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Waveform distortion by 2-step modeling ground vibration from trains

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Abstract. The 2-step procedure is widely used in numerical research on ground vibrations from trains. The ground is inconsistently represented by a simplified model in the first step and by a refined model in the second step, which may lead to distortions in the simulation results. In order to reveal this modeling error, time histories of ground-borne vibrations were computed based on the 2-step procedure and then compared with the results from a benchmark procedure of the whole system. All parameters involved were intentionally set as equal for the 2 methods, which ensures that differences in the results originated from the inconsistencies of the ground model. Excited by wheel loads of low speeds such as 60 km/h and low frequencies less than 8 Hz, the computed responses of the subgrade were quite close to the benchmarks. However, notable distortions were found in all loading cases at higher frequencies. Moreover, significant underestimation of intensity occurred when load frequencies equaled 16 Hz. This occurred not only at the subgrade but also at the points 10 m and 20 m away from the track. When the load speed was increased to 350 km/h, all computed waveforms were distorted, including the responses to the loads at very low frequencies. The modeling error found herein suggests that the ground models in the 2 steps should be calibrated in terms of frequency bands to be investigated, and the speed of train should be taken into account at the same time.

1. Introduction

As a train moves along a track, uneven contact occurs between the wheels and rails, which excite coupled vibrations of the train-track-subgrade system. This impacts the environment of the surrounding area in a negative way. Research on this vibration and its propagation in terms of mathematical modeling and numerical simulation has been progressively conducted in the past 2 decades. These models can be categorized into 2 groups according to the integrality of the vibration system involved. Typical works in the first group consist of mechanical models of the 3 fundamental parts, i.e., the vehicle, the track, and the ground. The vibrations of these 3 parts are then simulated by the complete whole-system model [1-5]. In the second group, the modeling procedure consists of 2 steps, each with a separate mechanical model, as shown in figure 1 [6-9]. In the first step, the train and the track are carefully modeled to simulate their dynamic interactions, while the ground is simplified to a manageable damped-springs system. These damped springs were not able to describe wave propagation in the subgrade. The ground vibration needs to be simulated by means of another refined model in the second step, such as a multi-layered half-space, finite element model, or finite difference model.

Theoretically, the whole-system procedure is more practical than the separated 2-step procedure. However, the former has to be formulated in a Cartesian system of reference moving with the train, which requires a time-invariant mechanical system in a moving coordinates system. It is common that engineering practices do not fulfill this rigorous requirement of a moving coordinate system, since any structure with a limited length in the track direction, a railway station for example, could make the model time-variant in the moving reference system. Therefore, in many cases, researchers utilize the separated modeling procedure.

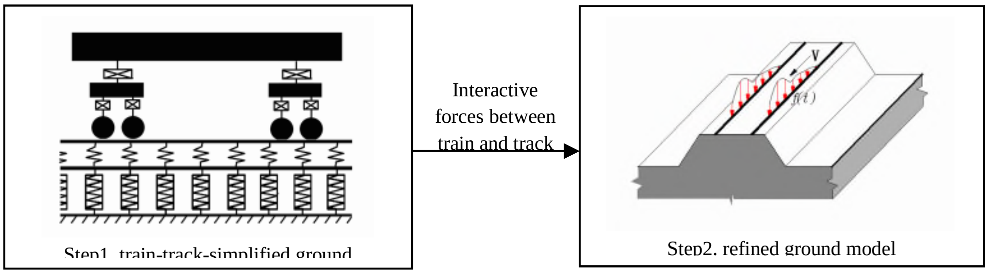


Figure 1. Computational scheme of the 2-step procedure.

One of the key conditions of the 2-step procedure is that the springs in the first step need to be equivalent to the ground model in the second step, since both of them represent the same physical ground. The equivalent stiffness of the springs is commonly computed by comparison of the static responses of both the ground models with an identical static load, which likely maintains equivalence in a static state. However, the train-induced vibrations are typically dynamic processes. Thus, it is uncertain whether the springs are still equivalent to the refined ground model in the dynamic case of vibration. If it is no longer equivalent, the calculated ground vibration is distorted and inaccurate. Numerical results may no longer be reasonable and can be interpreted incorrectly. This inaccuracy can be overcome if the influence of the modeling error is well understood.

