_1',\bar{i})+(\bar{l}_2,\bar{i})\varphi_2'+(\bar{l}_4,\bar{i})\varphi_3'\end{aligned}\right\}\quad(11)$$

where $\Delta=\left(\bar{l}_4\,\bar{j}\right)\left[\bar{l}_2\,\bar{k}\,\bar{l}_6\right]-\left(\bar{l}_2\,\bar{j}\right)\left[\bar{l}_3\,\bar{k}\,\bar{l}_6\right]$.

For definition of analogs of speedups of links we differentiate Eq. (10) on φ_1:

$$\left.\begin{aligned} \overline{\ell}_2\varphi_2'' + \overline{\ell}_3\varphi'' - \overline{\ell}_6\varphi_6'' &= -\overline{L}_1 \\ \overline{\ell}_2\varphi_2'' + \overline{\ell}_4\varphi_3'' - \bar{i}\,\ell_0'' &= -\overline{L}_2 \end{aligned}\right\} \tag{12}$$

where:

$$\begin{aligned} \overline{L}_1 &= \overline{k}\times\overline{l}_1 + \overline{k}\times\overline{l}_2\varphi_2'^2 + \overline{k}\times\overline{l}_3\varphi_3'^2 - \overline{k}\times\overline{l}_6\varphi_6'^2 \\ \overline{L}_2 &= \overline{k}\times\overline{l}_1 + \overline{k}\times\overline{l}_2\varphi_2'^2 + \overline{k}\times\overline{l}_4\varphi_3'^2 \end{aligned} \tag{13}$$

The solution of linear system Eq. (12) can be written down similarly in the following type:

$$\left.\begin{aligned} \varphi_2'' &= \frac{1}{\Delta}\{(\overline{L}_2, \bar{j})[\bar{l}_3, \bar{k}, \bar{l}_6] - (\bar{l}_4, \bar{j})[\overline{L}_1, \bar{k}, \bar{l}_6]\} \\ \varphi_3'' &= -\frac{1}{\Delta}\{(\overline{L}_2, \bar{j})[\bar{l}_2, \bar{k}, \bar{l}_6] - (\bar{l}_2, \bar{j})[\overline{L}_1, \bar{k}, \bar{l}_6]\} \\ \varphi_6'' &= \frac{1}{l_6^2}[(\bar{l}_2, \bar{l}_6)\varphi_2'' + (\bar{l}_3, \bar{l}_6)\varphi_3'' + (\overline{L}_1, \bar{l}_6)] \\ l_0'' &= (\overline{L}_2, \bar{i}) + (\bar{l}_2, \bar{i})\varphi_2'' + (\bar{l}_4, \bar{i})\varphi_3'' \end{aligned}\right\} \tag{14}$$

The usage of formulas Eq. (9), Eq. (11) and Eq. (14) quite just allows making an algorithm of the solution tasks of the kinematic analysis of the mechanism of the third class on the computer [5].

3. Results and Discussion

In this paper, a kinematic model of the third class Assur group is presented. The method has the property of visibility, which makes it suitable for use in engineering practice [7]. Simulation of the kinematical analysis of third class plain lever mechanism with output dwell is done using Model GIM software application that gives opportunity to develop discrete, continuous and hybrid models of complicated technical systems (see Fig. 2-5).

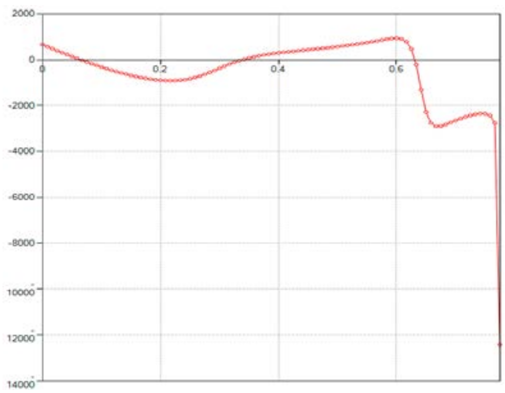

Figure 2. Coputed plot of the E point velocity module.

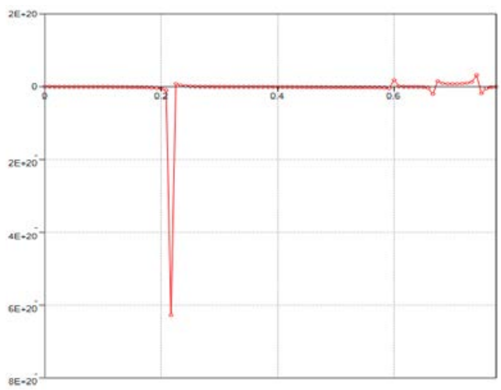

Figure 3. Coputed plot of the E point curvature center module.

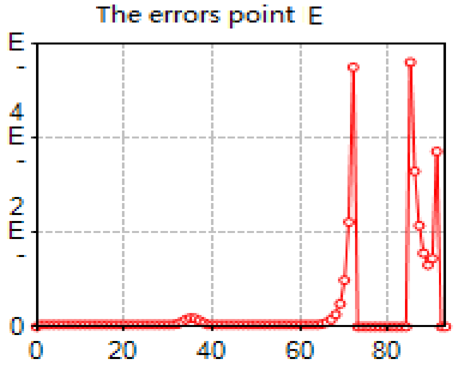

Figure 4. Computed plot of the errors point E.

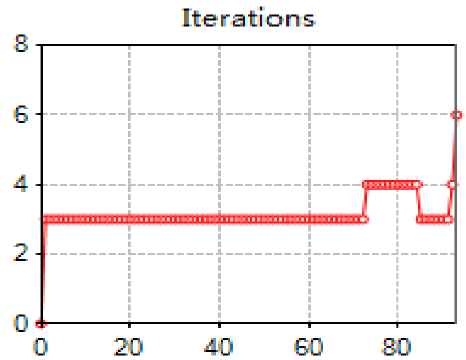

Figure 5. Computed plot of the iterations point E.

4. Conclusions

The combined analytic and graphic-analytical kinematic analysis method of the sixth class third order mechanism can be performed by accepting the inverse motion, considering the input element of the original mechanism as frame and one of the adjacent ternary element as drive element. The analysis of the mechanism is reduced to the study of third and second class Assur kinematic groups. The positional kinematic analysis shows that the most difficult problem is the solving of a trigonometric equations system by means of numerical methods. The kinematic analysis of a particular case indicated by Artobolevsky highlights the difficulties in computing the velocities in particular positions. However, some properties of the mechanism recommend it to be used in applications which need the preservation of parallel positions of the higher order couplers in self-locked extreme position and its compact size in folded position.

5. References

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