Minimization of Poisson's ratio in anti-tetra-chiral two-phase structure

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Abstract. One of the most important goal of modern material science is designing structures which exhibit appropriate properties. These properties can be obtained by optimization methods which often use numerical calculations e.g. finite element method (FEM). This paper shows the results of topological optimization which is used to obtain the greatest possible negative Poisson's ratio of the two-phase composite. The shape is anti-tetra-chiral two-dimensional unit cell of the whole lattice structure which has negative Poisson's ratio when it is built of one solid material. Two phase used in optimization are two solid materials with positive Poisson's ratio and Young's modulus. Distribution of reinforcement hard material inside soft matrix material in anti-tetra-chiral domain influenced mechanical properties of structure. The calculations shows that the resultant structure has negative Poisson's ratio even eight times smaller than homogenous anti-tetra chiral structure made of classic one material. In the analysis FEM is connected with algorithm Method of Moving Asymptote (MMA). The results of materials' properties parameters are described and calculated by means of shape interpolation scheme – Solid Isotropic Material with Penalization (SIMP) method.

1. Introduction

One of the most popular structures which exhibit the negative Poisson's ratio (auxetics) are periodic lattice structures. These structures are built of single unit cells. The main class of auxetic unit cells are: 2-dimensional and 3-dimensional re-entrant honeycombs – first suggested by Gibson [1], rotating rigid and semi-rigid units (triangles or rectangles) – proposed by Grima et al. [2], chiral structures – first introduced by Prall and Lakes [3]. During the last three decades the researchers developed more sophisticated and complicated geometries of auxetic unit cell. Furthermore, there are also known structures which are the combination of two or more different unit cells. This work will focus on anti-tetra-chiral structures.

The adjective "chiral" means physical property to spin. Basic unit of this kind of structures consists of straight ligaments connecting to central nodes (see figure 1). The node may be of any geometrical form, usually circles or rectangles. The mechanism of "auxeticity" is obtained by the wrapping and unwrapping of the ligaments around the nodes (circles or rectangles). The whole chiral network is built of many single unit cells joined together (see figure 1a). In the papers devoted to chiral structures the value of Poisson's ratio is estimated close to -1. The geometry of unit cell is named by the Greek numbers: tri-, tetra-, penta- or hexa-chiral – the number depends on amount of ligaments joined to one node. Apart from chiral lattices it is possible to develop anti-chiral structures. Anti-chiral networks are generated if any two adjacent nodes share the same side of the common tangential ligament. Chiral and anti-chiral geometries can be clearly defined by ratio of the parameter of node (e.g diameter for the circle) and the length of ligament. The behaviour, deformation process and mechanical properties of chiral geometries are investigated in many works of researchers.

Grima et al [4] described in details the mechanism of deformation responsible for auxetic functionality. The discussion of results was made for the whole family of the chiral structures and their auxeticity. The Poisson's ratio and other in-plane elastic constants were various for the different amount of ligaments connecting to nodes and different shape of node (circle, rectangle and other). Alderson et al. [5] studied the in-plane linear elastic response and process of out-of-plane bending of tri- and anti-tri-chiral honeycombs and their re-entrant counterparts using FE analysis and experiments.

The out-of-plane linear elastic mechanical properties of tri-, tetra- and hexa-chiral honeycomb configurations was described by Loreto et al. [6]. Analytical models were developed to compute the transverse Young's modulus and the Voigt and Reuss bounds for the transverse shear stiffness. Besides the analytical solutions numerical models was created for validation the analytical results, and to find the correlation between the transverse shear stiffness and the gauge thickness of the honeycombs. Miller et al. [7] investigated the flatwise compressive behaviour of tetra- and hexa-chiral honeycombs using FEM.

The researchers investigated also chiral structures for their use in the industrial applications. The best way to obtain this goal is to optimize specified mechanical properties using numerical methods. Kolla et al. [8] investigated auxetic structures for its use in shear layer of non-pneumatic wheel. The objective of the study was to find an ideal geometry for the shear layer while meeting its requirements of shear properties with polycarbonate as base material. Joshi [9] studied the mechanical, acoustic and vibration properties of many chiral and anti-chiral structures with various geometrical parameters, so the dynamic response of shapes in chiral fulfilments was investigated.