The objective of this research is to reveal the modeling error of the 2-step procedure in the computed vibration of the ground and subgrade. A whole-system model is also formulated as a benchmark. Their results are compared, and some features of the vibrations are subsequently discussed.

2. Whole-system model of vehicle-track-ground

Figure 2 illustrates a well-developed whole-system model of the vibration for the vehicle-track-ground [4]. The vehicle is simplified to be a mass-spring-dashpot system, while the matrix $\boldsymbol{H}_{\mathrm{w}}$ represents its receptance at the M axles. The track is infinitely long in the x direction, and has a contact width of 2b with the ground. The rails are represented by an Euler beam that has a mass per unit length of m_{R}, and the flexural rigidity of EI. The sleepers are modeled by a distributed mass under the rail beam with a mass per unit length of m_{s}. The rail pads are approximated by a continuous spring distributed along the x direction with a stiffness of k_{p}. Consistent mass approximation is leveraged to simplify the embankment with a mass per unit length m_{B}, while only the vertical stiffness k_{B} is taken into account. Material damping effects are represented by a loss factor, and their imaginary parts are incorporated into the complex stiffness and modulus. Only the vertical dynamics of the vehicle and track are accounted for, and the interactive stress between the embankment and the ground is assumed to be vertical and uniformly distributed along the y axis

Only the harmonic wheel/rail unevenness with respect to the x-coordinate is used in this paper. Any random unevenness is composed of such fundamental ingredients according to Fourier analysis. The spatial wavelength of the unevenness can be written as $\lambda=2\pi c/\Omega$, thus the axles oscillate up and down, with respect to the z axis, in a time-harmonic manner with the frequency of $\Omega/2\pi$ when the train is moving at the speed of c. The motions of the axles result in a series of time-harmonic forces acting on the wheel-rail contacts moving along the track. Regarding the equilibrium of forces and compatibility of displacement, the amplitudes of the interactive forces at the contacts can be obtained and written in a vector form as

$$\boldsymbol{P} = -\left[\boldsymbol{H}_{\mathrm{R}} + \boldsymbol{H}_{\mathrm{w}} + \boldsymbol{K}_{\mathrm{h}}^{-1}\right]^{-1}\boldsymbol{Z} \tag{1}$$

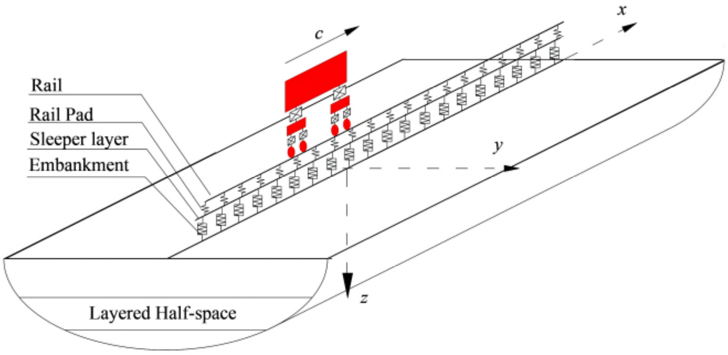


Figure 2 The Whole-system model of the coupled vibrations of vehicle-track-ground

The vector $\boldsymbol{Z}=\{e^{i\Omega a_1/c}, e^{i\Omega a_2/c},\ldots, e^{i\Omega a_M/c}\}^{\mathrm{T}}$ collects the wheel/rail unevenness perceived by all the wheels, and a_1, a_2,…, a_M are the initial x-coordinates of the M axles at time t=0. $\boldsymbol{K}_{\mathrm{h}}$ is a matrix representing the stiffness of the linearized Hertz spring between the rails and the wheels. The matrix $\boldsymbol{H}_{\mathrm{R}}$ represents the transfer function of the track-ground subsystem. The elements at the lth row and mth column are

$$H_{lm}^{\mathrm{R}} = w(a_l - a_m),\ l,\ m=1,M, \tag{2}$$

where (a_l-a_m) is the horizontal distance of the 2 wheel/rail contacts with the initial position $x=a_l$ and $x=a_m$. The function $w(x_*)$ in the moving system of reference ($x_*=x-ct$, y) can be obtained by the inverse Fourier transform of,

