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Application of trilinear softening functions based on a cohesive crack approach to the simulation of the fracture behaviour of fibre reinforced cementitious materials.

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Abstract. The relevance of fibre reinforced cementitious materials (FRC) has increased due to the appearance of regulations that establish the requirements needed to take into account the contribution of the fibres in the structural design. However, in order to exploit the properties of such materials it is a key aspect being able to simulate their behaviour under fracture conditions. Considering a cohesive crack approach, several authors have studied the suitability of using several softening functions. However, none of these functions can be directly applied to FRC. The present contribution analyses the suitability of multilinear softening functions in order to obtain simulation results of fracture tests of a wide variety of FRC. The implementation of multilinear softening functions has been successfully performed by means of a material user subroutine in a commercial finite element code obtaining accurate results in a wide variety of FRC. Such softening functions were capable of simulating a ductile unloading behaviour as well as a rapid unloading followed by a reloading and afterwards a slow unloading. Moreover, the implementation performed has been proven as versatile, robust and efficient from a numerical point of view.

1. Introduction and background

Cementitious materials share a common characteristic among them which is the preponderantly proportion of cement in their composition. Therefore, most of their properties are highly dependent on those of cement being its good compressive strength and its high modulus of elasticity two of them. Nevertheless, some other characteristics of concrete are not so adequate for constructing such as its limited flexural and tensile strength. Concrete, which has been the most widespread construction material, is a clear example of what has been previously mentioned. It boasts a noticeable compressive strength but its tensile strength might be considered about the tenth of such value. Consequently, in structural elements subjected to bending moments, the tensile stresses may exceed the tensile strength of concrete and cracks would appear, producing eventually the fracture of the element and causing both an economic damage and increasing the risk of physical harm in humans. One possibility to solve that issues is adding a continuous reinforcement to the cementitious material, in the way of randomly distributed fibres. This approach was firstly considered in the nineteenth century but due to its success it is still applied.

When randomly distributed fibres are added to the concrete mix the material is commonly known as fibre-reinforced concrete (FRC). Traditionally, the most employed fibres in concrete structural elements have been steel ones and consequently the material formed has been named steel fibre reinforced concrete (SFRC). However, due to the rise of the steel price and the possible corrosion that steel might suffer when in potentially hazardous environments, the use of structural macro-polyolefin fibres as a concrete addition has become a possibility. The latter material has been termed polyolefin fibre reinforced concrete (PFRC). Another cementitious material that has been reinforced with fibres is cement mortar. It has been mainly used in elements that have a reduced thickness which tend to crack as a result of the shrinkage. In order to avoid such cracking, randomly distributed glass fibres have been added to the cement mortar forming glass fibre reinforced cement (GRC). The presence of glass fibres increase not only the flexural and tensile strength but also provide certain degree of ductility.

In order to fully exploit the mechanical properties of fibre reinforced cementitious materials several models that deal with the physical process of cracking in concrete under tensile stresses have been postulated. One remarkable one is the smeared crack approach [1] that has been commonly used when there is no localisation of the cracks and when the crack opening is reduced. Another, used when crack is localised is the so-called discrete approach which has provided more accurate results. One of the most used cracking models for plain concrete was developed by Hillerborg [2] and named the cohesive crack model. The applicability of this model has been analysed in depth by several authors and has been applied not only to plain concrete, but also to other brittle or quasi-brittle materials such as brick masonry [3-5]. In addition, in certain circumstances this model represents with accuracy both fracture under Mode I and a fracture process generated under a Mode II without the need of using a tracking algorithm [6]. A more detailed explanation of this model may be found in any of the references provided.

The cohesive crack model is able to reproduce the fracture behaviour of brittle materials by using as an input several mechanical properties that can be obtained by performing standard laboratory tests. In the case of plain concrete, these properties are the tensile strength (obtained by means of the indirect tensile strength test) [7] and the fracture energy (by following any of the several recommendations commonly adopted) [8]. Using these parameters, the cohesive crack model replicates the fracture tests of plain concrete only if another characteristic of concrete is defined, that is to say, the type of softening function [9]. Several authors have studied the suitability of using diverse softening functions, which may vary from an exponential function to a linear or bilinear one. The latter has been profusely used due both to its simplicity and the accurate results that it provides.

However, none of these functions can be directly applied to reproduce the fracture behaviour of the various types of FRC produced nowadays. Recently, the application of multilinear linear softening functions to FRC and GRC have been shown as a suitable and simple option that provides accurate results [10-14]. However the changes that should be applied to such softening functions in order to widen their applicability to those materials have not studied in depth. Therefore, the present paper seeks to fulfil that gap by analysing the suitability of such softening functions in order to obtain accurate simulation results in numerical analysis of fracture tests of a wide variety of FRC and GRC mixes. In that sense, the changes introduced by the variation of behaviour of the amount of fibres added in a PFRC will be studied. The analysis of the softening functions of three types of GRC will be also analysed. Lately, all functions will be compared and some conclusions from such comparison will be extracted.

2. Softening behaviour of fibre reinforced cementitious materials

The embedded cohesive crack model that has been extensively used for simulating the fracture of concrete is based on a central forces model and has been extensively explained in references [13 14].

Once this implementation has been performed, the most important issue to address the fracturing behaviour of the fibre reinforced cementitious material is defining the softening function. The cohesive fracture approach considers the softening function of concrete as a material property [15].