Chiral lattices were optimized also to combine the best mechanical properties with the best technology of production. Airoldi et al. [10] presents the advancements in the technological processes developed to produce chiral honeycombs made of thin composite laminates. The objective of such approach is the production of thin-walled chiral composite structures with enhanced strength properties by using a more feasible technology.

Mousanezhad et al [11] used FEM and optimization method to optimize several chiral, anti-chiral, and hierarchical honeycombs with hexagon and square based networks. Simple closed-form expressions were derived for the optimization of elastic moduli.

Undoubtedly, topology optimization is big a challenge for researchers and many works have already been devoted to this subject. Important aspect of optimization is spatial distribution of material with the gaol to obtain optimal structure which fulfils assumed criteria [12, 13]. This definition of properties' optimization is made by means of material density-like function. Single material and multi-material topological design in elasticity were solved using continuous control variable function [14]. The distribution of two materials with non-vanishing stiffness and voids in composite-like structure was investigated by Thomson [15]. Sigmund [16] also optimized the two-phase composite structure to obtain the structure with extreme behaviour parameters. The multi-phase composite structure can also be characterized by the negative Poisson's ratio. This possibility has been investigated by Evans [17] and Milton [18]. Milton in his work presented that negative Poisson's ratio materials can be built of symmetrized laminates as well as in two as in three dimensions. Evans et al. described that a negative Poisson ratio can be obtained by insertion in a network-embedded composite using materials that have the large ratio of the reinforcement to matrix modulus.

Lim [19] has been presented mechanical properties of auxetic sandwich panels made by different materials. It has been built also topologies which have effective properties for vibration damping inserts in honeycomb structures (e.g. Boucher [20]). Strek et al. [21] have presented that the sandwich composite structure with the two-phase auxetic core can exhibit auxetic properties even if all its are characterized by positive Poisson's ratio. The subject of research in the field of properties of auxetic composites was also the influence of temperature on the effective Poisson ratio (thermoauxeticity) of the composites made of solid matrix material with elliptical fibres [22]. Pozniak et al. [23] showed planar auxeticity of materials with various inclusions. Strek et al. [24] presented the simulations of torsional behaviour of a composite beam with auxetic phase.

Topology optimization method were used by many researchers to present new types of auxetic structures with optimal stiffness or damping characteristic of material or two or multi-phase auxetic

materials, e.g. Gibiansky and Sigmund [25], Czarnecki and Wawruch [26], Lukasiak [27], Yi et al. [28], Whitty et al. [29], Schwerdtfeger et al. [30], Kaminakis et al. [31], Strek et al. [32].

In this work Finite Element Method (FEM) combined with Method of Moving Asymptotes (MMA) algorithm and Solid Isotropic Material with Penalization (SIMP) method is used to minimize the effective Poisson's ratio (PR) of two-phase anti-tetra-chiral structure. The domain of optimization is the shape of anti-tetra-chiral unit cell, which is characterized by the Poisson's ratio close to minus one. The Poisson's ratio of anti-tetra-chiral structure can be decreased by proper distribution of two different materials (two phases) inside the chiral geometry. Distribution of hard and soft material with given ratio of Young's modulus in anti-tetra-chiral domain influenced mechanical properties of structure. The calculations shows that the resultant anisotropic structure can exhibit greater values of negative Poisson's ratio than homogenous isotropic anti-tetra chiral structure made of one positive material. Topology optimization allows to find distribution of materials in anti-tetra-chiral unit cell creating original new structures with auxetic properties beyond the limits of isotropic one. The results are presented in the second section of this paper.

