

SDRE controller for motion design of cable-suspended robot with uncertainties and moving obstacles

Ahad Behboodi¹ and Seyedmohammad Salehi²

¹Biomechanics and Movement Sciences University of Delaware, USA

²Computer and Information Sciences University of Delaware, USA

Abstract. In this paper an optimal control approach for nonlinear dynamical systems was proposed based on State Dependent Riccati Equation (SDRE) and its robustness against uncertainties is shown by simulation results. The proposed method was applied on a spatial six-cable suspended robot, which was designed to carry loads or perform different tasks in huge workspaces. Motion planning for cable-suspended robots in such a big workspace is subjected to uncertainties and obstacles. First, we emphasized the ability of SDRE to construct a systematic basis and efficient design of controller for wide variety of nonlinear dynamical systems. Then we showed how this systematic design improved the robustness of the system and facilitated the integration of motion planning techniques with the controller. In particular, obstacle avoidance technique based on artificial potential field (APF) can be easily combined with SDRE controller with efficient performance. Due to difficulties of exact solution for SDRE, an approximation method was used based on power series expansion. The efficiency and robustness of the SDRE controller was illustrated on a six-cable suspended robot with proper simulations.

1. Introduction

A new class of highly applicable crane-like robots, driven by cables, is designed with advantages including convenient assembly, simple structure, lightness, acceptable accuracy, huge workspace, and relatively high Dynamic Load Carrying Capacity (DLCC) [1]. Using cables as actuators in this kind of robot makes the dynamic equations more complicated and nonlinear. Therefore, a controller is needed to capture nonlinearities as well as to guarantee optimal performance in keeping DLCC at a maximum level. Another important feature of cable-suspended robots is their huge workspace, which might contain moving or fixed obstacles. Thus, an appropriate controller must be capable of capturing supplementary techniques for motion design in the presence of obstacles in a convenient and efficient manner.

Controlling and regulating the performance of dynamical systems based on the desired task is the key objective in almost all mechanical engineering research fields not only in vibrations [2], dynamics and robotics [3-5] as the most common ones, it is also extended to fluid mechanics [6, 7] and boundary layer analysis [8, 9]. For cable-suspended robots, many control methods have been proposed; open-loop controllers like Hamilton-Jacobi-bellman (HJB) [10] (which is weak against noises and uncertainties), closed-loop controllers like feedback linearization [11] and Sliding Mode Control (SMC) [12] (which are not optimal), Optimal linear and closed-loop controllers like Linear Quadratic Regulator (LQR) with feedback linearization basis [13] (which is unable to capture nonlinearities properly).

SDRE method provides an optimal controller that can be applied to complicated nonlinear dynamic systems easily and systematically [14]. Authors in [15] has done a thorough survey of SDRE that contains details like formulation and structure, optimality concept, state dependent parameterization



and stability conditions. In comparison with LQR, SDRE is proved to be more robust [16]. Difficulties of SDRE's exact solution has given rise to numerical techniques like power series [17], which are developed for the most general cases [14].

Motion design for cable-suspended robots that perform in huge workspace might be problematic and are usually subjected to different constraints like obstacles [13] and moving boundaries [18]. In this paper, obstacles were considered based on their more commonalities over other constraints. Most of the existing obstacle avoidance techniques are based on the APF concept [19]. The potential term created by this method is usually nonlinear which is problematic for linear based control methods like LQR and makes them more complex and restricted [13]. While SDRE is shown to be capable of being combined with this technique very conveniently and perform efficiently in the presence of fixed obstacles [14], many other potentials of such controllers are not yet studied. In this paper, by proper simulations, we show that the SDRE combined with APF techniques is able to avoid moving obstacles in motion design. Robustness of this controller against uncertainties is also illustrated with simulation results on six-cable-suspended robot.

2. Controller formulation

A nonlinear dynamical system equation can be always factorized and be expressed in the form below:

$$\dot{x}(t) = A(x)x(t) + B(x)u(t) \quad (1)$$

where $x \in R^n$ and $u \in R^m$ are state and input vectors respectively. The input $u(t)$ is designed to minimize the objective function:

$$W = \int_0^\infty [x^T(t)Q(x)x(t) + u^T(t)R(x)u(t)]dt \quad (2)$$

SDRE method suggests the optimal control law as:

$$u(x) = -R^{-1}(x)B^T(x)K(x)x \quad (3)$$

where $K(x)$ is unique symmetric and positive definite solution of the Riccati equation below:

$$K(x)A(x) + A^T(x)K(x) - K(x)B(x)R^{-1}(x)B^T(x)K(x) + Q(x) = 0 \quad (4)$$

Solving state dependent Riccati equation is very difficult and even impractical when system has complicated dynamical equation. Consequently, an approximate solution approach is needed. Power series method has been developed for dynamic systems for the case where A, B and even Q matrix is state dependent [14]. The same method has been used in this paper.

