

Transverse vibration analysis of stiffened plates with elastic support

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Abstract. The transverse vibration of stiffened plates with elastic support boundary conditions is studied in this paper. First, strain energy and kinetic equations of the deck, girders and ribs are respectively established. Second, governing equations for the structure are obtained according to the theorem of conservation of energy and solved using Ritz method. According to the relationship between the motherboard and the main girder deformation coordination, the displacement function of the deck can be got based on the displacement function of the main girder. At last, finite element model is built to check the accuracy of theoretical analysis. It can be found that the theoretical solution and the finite element solution have the same variation trend, and the error between the two methods is small. Theoretical analysis method is reliable, and it has certain significance for further research and engineering design.

1. Introduction

In practical engineering, the orthotropic bridge deck is used more and more widely. The deck slab is the most important bearing structure in the bridge, and the load on it is complex. So we need to conduct the thorough research to the characteristics of the bridge deck. There are several kinds of methods, such as grillage method [1], Raileigh-Ritz method [2], finite element method [3], differential quadrature method [4] and no element method [5]. However, the work has been done confined to the boundary conditions of four edges simply supported or clamped, and the research to the boundary conditions for elastic supports is very rare. Zhong W.H et al. [6] considered the material nonlinearity and static load in their paper. Han Q. [7] studied the free vibration problem of rectangular plates with four edges simply supported. Ma N.J. et al. [8, 9] studied the local vibration of orthotropic steel bridge deck and the free vibration problem of rectangular plates with four boundary fixed.

In this paper, the transverse vibration of stiffened plates with elastic support is analyzed. The law of conservation of energy is used to establish the control equations, and Ritz method is used to solve it. In order to check the accuracy of theoretical analysis, finite element model is built by Midas Civil.

2. Building energy formula alone

The energy contains the energy in the orthotropic deck and in the main beam. The strain energy in the deck is

$$V_p = \frac{D}{2} \int_0^a \int_0^b \left\{ \left(\nabla^2 w_p \right)^2 + 2(1-\mu) \left[\left(\frac{\partial^2 w_p}{\partial x \partial y} \right)^2 - \frac{\partial^2 w_p}{\partial x^2} \frac{\partial^2 w_p}{\partial y^2} \right] \right\} dx dy \quad (1)$$



Where

$$D = \frac{Eh^3}{12(1-\mu^2)} \quad (2)$$

where, a and b are length and width, E is the Young modulus of the material, h is the thickness of the deck, μ is the Poisson ratio of the material, w_p is the deflection of the deck.

The strain energy in the section i stiffening rib in the x -direction and y -direction is

$$\left. \begin{aligned} V_{sxi} &= \frac{1}{2} EI_x \int_0^a \left(\frac{\partial^2 w_p}{\partial x^2} \right)^2 dx \\ V_{syj} &= \frac{1}{2} EI_y \int_0^b \left(\frac{\partial^2 w_p}{\partial y^2} \right)^2 dy \end{aligned} \right|_{\substack{y=y_i \\ x=x_j}} \quad (3)$$

The strain energy in the main beam is

$$\left. \begin{aligned} V_{gx} &= \frac{1}{2} EI_{gx} \int_0^a \left(\frac{\partial^2 w_x}{\partial x^2} \right)^2 dx \\ V_{gy} &= \frac{1}{2} EI_{gy} \int_0^b \left(\frac{\partial^2 w_y}{\partial y^2} \right)^2 dy \end{aligned} \right\} \quad (4)$$

From Equations (1) and (4), the strain energy in the whole structure is obtained

$$V = V_p + 2V_{gx}(x=0, a) + 2V_{gy}(y=0, b) + V_s \quad (5)$$

where,

$$V_s = \sum_{i=1}^{N_x} V_{sxi} + \sum_{j=1}^{N_y} V_{syj} \quad (6)$$

N_x and N_y are the number of stiffening ribs in the x -direction and y -direction.

The kinetic energy in the deck is

$$T_p = \frac{1}{2} \rho h \int_0^a \int_0^b \left[\left(\frac{\partial w_p}{\partial t} \right)^2 \right] dx dy \quad (7)$$

The kinetic energy in the section i stiffening rib in the x -direction is

$$T_{sxi} = \frac{1}{2} \rho A_x \int_0^a \left(\frac{\partial w_p}{\partial t} \right)^2 dx \bigg|_{y=y_i} \quad (8)$$

where, ρ is density of the material, A_x is sectional area of the stiffening ribs.

The kinetic energy in the section j stiffening rib in the y -direction is

$$T_{syj} = \frac{1}{2} \rho A_y \int_0^b \left(\frac{\partial w_p}{\partial t} \right)^2 dy \bigg|_{x=x_j} \quad (9)$$

The kinetic energy in the main beam is

$$\left. \begin{aligned} T_{gx} &= \frac{1}{2} \rho A_{gx} \int_0^a \left(\frac{\partial w_x}{\partial t} \right)^2 dx \\ T_{gy} &= \frac{1}{2} \rho A_{gy} \int_0^b \left(\frac{\partial w_y}{\partial t} \right)^2 dy \end{aligned} \right\} \quad (10)$$

From equations (7) and (10), we have the kinetic energy in the whole structure, as following

$$T = T_p + 2 \cdot T_{gx}(x=0, a) + 2 \cdot T_{gy}(y=0, b) + T_s \quad (11)$$

where,

$$T_s = \sum_{i=1}^{N_x} T_{sxi} + \sum_{j=1}^{N_y} T_{syj}$$

3. Displacement function

To apply Ritz method, we need first select a Displacement function for the orthotropic deck. It has been mentioned before that the deck is laid on four girders simply supported at their ends.