1.1. Method of evaluation of effective properties – SIMP interpolation
An analysis of effective properties of two-phase chiral shape uses the methods of evaluation of generalized parameters such as Young's modulus, Poisson's ratio and density. These generalized properties were written by means of Solid Isotropic Material with Penalization (SIMP) scheme. SIMP is very often used for material interpolation in geometry optimization. The main part of this method is the introduction of an interpolation function that expresses various generalized physical quantities as a function of continuous variable. The parameters in SIMP method fulfils the equations:

$$E(r) = E_1 + (E_2 - E_1) \cdot r^p, \tag{1}$$

$$\vartheta(r) = \vartheta_1 + (\vartheta_2 - \vartheta_1) \cdot r^p, \tag{2}$$

where $r = r(x)$ is the control variable, p is the penalization parameter, E_1 and E_2 are the Young's moduli and ν_1 and ν_2 are the Poisson's ratios for the first and the second material ($E_1 \leq E_2$ and $\nu_1 \leq \nu_2$). In all considered cases materials differ only with respect to the values of Young's modulus. All other parameters are the same for both materials. The density of material is represented by continuous variable $0 < r < 1$. The interpolating functions (1-2) are used to describe the distribution of two phases with different properties inside the analysed domain. The most important goal is to obtain such distribution for which the anti-tetra-chiral shape exhibit the minimal value of effective Poisson's ratio. The values of elastic properties of materials (E_1, E_2, ν_1, ν_2) of both phases were chosen as follows: structural steel (hard material) and computational artificial material (soft material). More detailed simulation would optimize the interpolation of bulk and shear moduli, but it isn't main subject of this work.

1.2. Computing of effective Poisson's ratio and FEM analysis
The commonly used definition of Young's modulus for a homogeneous, isotropic elastic solid domain is specified as the ratio of longitudinal stress to longitudinal strain. Respectively the Poisson's ratio for the same homogenous, isotropic elastic solid material is the negative ratio of transverse to longitudinal strain at every point in a body under longitudinal tension. In a material with two phases where the ratios may be changed from point to point both classic definitions of Young modulus and Poisson's ratio isn't still possible to use. The new definitions of the ratios were checked by many authors. The most common definition of Poisson's ratio is based on the assumption of small deformation and it is computed as a negative ratio of the average transverse to longitudinal strains [33]:

$$\nu_{eff} = -\frac{\bar{\varepsilon}_t}{\bar{\varepsilon}_l}, \tag{3}$$

where: $\bar{\varepsilon}_t$ is the average strain in transverse direction and $\bar{\varepsilon}_l$ is the average strain in longitudinal direction. If the force is applied along the y-axis, the average transverse strain is defined by equation:

$$\overline{\varepsilon}_t = \frac{\int_{G_1} u_1 dG}{L_x \cdot \int_{G_1} dG}, \tag{4}$$

where: G_1 is the boundary parallel ($x = L_x$) to the boundary with applied prescribed displacement and u_1 is displacement in x-axis. The average longitudinal strain is defined by equation:

$$\overline{\varepsilon}_l = \frac{\int_{G_2} u_2 dG}{L_y \cdot \int_{G_2} dG}, \tag{5}$$

where: G_2 is the boundary parallel ($y = L_y$), where a load is applied and u_2 is displacement in y-axis. Because of using SIMP scheme effective Poisson's ratio must be dependent on control variable r. In this way equation receive objective functions for the optimization problem of the two-phase chiral shape:

$$\vartheta_{eff}(r) = -\frac{\overline{\varepsilon_t(r)}}{\overline{\varepsilon_l(r)}}, \tag{6}$$

As a control variable function in SIMP scheme two constraints are applied - pointwise inequality (7) and integral inequality (8), which are given by formulas:

$$0 \le r(x) \le 1 \text{ for } x \in S, \tag{7}$$

$$0 \le \int_S r(x) dS \le A_f \cdot S, \tag{8}$$

where x is defined coordinate, A_f – is a percentage fraction of the second material in the domain S. The process of optimization is defined in following order: FEM – discretization, redefinition of function minimization with applied constraints as standard finite dimensional nonlinear programmable problem. Then the value of control variable is evaluated in every mesh node as:

$$r(x) = \sum_{i=1}^{N} r_i \cdot \phi_i(x), \tag{9}$$

where: $\phi_i(x)$ are shape functions and N is the number of an element node.

