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# Dynamic analysis of suspension cable based on vector form intrinsic finite element method

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**Abstract.** A vector finite element method is presented for the dynamic analysis of cable structures based on the vector form intrinsic finite element (VFIFE) and mechanical properties of suspension cable. Firstly, the suspension cable is discretized into different elements by space points, the mass and external forces of suspension cable are transformed into space points. The structural form of cable is described by the space points at different time. The equations of motion for the space points are established according to the Newton's second law. Then, the element internal forces between the space points are derived from the flexible truss structure. Finally, the motion equations of space points are solved by the central difference method with reasonable time integration step. The tangential tension of the bearing rope in a test ropeway with the moving concentrated loads is calculated and compared with the experimental data. The results show that the tangential tension of suspension cable with moving loads is consistent with the experimental data. This method has high calculated precision and meets the requirements of engineering application.

## 1. Preface

The freight ropeway has simple structure and high tensile strength. During working it can fully exert its advantages of small weight, low cost, easy construction and strong transportation ability. It has been widely used in fields such as mining, forestry and aerial transmission line construction, etc.

At present, the dynamic analysis of suspension cable has also been studied. In the papers [1, 2] the discrete method and direct method for nonlinear dynamic analysis are used to analyse the suspension structure. It is found that the result of discrete method may be wrong. In the paper [3] the dynamic model of the cable with large sag is established by multi rigid body system dynamic theory. The results show that the method is accurate. The dynamics of an infinite string on an elastic foundation subjected to a moving load is under investigation in the paper [4]. The free vibration of suspension cable is analysed by using Wavelet-Galerkin method in the paper [5]. In the paper [6, 7] the dynamic study of the fixed load on the suspension cable is carried out. However, the study on suspension cable under moving concentrated loads is rare.

In this paper, the dynamic analysis method of suspension structure under moving concentrated load is established based on the vector form intrinsic finite element (VFIFE) theory [8]. Then the method is applied to calculate the cable tension of an engineering ropeway under the test condition.

## 2. VFIFE of suspension structure

In the ropeway, the bearing rope can be regarded as the flexible suspension cable, and it follows the assumption of elastic cable.

The truss element is used to analyse the dynamic response of suspension structure by the VFIFE method. The VFIFE method for suspension structure is deduced below.

### *2.1. The description of space points in suspension structure*
In the calculation of VFIFE, the suspension structure is discretized into a set of space points, the points are combined with the truss element. The shape of suspension cable is reflected by the motion trajectory of space points, shown as figure 1. The mass of the structure is shared by the space points, and the elements have no mass. The motion of the space points follows Newton's second law.

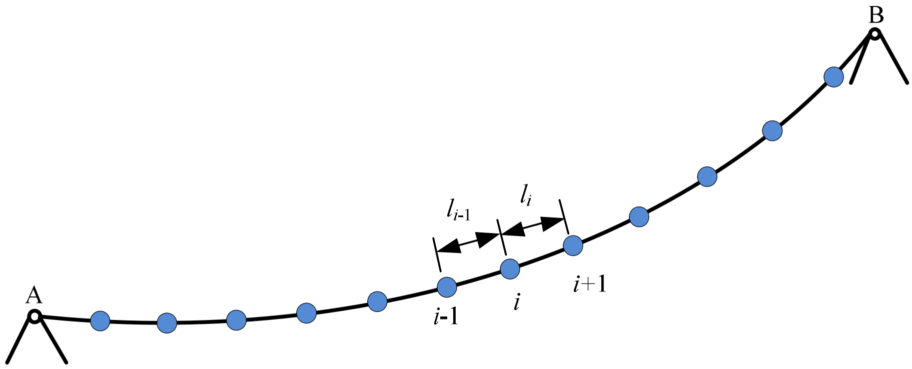


**Figure 1.** The Description of Space Points in Suspension Structure.

The suspension cable has $n$-1 elements, and the length of element is $l_i$. The mass of the suspension cable is evenly distributed on the $n$ space points, so

$$m_i = \frac{1}{2} q l_{i-1} + \frac{1}{2} q l_i \tag{1}$$

Here, $m_i$ is the mass of the space point $i$, $q$ is the mass of suspension cable per unit length.

Any point on the cable follows Newton's second law in the movement, so its equation of motion can be written as

$$m_i \frac{\mathrm{d}^2 \boldsymbol{x}_i}{\mathrm{d}t} = \mathbf{P}_i + \boldsymbol{p}_i + \boldsymbol{f}_i \tag{2}$$

Here, $x_i$ is the position vector function for space point $i$, $\mathbf{P}_i$ is the external force acting on the space point $i$ directly, $\boldsymbol{p}_i$ is the equivalent external force acting on the space point $i$ which is transformed from the external force acting on the elements connected to the point, $\boldsymbol{f}_i$ is the resultant force acting on the space point $i$ that connected with the truss elements.