$$\tilde{w}(\beta) = \frac{\Delta_1\Delta_2 - \Delta_4^2 H}{\Delta_1\Delta_2\Delta_3 - \Delta_3\Delta_4^2 H - \Delta_1 k_{\mathrm{p}}^2} \tag{3}$$

in the wave number domain (β, γ). In equation (3), $\Delta_1=1+k_{\mathrm{B}}H-\omega^2 m_{\mathrm{B}}H/3$, $\Delta_2=k_{\mathrm{p}}+k_{\mathrm{B}}-\omega^2(m_{\mathrm{s}}+m_{\mathrm{B}}/3)$, $\Delta_3=EI\beta^4-\omega^2 m_{\mathrm{R}}+k_{\mathrm{p}}$, $\Delta_4=k_{\mathrm{B}}+\omega^2 m_{\mathrm{B}}/6$. ω denotes the circular frequency of the particles in the fixed frame of reference

$$\omega=\Omega-\beta c. \tag{4}$$

The function H is the receptance of the ground

$$H = -\frac{1}{2\pi}\int_{-\infty}^{+\infty}\delta(\beta,\gamma)\frac{\sin\gamma b}{\gamma b}d\gamma \tag{5}$$

in which $\delta(\beta, \gamma)$ is the dynamic Green's function of the layered half-space in the wave number domain (β, γ) [10].

Excited by the wheel/rail forces $\boldsymbol{P}=(P_1, P_2,\ldots,P_M)$, the vibration pattern of displacement in the wave number domain (β, γ) can be computed in terms of

$$\tilde{U}_{\mathrm{p}}(\beta,\gamma) = \tilde{U}(\beta,\gamma)\sum_{l=1}^{M} P_l e^{-i\beta a_l} \tag{6}$$

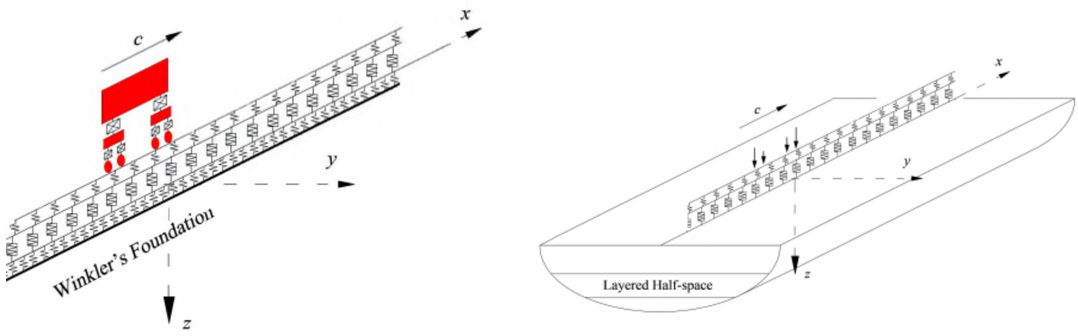
where $\tilde{U}(\beta,\gamma)$ is the displacement pattern exited by the time-harmonic load moving along the track [11]. For a particle located at (x, y, z) of the ground, the displacement time history is

$$U_{\mathrm{P}}(x,y,t) = e^{i\Omega t}U_{\mathrm{p}}(x-ct,y) \tag{7}$$

where the vibration pattern $U_{\mathrm{p}}(x_*,y)$ in the moving reference frame ($x_*=x-ct$, y) can be obtained by the inverse transform of $\tilde{U}(\beta,\gamma)$ with respect to β and γ.

3. 2-step model of train induced vibration

For the purpose of comparison, the 2-step model in this paper has been intentionally set to be the same as the above whole-system, except for the ground representation in the first step. As shown in figure 3a, the ground is replaced by the damped Winkler spring.



a) b)

Figure 3. The 2-step model with a) for the Tain-track-simplified ground model of the first step, and b) for the Track-ground model of the second step.