When a thin plate free vibrate at a certain frequency ω and vibration mode $W(x, y)$, its instantaneous deflection can be expressed as

$$w = A \cos(\omega t) W(x, y) \quad (12)$$

where,

$$W(x, y) = A_1 \left(-\frac{1}{2} x^4 + ax^3 - \frac{a^3}{2} x \right) + B_1 \left(-\frac{1}{2} y^4 + by^3 - \frac{b^3}{2} y \right) + C_1 \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \quad (13)$$

4. Approximate solutions from energy method

According to the principle of conservation of energy, the maximum value of the strain energy is equal to the maximum of the kinetic energy. We have

$$V_{\max} = T_{\max} \quad \text{that is} \quad Q = V_{\max} - T_{\max} = 0 \quad (14)$$

Based on the assumption of the vibration type function and different constants, Q obtain the minimum value, as follow

$$\frac{\partial Q}{\partial A} = \frac{\partial Q}{\partial B} = \frac{\partial Q}{\partial C} = 0 \quad (15)$$

From equations (15), we have

$$\frac{5}{32} A + \frac{5}{32} B + C = W \quad (16)$$

From equations (16), constants A B C and lowest order natural frequency ω can be obtained. Due to limited space, the results of the formula will not be listed.

5. Numerical calculations and discussions

The geometry of the finite element model of the orthotropic bridge deck is shown in figure 1 with $b_r = 0.05m$ and $h_r = 0.1m$. The size of the main girder is also shown in figure 1 with $b_g = 0.1m$ and $h_g = 0.2m$. The thickness of the deck is $0.05m$. The model is simply supported at the four corners illustrated in figure 2 (b). The main girders and ribs are simulated by beam element. The bridge deck is simulated by plate element. Figures 2 (a) illustrates the geometry of this bridge system established by MIDAS Civil. The width of the plate is $b=2m$. The aspect ratio a/b is a parameter. Three longitudinal ribs located in $3b/4$, $b/2$ and $b/4$ and three transverse ribs located in $3a/4$, $a/2$ and $a/4$ are selected.

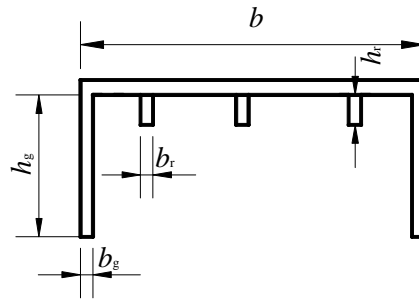


Figure 1. Geometry of the cross section in breadth direction.

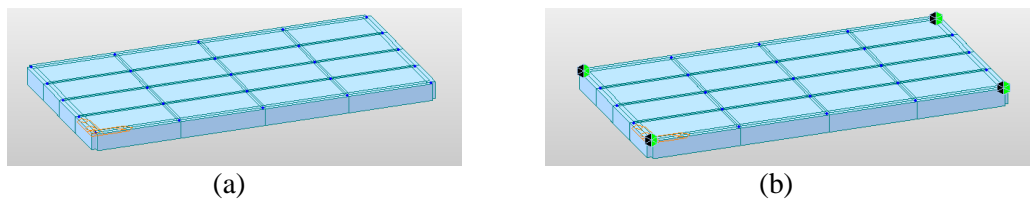


Figure 2. Model established by MIDAS Civil.

(a) bridge system (b) boundary conditions

All parameters of the material and structural were brought into theoretical analysis and MIDAS Civil analysis. The natural frequency " ω " of two methods and MIDAS Civil and errors between them are listed in table 1. The aspect ratio a/b of the bridge deck is usually in the range of 1 to 10 in application.

Table 1. Natural frequency of two methods and errors between them.

a/b	The natural frequency " ω " (hz)							
	1	1.5	2	2.5	3	4	5	10
Theoretical solution	71.83	41.24	25.36	17.02	12.19	7.12	4.66	1.22
MIDAS Civil	72.2	41.36	25.23	16.7	11.8	6.75	4.36	1.11
The error (%)	-0.5	-0.3	0.5	1.9	3.3	5.5	6.9	10.3

From table 1, we can get some conclusions as following:

- (1) The trend of two methods that the natural frequency decrease with the increase of the aspect ratio a/b is the same. The main cause for such trend is that the stiffness of the bridge deck decrease with the increase of the aspect ratio a/b and is proportional to the natural frequency, such as the hypothesis of the small deflection of the sheet.
- (2) The error between the theoretical solution and finite element solutions increases with the increase of the aspect ratio a/b . First reason is some assumptions or simplified are taken in the process of theoretical analysis. Second, the nature of the orthotropic plate is more significant with the increase of aspect ratio. But energy method did not reflect this characteristics. Third, the finite element model also has some errors itself. For example, multiple Ritz vector method is taken to calculate the natural frequency, and it will be affected by the number of units.

- (3) The errors between two methods are relatively small when the aspect ratio is from 1 to 5. It reflects the theoretical analysis method has higher precision and the displacement function satisfies the boundary conditions and meet the accuracy requirements.

6. Conclusions

Transverse vibration of stiffened plates with elastic support is analyzed in this paper. Theoretical model and finite element model are built and compare with each other. From section 6 in this paper, we can get that the errors between two methods are relatively small. Displacement function that is selected to simulate the elastic boundary conditions is reliable, and it can be provide reference for further research.

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