APF was used as obstacle avoidance technique in this paper. The blow potential term was added to the objective function.

$$\sum_{i=1}^n w'_i \left(\frac{1}{r_{o_i} + r_m + S_p - d_i} \right)^2 \quad (5)$$

where n is the number of the obstacles, r_{o_i} is the radius of circumscribed sphere to the i -th obstacle, r_m is the radius of safe zone sphere for the end-effector, S_p is the safety factor, w'_i is weight of the potential term and d_i is the distance between end-effector and the i -th obstacle. After adding this term to the objective function, since Q can be state dependent, it will be resolved to the Q matrix to keep the standard format of SDRE objective function. In this way, all the same formulation of SDRE can be used for motion design with obstacle avoidance. This is a very convenient capability of SDRE method that can be easily combined with complementary techniques like APF.

3. Simulations

The schematic of spatial cable-suspended robot is presented in figure 1. Dynamic equations of the robot were derived in [20] and dynamic equations that were adapted to use in SDRE formulation are presented in [14]. There were six motors in A, B, C (two in each one) and six cables to control the end-effector DEF .

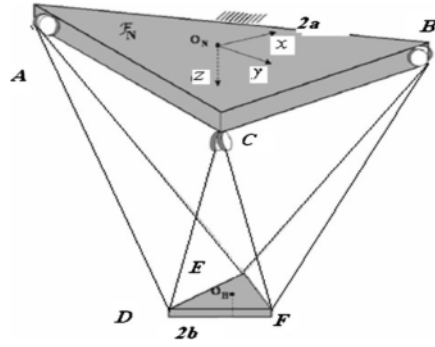


Figure 1. Schematic of six-cable-suspended robot [14].

3.1. Trajectory tracking

In addition to SDRE controller's capability in acceptable trajectory tracking, the parameters of the controller can be tuned to compensate for the uncertainties in the initial position. In the conducted simulation, end-effector should follow a predefined path while at the beginning it was not on the desired path. Thus it should first go to the desired path as quickly as possible and then follow it. The predefined trajectory is given by:

$$x = 0.035 \cos(t) - 0.035 \sin(t); y = 0.035 \sin(t); z = 1 + 0.025 \cos(t) + 0.025 \sin(t) \quad (6)$$

where $0 \leq t \leq 2\pi$. Comparing the results with LQR method demonstrated how capturing nonlinearities of the system helps the controller to be more precise. Trajectories of SDRE and LQR are shown in figure 2 and motor torques for the first three motors is demonstrated in figure 3.

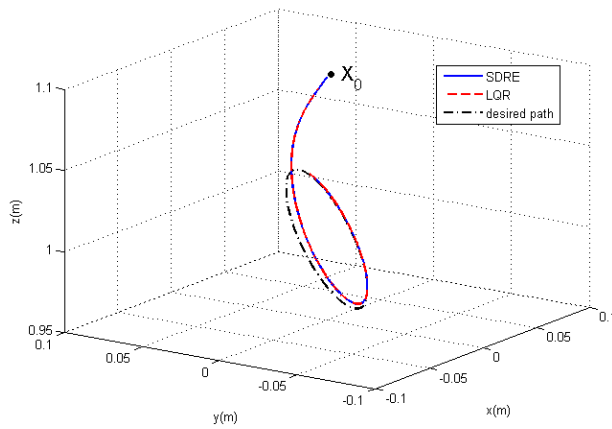


Figure 2. Trajectories of the end-effector.

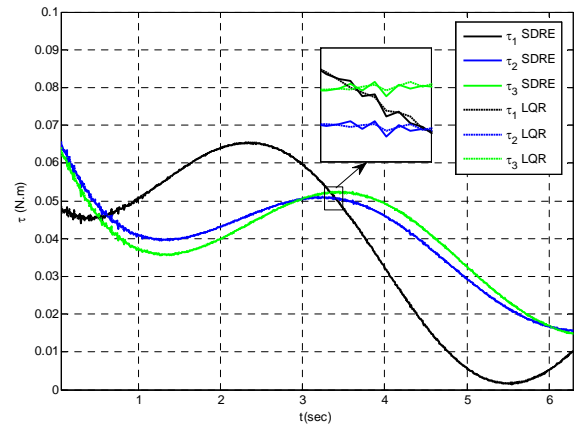


Figure 3. Motor torques as control inputs.