The receptance of the ground is now

$$H = \frac{1}{2bk_s} \tag{8}$$

in which k_s denotes the foundation modulus and is NOT frequency-dependent. Regarding the receptances in equation (5) and equation (8) yields,

$$k_s = -\frac{\pi}{b\int_{-\infty}^{+\infty} \delta(\beta,\gamma)\frac{\sin\gamma b}{\gamma b}d\gamma} \tag{9}$$

Equation (9) is used to maintain the static equivalence of the 2-step model and the whole-system model. Note that the Green's function in equation (9) is of the static state with frequency ω=0 and is simply a special case of the one in equation (5). The wheel/rail interaction forces $\boldsymbol{P}$ are also obtained by the equation (1), but the function H in equation (3) should be calculated in terms of equation (8) instead of equation (5).

The model of the second step, shown in figure 3b, is identical to the whole-system model without the vehicle. The wheel/rail forces $\boldsymbol{P}$ generated in the first step are inputted as a series of moving excitations on the rail beam. Responses in time domain are still calculated according to equation (7).

All other parameters involved are the same as those in the whole-system model, which helps ensure that every difference of the response originates from separate modeling.

4. Numerical results and discussion

An example was computed according to the above 2 models respectively. The model's ground had 2 upper sand layers overlaid on a harder half-space of Berea sandstone. The properties of the ground, the 2-suspension vehicle, and the track structure are detailed in references [10] and [11].

It was assumed that the amplitude of harmonic unevenness was 1 mm, thus the frequency perceived by the wheels, $\Omega/2\pi$, equaled 0.1, 4, 8, 16, 31.5, 63 Hz. Then the vibration of a particle in the ground was calculated in terms of equation (7). The computed time histories of the vertical displacement at x=0, y=0, and z=0.2m, located directly below the midline of the track within the sandy subgrade, are presented in figure 4. t=0 corresponds to the moment that the first wheelset passed over the particle, while the speed of the vehicle was c = 60 km/h.