After discretization, pointwise inequality condition is expressed as follows (where: M – the number of all nodes of all mesh elements, where the value of control variable is derived):

$$0 \le r_i \le 1 \text{ for } x = 1, \dots, M. \tag{10}$$

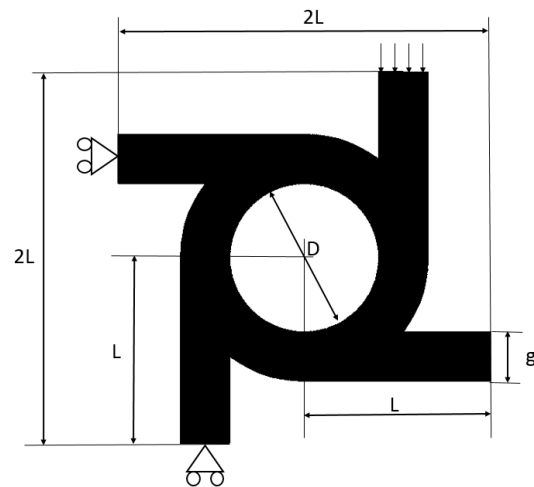*1.3. Definition of mechanical problem and boundary conditions*
The analysis is made for anti-tetra-chiral (ATC) unit cell with circular nodes subjected to compression load from the top. Considered shape is defined by the size of the unit cell ($2L$ on $2L$), L – length of ligament, D – diameter of node and g – thickness of unit cell. In considered case $L_x = L_y = 2L$ (see figure 1b).

Due to symmetry of considered problem only quarter of the structure (the whole optimized geometry consists of four unit cells) is analysed. The boundary conditions (BC) used in computations (Figure 1B) are as follows: right boundary: x=L - free BC; left boundary (symmetry): x = -L - $\boldsymbol{n} \cdot \boldsymbol{u} = 0$; bottom boundary (symmetry): y =-L $\boldsymbol{n} \cdot \boldsymbol{u} = 0$; top boundary: y = L - prescribed displacement, $u_2 = w$, where: $w = 0.01$ and L=0.15.

The governing equations for the solid stress–strain mechanical problems by the displacement formulation are steady state Navier's equation with linear constitutive equation and zero body load.
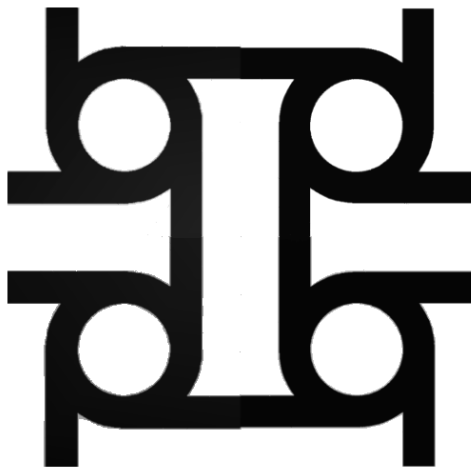
Figure 1A. Geometry of ATC structure.   **Figure 1B.** Dimensions and boundary conditions.

2. Numerical results

In order to obtain the goal of topology optimization of a two-phase anti-chiral structure the correct and effective algorithm needs to be applied [29]. If the number of design variables is very large simple mathematical programming methods usually can't bring correct results. In this article the method of moving asymptotes (MMA) algorithm is used to find optimal distribution of two positive materials in anti-tetra-chiral shape. This algorithm was firstly used and described in 1986 by Svanberg [34, 35]. MMA method is constructed on the thing that it is used the changed Lagrangian function to make a convex approximation and solving a sequence of convex approximations. The base of MMA algorithm is the method of CONLIN (convex linearization) and it was used in a wide range of computational problems with the great successes. In this paper the MMA algorithm is used to solve the optimization problem, which distributes two materials in the chiral shape of single unit cell. As result it is created the whole structure with negative Poisson's ratio smaller than value of anti-chiral cell (close to -1). MMA algorithm minimizes the objective function which defines the effective PR of a structure with some appropriate constraints and finds the optimal distribution characterized by the minimal Poisson' ratio. All numerical computations were conducted by means of FEM software (Comsol Multiphysics). Control variable shape functions in optimizations were bilinear Lagrange polynomials. Quadratic Lagrange polynomials were defined as shape functions for displacement fields. The value of penalization parameter in SIMP method was $p=3$.