### *2.2. The calculation of element internal force*
When the space points are moving, they will be affected by the internal force of the element connected with them. Take one element for force analysis. Define that the cross-sectional area is $A$, the elastic modulus is $E$, the length of element is $l$. In the process of deformation, the cross-sectional area and elastic modulus of the truss element remain unchanged.

The truss element is a kind of axial force element, the axial stress is $\sigma^0$, thus the internal force of the element is

$$-\boldsymbol{f}_1^0 = \boldsymbol{f}_2^0 = f^0 \boldsymbol{e}^0 = (\sigma^0 A) \boldsymbol{e}^0 \tag{3}$$

Here, $\boldsymbol{f}^0$ is the internal force of the truss element at time $t_0$, $\boldsymbol{e}^0$ is the unit vector of the truss element at time $t_0$.

The element moves as a rigid body from time $t_0$ to $t$, so it is still straight. The deformation of the element:

$$\Delta \boldsymbol{u} = (l - l_0) \boldsymbol{e}^0 \tag{4}$$

Here, $l_0$ is the length of the truss element at time $t_0$, $l$ is the length of the truss element at time $t$.

So the internal force of the truss element is

$$\boldsymbol{f}_{elem} = \left( f_{elem}^{0} + \frac{EA}{l_0}(l - l_0) \right) \boldsymbol{e}^{t} \tag{5}$$

The suspension cable is absolutely flexible, so it can't bear compressive stress. If $\boldsymbol{f}_{elem}<0$, then the element internal force is defined as zero, $\boldsymbol{f}=0$.

*2.3. The external force calculation of the space point*

The truss element is axial force element, the external force of the element can be converted into the concentrated force acting on the space point according to the principle of equivalence.

*2.4. The process of moving loads*

When loads moving along the suspension cable, the gravity forces of loads are constant and the directions of gravity forces are vertical. Assumed that the length of suspension cable changes very small, the loads move from the starting point to the next adjacent space point during the integration step.

## 3. Solution of the motion equation

In the VFIFE, the central difference formula is used to solve the motion equations. The velocity and acceleration of the space points can be obtained by the central difference formula:

$$\dot{x} = \frac{1}{2h}\left(x_{n+1} - x_{n-1}\right) \ (n \geq 1) \tag{6}$$

$$\ddot{x} = \frac{1}{h^2}\left(x_{n+1} - 2x_n + x_{n-1}\right) \ (n \geq 1) \tag{7}$$

Here, $x_{n+1}$, $x_n$ and $x_{n-1}$ is the position vector of space points at step $n$+1, $n$ and $n$-1 respectively. $h$ is time increment step. Substituting the formula (6) and (7) into the motion equations of the space points, $x_{n+1}$ can be obtained:

$$\boldsymbol{x}_{n+1} = \frac{\boldsymbol{F}^n}{m}h^2 + 2\boldsymbol{x}_n - \boldsymbol{x}_{n-1} \ (n \geq 1) \tag{8}$$

Here, $\boldsymbol{F}^n$ is the resultant force acting on the space points at the time step $n$, $\boldsymbol{F}^n = \mathbf{P}^n + \boldsymbol{p}^n + \boldsymbol{f}^n$.

Introducing the energy dissipation mechanism, the structure can reach static balance. In the equation of motion, the virtual damping force is increased, thus the motion equation of the space points is:

$$m\ddot{\boldsymbol{x}} = \boldsymbol{F}^n + \boldsymbol{f}_{dmp} \tag{9}$$

Here, $\boldsymbol{f}_{dmp}$ is the virtual damping force, $\boldsymbol{f}_{dmp} = -\xi m\dot{\boldsymbol{x}}$, $\xi > 0$ is the damping factor.

Substituting the formula (6) and (7) into (9), it can obtain:

$$\boldsymbol{x}_{n+1} = \frac{h^2}{m}c_1\boldsymbol{F}^n + 2c_1\boldsymbol{x}_n + c_1c_2\boldsymbol{x}_{n-1} \ (n \geq 1) \tag{10}$$

Here, $c_1 = \dfrac{2}{2+\zeta h}$, $c_2 = \dfrac{\zeta h}{2} - 1$.

When $n$=0, by the formula (6), (7) and (10), the initial position vector can be obtained:

$$\boldsymbol{x}_1 = \frac{\boldsymbol{F}^n}{2m}h^2 + \boldsymbol{x}_0 - c_2h\dot{\boldsymbol{x}}_0 \tag{11}$$

## 4. Example

To verify the calculation method, the field test of freight ropeway is finished in Hanzhong, Sichuan in China, shown as figure 2. Here a working case is selected to be calculated by the method and the results are compared with the experimental data.