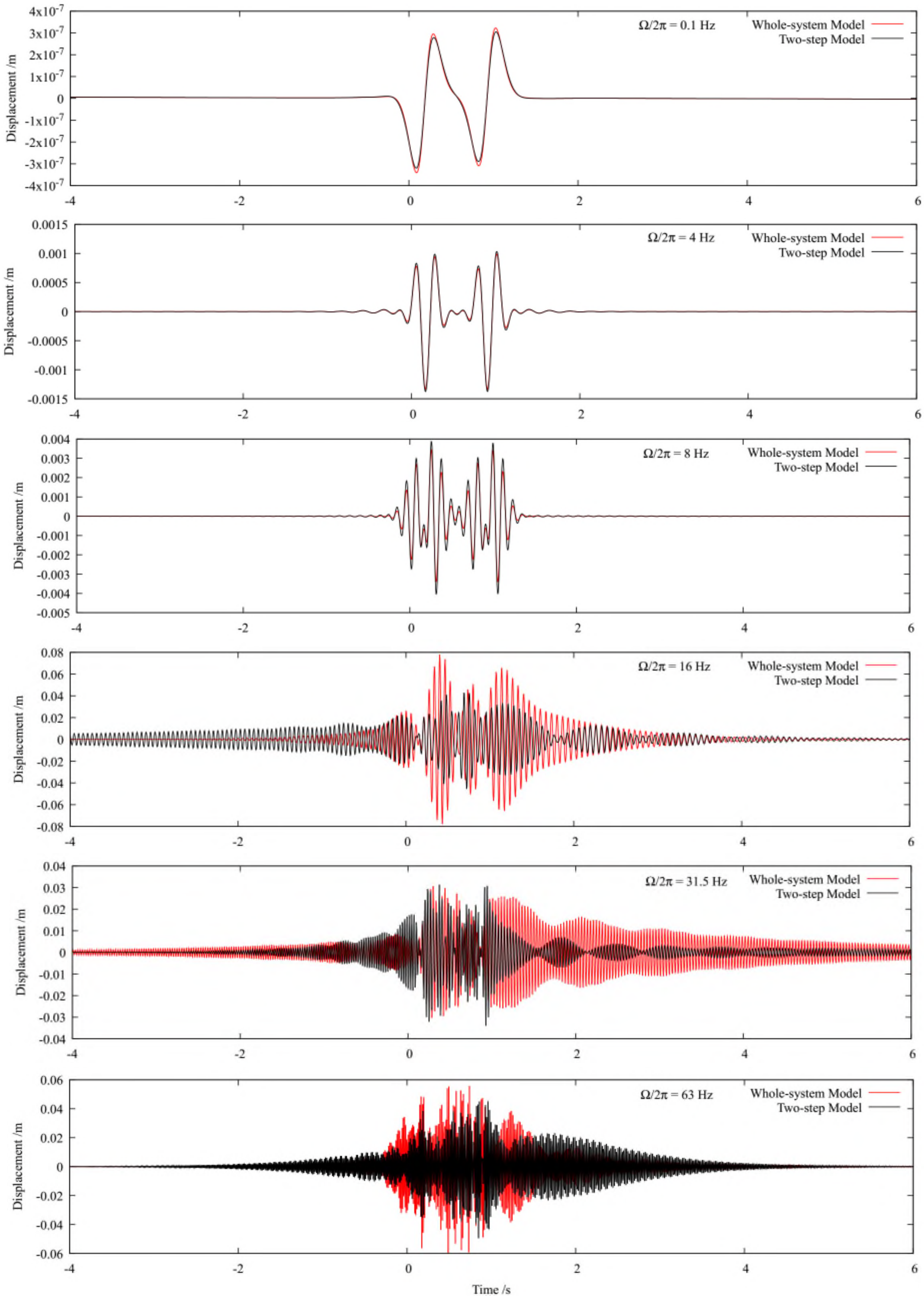


Figure 4. Displacement time histories of the particle 0.2 m below the midline of the embankment-ground interface, which was excited by a vehicle with a speed of 60 km/h.

In the cases of low excitation frequencies such as 0.1 Hz and 4 Hz, the curve obtained by the 2-step model coincides with that of the whole-system model. As the frequency increased to 8 Hz, subtle differences were found between the 2 results. When the frequency approached 16 Hz, remarkable distortion of the 2-step modeling waveform occurred. For higher frequencies of 31.5 Hz and 63 Hz, this discrepancy was still evident enough that it should not be ignored. The good agreement between models at very low excitation frequencies was expected, since the 2 models are equivalent under the static conditions where the vibration frequency equals zero. The distortion of the waveforms of 16 Hz, 31.5 Hz, and 63 Hz indicates that separate modeling may bring significant error to the simulation, since the main frequency range of the ground vibration approaches 80 Hz according to field measurements [12-14].

The computed results also showed that the distortion can be found not only in the immediate vicinity of the track but also in the far field of the ground, as shown in figure 5. This is where the time histories at the 2 points with horizontal distances 10 m and 20 m to the track were presented. Compared with the magnitude of curves from the whole-system model, the maximum displacement computed by the 2-step model was much lower, which indicates that the error of separate modeling may lead to an unfavorable underestimation of the vibration in certain circumstances.