As described below, two phases were used characterized with positive Poisson's ratio and real Young moduli. One of these materials can be called "soft material" with the lower Young's modulus and second: "hard material" with the greater Young's modulus:

- soft material: Young's modulus is $E_1 = 10^7$ Pa and Poisson's ratio $v_1 = 0.33$;
- hard material (structural steel): Young's modulus is is $E_2 = 2 \cdot 10^{11}$ Pa and Poisson's ratio $v_2 = 0.33$.

In all simulations initial value of A_f is defined which means the percentage value of hard material in the whole structure and it changes between 20% and 50%.

The dimensions of optimized structure is $L = 0.15$ m, $D = 0.1$ m and thickness of ribs is $g = 0.05$ m and $g = 0.025$ m.

In the figure 2A the optimized distribution of materials in anti-tetra-chiral structure with $A_f = 20\%$ is presented. In figures 2-5 the green colour represents soft material and the blue one the hard material. The whole structure after optimization is characterized by the effective Poisson's ratio equal to -6.4188. The anti-tetra-chiral structure made of four unit cells is shown in the figure 2B.
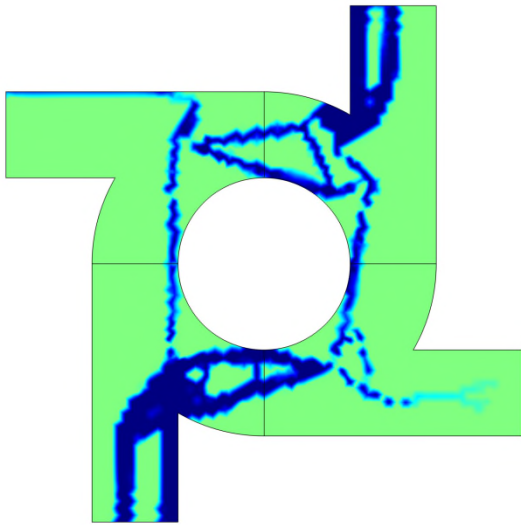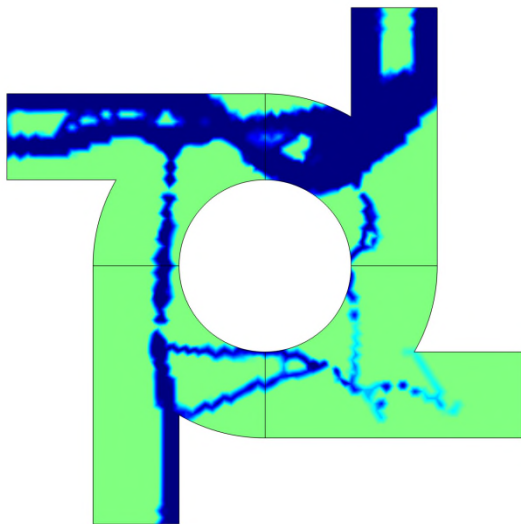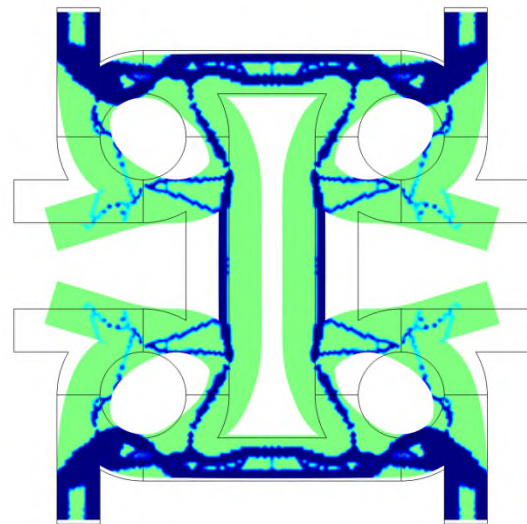
Figure 2A. Two-phase material with A_f=20%.
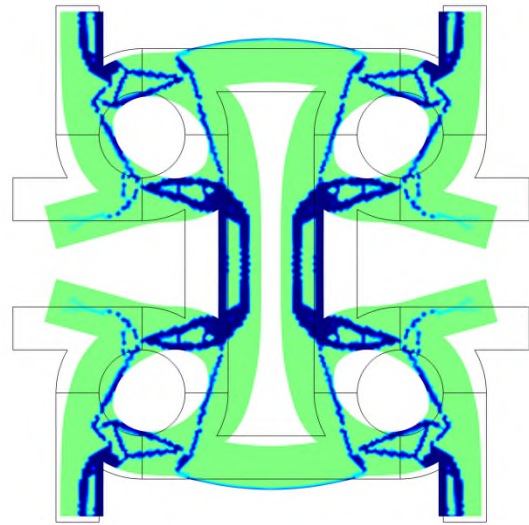
Figure 2B. Deformed shape of structure I.