**Figure 2.** Testing Site and Concentrated Load.

The test ropeway is a single span ropeway, shown as figure 3. The span is 218.183m, height is 118.668m, diameter of bearing rope is 20mm, the elastic modulus is 80GPa and the mass of suspension cable per unit length is 1.6kg/m. The initial length of bearing rope calculated is 249.06m.

The starting position of the ropeway is marked as A, the end is marked as B. The number of elements of bearing rope is 60, the speed of loads is 0.3m/s.

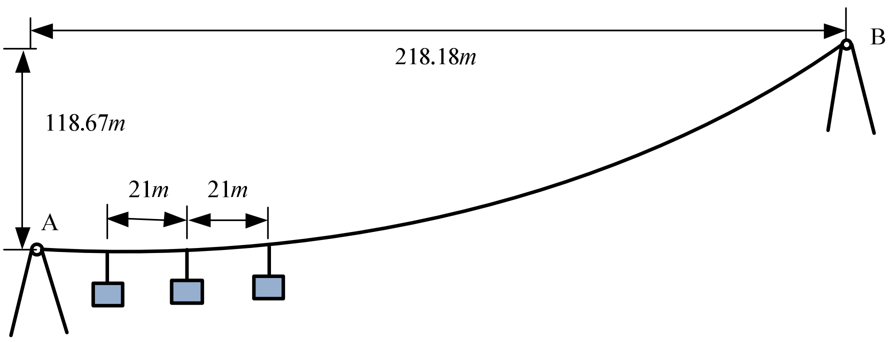


**Figure 3.** Working Case.

In the test, there are three concentrated loads (640kg, 640kg, 425kg), and the first load is 640kg, the distance between the loads is 21m. The first load starts from the starting position (away A point 53m) and stops at the position away A point 125m. The tangential tension of bearing rope at A and B is calculated by the method presented in this paper during the loads moving.

As shown in figure 4, the tangential tension at position A and B of the ropeway is agreed with the test data. The maximum tangential tension at position A by test is 74.503kN, then the calculation result is 73.106kN, 1.88% smaller than the test result. The maximum tangential tension at position B by test is 75.031kN, then the calculation result is 74.785kN, 0.33% smaller than the test result.

The results show that the relative error between the calculated results and the test results is very small and the method has high precision.

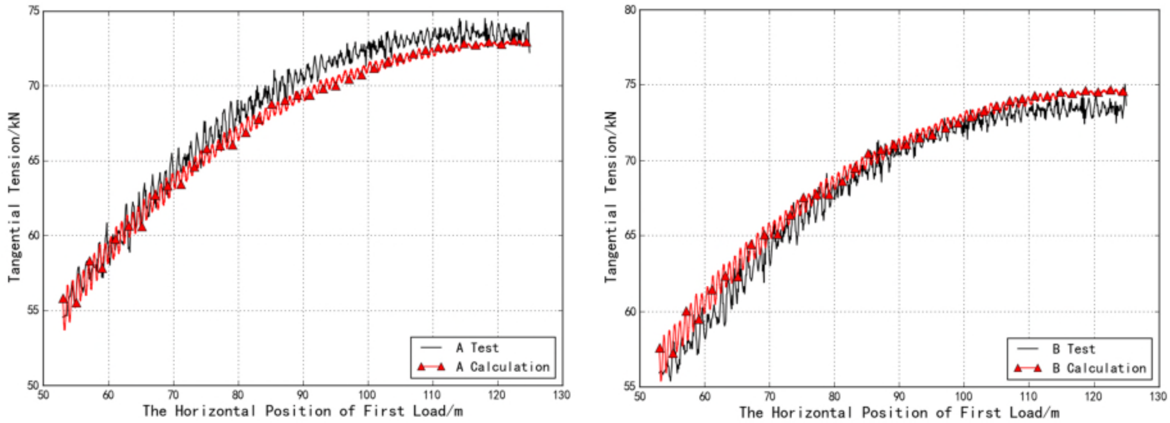


**Figure 4.** The Tangential Tension at Position A and B during Loads Moving.

## 5. Conclusions

According to the VFIFE theory and the mechanical characteristics of suspension structure, a vector finite element method for the dynamic analysis of cable structures is presented. A working case of an engineering ropeway verifies the calculation method.

The comparison between the calculated results and experiment data shows that:

1) The calculated tangential tension curves presented in this paper agree with the test curves;

2) The maximum of the relative error is 1.88%.

Therefore, this method has high precision and is easy to be programed in computer, and it is very convenient for engineering application.

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