The results show that how the proposed controller successfully overcomes the uncertainties at initial position and makes end-effector follow the desired trajectory precisely. Errors from real trajectories to the desired one are shown in figure 4. Clearly SDRE performed more accurate than LQR since it captures nonlinear behavior of the dynamic system.

3.2. Moving obstacle

To make the situation more difficult and realistic and to assess the performance of the proposed controller in combination with APF obstacle avoidance technique, a moving obstacle was considered during the robot performance. The motion of the obstacle of radius 2 cm is characterized by:

$$x_{obs} = (0.0332)t; y_{obs} = (0.0332)t - 0.81; z_{obs} = 0.7595 \quad (7)$$

Simulations were conducted for three cases; first, no obstacle avoidance technique, in second and third case the obstacle avoidance technique was added but the weight of the potential term was different and set to 0.05 and 0.15 respectively in order to show the effect of this parameter.

Trajectories and the distance between end-effector and obstacle during motion are shown in figure 5 and figure 6 respectively. The results illustrated how a complicated technique like APF can be easily combined with SDRE formulation to successfully design an optimal motion for the robot to avoid moving obstacles.

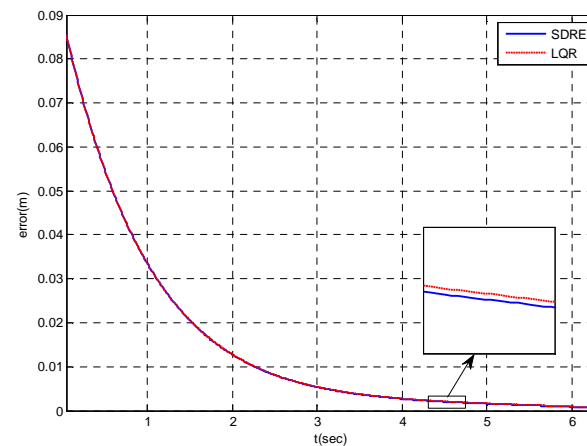


Figure 4. Comparison of trajectory errors of SDRE and LQR.

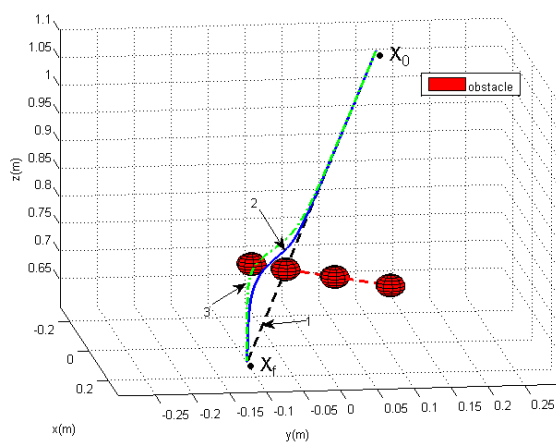


Figure 5. Obstacle avoidance of the end-effector.

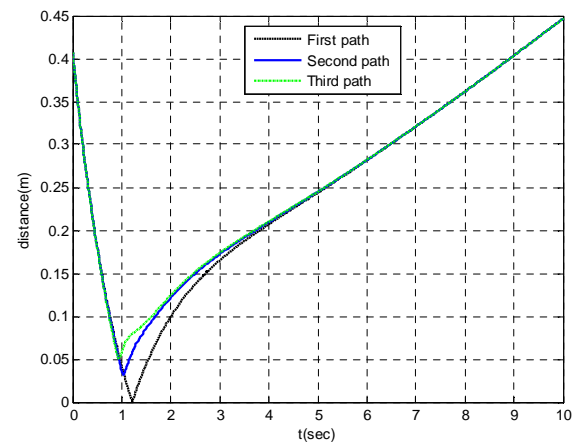


Figure 6. End-effector and obstacle distance.

4. Conclusion

In this work, capabilities of SDRE controller in handling uncertainties were investigated. The simulation results on trajectory tracking of cable-suspended robot with uncertainty in initial position illustrated that SDRE method can provide a robust and systematic basis for controlling nonlinear dynamical systems. One advantage of SDRE method is the ability to be easily combined with supplementary techniques for motion design such as APF-based obstacle avoidance techniques. Results in difficult and realistic situation i.e. moving obstacles in the robot workspace showed the efficiency of the controller in designing an optimal motion for the robot in the presence of obstacles.

5. References

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