In the figure 3A the distribution of materials in anti-tetra-chiral structure with $A_f = 30\%$ is shown. The whole structure after optimization exhibits the effective Poisson's ratio equal to -8.7017. In the figure 3B the deformed shape consisted of four unit cells is shown.
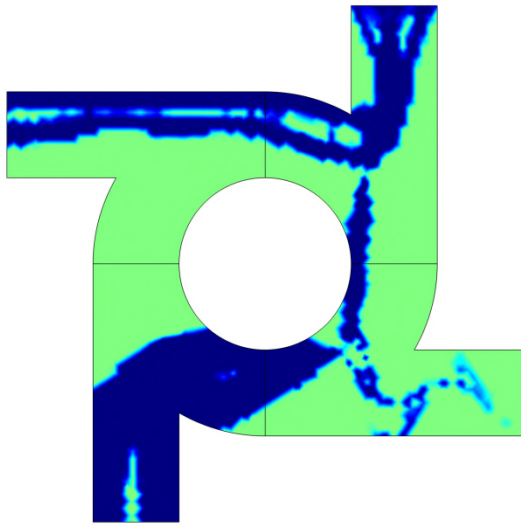

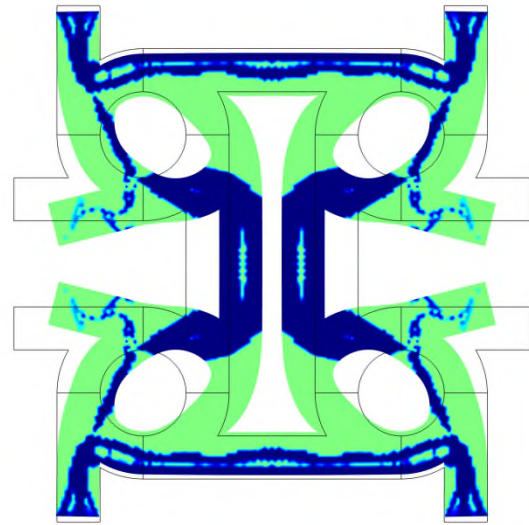**Figure 3A.** Two-phase material with A_f=30%.

Figure 3B. Deformed shape of structure II.

In the figure 4A the next optimized distribution of materials is presented in anti-tetra-chiral structure with $A_f = 40\%$. The whole structure after optimization exhibit the effective Poisson's ratio equal to -6.9386. In the figure 4B the network consisted of four optimized unit cells is presented.