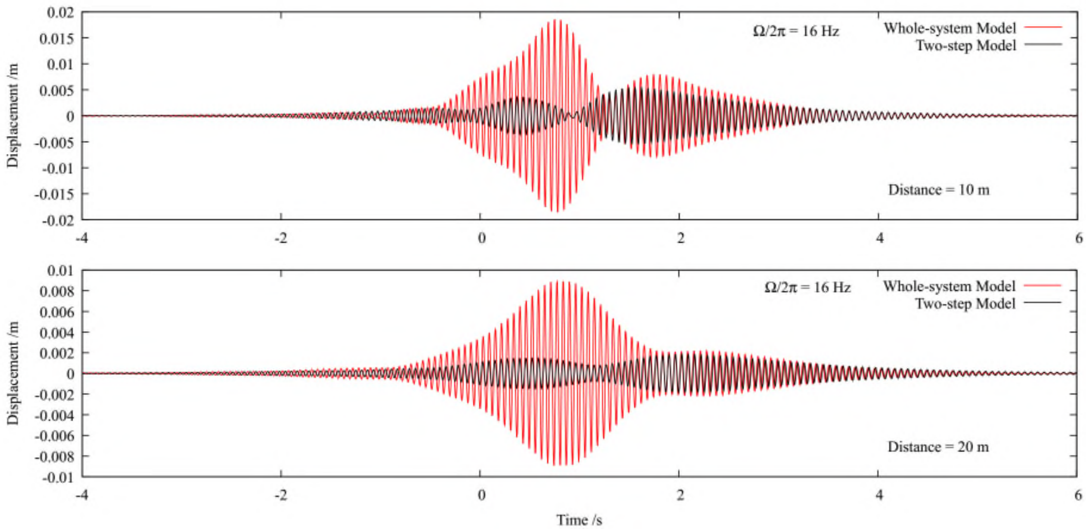


Figure 5. Displacement time histories of the 2 particles 0.2 m below the ground surface with horizontal distances to the midline of the track of 10 m and 20m, which were excited by a vehicle with a speed of 60 km/h.

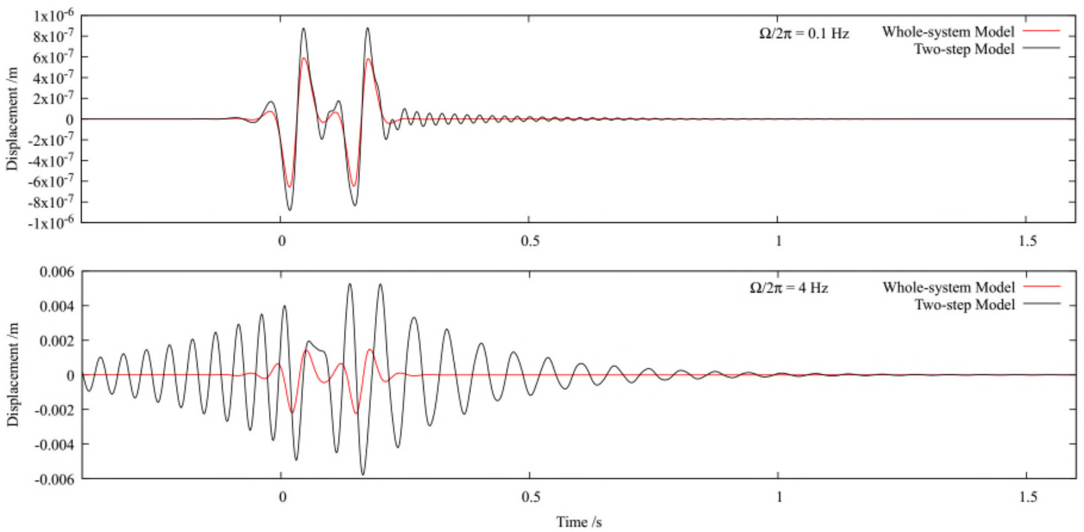


Figure 6. Displacement time histories of the particle, 0.2 m below the midline of the embankment-ground interface, excited by a vehicle with the speed of 350 km/h.

When the speed of the vehicle was increased to 350 km/h, the routine speed of CRH in China, the modeling error was more severe than the lower speed cases. Distortions were clearly found in the curves of vibration responses to all excitations of unevenness, even in the low frequencies of 0.1 Hz and 4 Hz, as shown in figure 6. The 2-step model failed to retain good performance under very low frequencies in the low-speed cases, which might be attributed to the high frequency content of ground vibrations generated by the high speed of the wheel/rail loads.

Conclusion

As the whole-system model of the coupled train-track-ground vibrations is unable to represent some complicated engineering conditions, the separated 2-step procedure has been commonly employed to numerically investigate ground vibration. The major concern of this method is the inconsistency of the ground model in the 2 steps, which may lead to modeling errors in the simulated results.

It was found that the modeling error depends on both the train's speed and the frequency of the wheel/rail excitation. In the case of low-speed trains and very low frequencies, the 2-step model generated satisfactory simulation results. However, for higher frequencies of excitation, the distortion of the waveforms was remarkable. This modeling error was not only found in the immediate vicinity of the track but also in the far field of the ground. Moreover, underestimation of the vibration intensity might take place because of the separate modeling. For high-speed trains, the distortion was found in the waveforms of vibration excited by every frequency of unevenness, which was more common than in the low-speed cases.

Regarding the modeling error of the 2-step method, it is strongly recommended that the stiffness of ground model in the first step be calibrated by the frequency response of the ground model in the second step, thus accounting for the influence of the train's speed.

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Acknowledgments

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