In the figure 5A the next optimized distribution of materials in anti-tetra-chiral structure with $A_f = 50\%$ is presented. The whole structure after optimization is characterized by the effective Poisson's ratio equal -7.0516. In the figure 5B the network consisted of four optimized unit cells is presented.
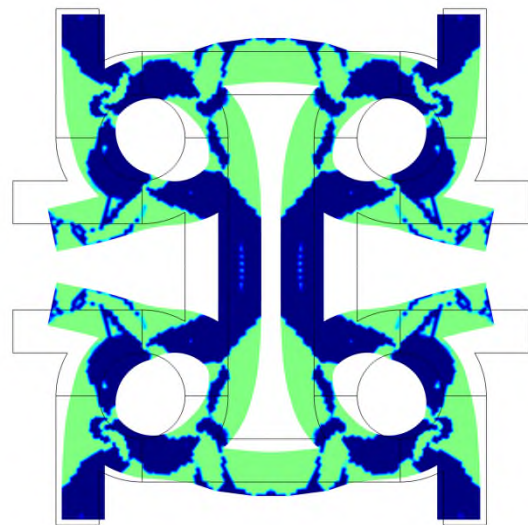
Figure 4A. Two-phase material with A_f=40%.

Figure 4B. Deformed shape of structure III.
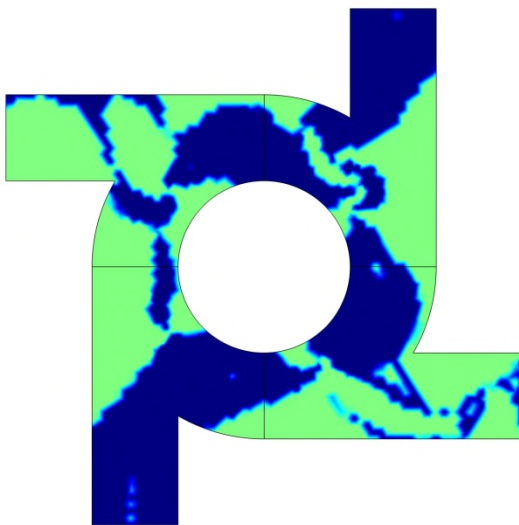
Figure 5A. Two-phase material with A_f=50%.

**Figure 5B.** Deformed shape of structure IV.

In the table 1 numerical values of Poisson's ratios for anti-tetra-chiral structures are presented for different values of A_f from 20% to 50% and g = 0.05 m. In table 2 numerical values of the Poisson's ratios for anti-tetra-chiral structures with smaller thickness of ribs g = 0.025 m are presented. All values of the Poisson's ratio are few times smaller than the value of the Poisson's ratio for homogenous isotropic structure.

Table 1. Poisson's ratio for anti-tetra-chiral structures for different values of A_f and g = 0.05 m.

Structure	I	II	III	IV
A_f	20%	30%	40%	50%
PR	-6.4188	-8.7017	-6.9386	-7.0516

Table 2. Poisson's ratio for anti-tetra-chiral structures for different values of A_f and $g = 0.025$ m.

Structure	V	VI	VII	VII
A_f	20%	30%	40%	50%
PR	-4.4994	-7.8173	-7.3032	-7.5293

A

B

C

D
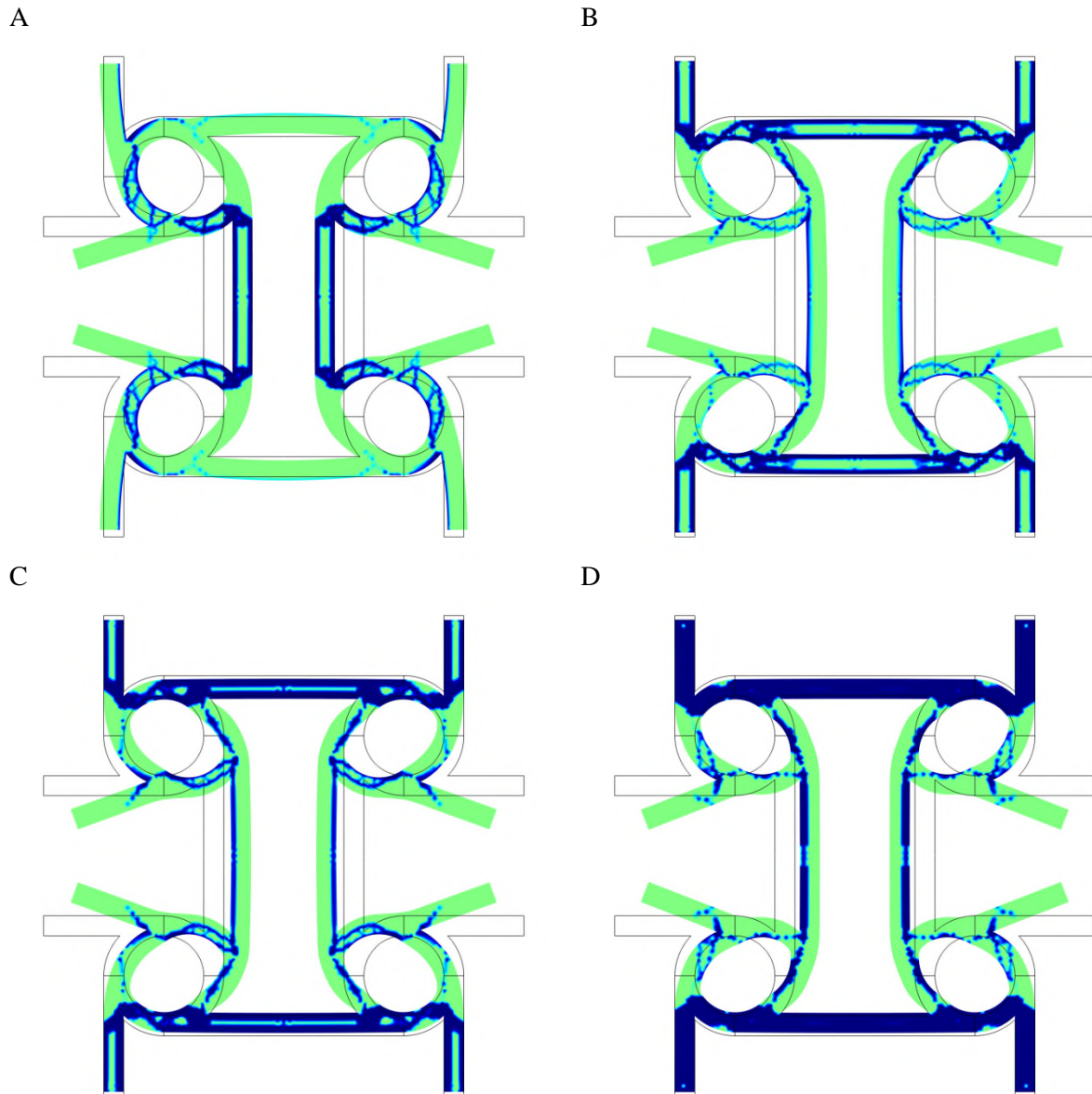
Figure 6. Deformed structures with $g = 0.025$ m and A_f (A) 20%, (B) 30%, (C) 40% and (D) 50%.

All presented optimized distributions of two-phase materials (Figures 2A-5A, 6) show that hard material (structural steel) represented by blue colour creates complicated shapes giving possibility to achieve anisotropic structures with characterized by great auxetic behaviour. One can find lattice-like structures created by hard material inside some parts of anti-tetra-chiral domain.

3. Conclusions
Normally, the value of the Poisson's ratio of anti-tetra-chiral structures made of one material approach to minus one and structure is isotropic. It is the lower bound for the Poisson's ratio of isotropic material. One-phase sample of considered structure is isotropic with Poisson's ratio equal -0.91099. However, the anti-tetra-chiral structure can be anisotropic if we use two materials to build the structure.

Optimisation of distribution of these materials allows to obtain structures characterized by the Poisson's ratio in the range from -4.5 to -8.7. The value depends on A_f parameter and mechanical properties of materials. The distribution of two materials causes that the whole structure is more auxetic and anisotropic. Also the distribution of reinforcing hard material inside soft matrix material in anti-tetra-chiral domain influenced mechanical properties of structure. This composite structure will exhibit great auxetic effect not known for recently built networks with anti-tetra chiral geometries of unit cells. The optimized structures may be in the future made by methods of 3-dimensional printing.

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