Such function is defined between the strain were the tensile strength of the material is reached, where the width of the crack is still zero and the critical crack opening where the material is no longer capable of withstanding any loads. The area below the softening function set the fracture energy and therefore the shape of the function determines the behaviour of the material. Such area can be obtained by integrating the softening function from a zero crack width and the critical crack width, w_c where cohesive stress becomes zero [15] being *f(w)* the tensile stress at *w* crack opening. This expression can be seen in (1)

$$G_F = \int_0^{w_c} f(w)dw \quad (1)$$

When the maximum strength of the material equals the tensile strength (f_{ct}) the cracking behaviour onsets and expression (2) is validated.

$$f_{ct} = f(0) \quad (2)$$

Consequently, it is the softening function the main characteristic of the material while fracturing and therefore the one that defines the behaviour while cracking. In this regard it has to be outlined that when dealing with plain concrete cracking there are several types of functions that have been used successfully. For instance, linear, bilinear or exponential curves have been used obtaining accurate results [2, 9, 16-18]. One possibility of an exponential softening function can be seen in (3).

$$\boldsymbol{\sigma = f_{ct} \cdot e^{\left(-\frac{f_t \omega}{G_F}\right)}} \quad \mathbf{(3)}$$

where G_F is the specific fracture energy and f_{ct} is the tensile strength. A way of obtaining such softening functions is by what has been termed inverse analysis, adjusting experimental response of a notched specimen by trial-and-error optimisation through use of finite element methods [19]. Based on the promising results obtained using a bilinear softening function both in accuracy to the experimental results and in numerical calculus efficiency, the material improvements provided by the presence of fibres has been introduced in several studies as modifications of multilinear functions [our references of simulation articles]. Such relations were implemented in a commercial finite element programme by means of a user subroutine for material. Hence, the numerical simulations were performed using ABAQUS code and one UMAT subroutine to model the fracture behaviour of PFRC and GRC. In such a sense, the non-linear fracture process zone emerges in the elements placed on the crack. Given that the behaviour of the fracturing elements depends on a constitutive relation that needs to be iteratively fit the scheme that can be seen in Figure 1 was followed.

Applying the multilinear approach to the softening functions that are capable of introducing the effect in the fracture behaviour of PFRC and GRC such functions can be defined as in expressions (4) and (5) respectively. In these expressions the only difference can be observed in the first unloading part. In this regard, both approaches; the one related with PFRC using an exponential function and the one related with GRC which uses a linear function provide a remarkable degree of accuracy.

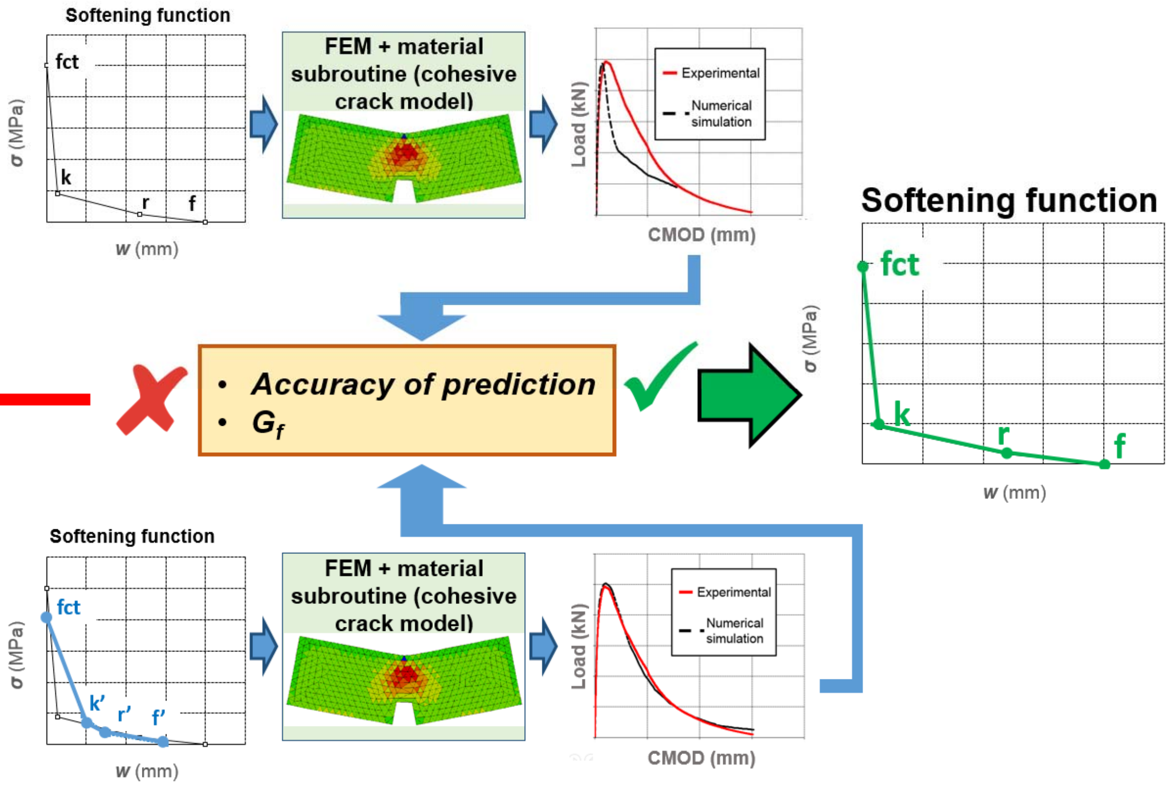

Figure 1: Inverse analysis used to reproduce the fracture test results

$$\begin{cases} \sigma = f_{ct} \cdot e^{\left(-\frac{f_{ct} w}{G_F}\right)} & \text{if} \quad 0 < w \le w_k \\ \sigma = \sigma_k + \left(\frac{\sigma_r - \sigma_k}{w_r - w_k}\right)(w - w_k) & \text{if} \quad w_k < w \le w_r \\ \sigma = \sigma_r + \left(\frac{-\sigma_r}{w_f - w_r}\right)(w - w_r) & \text{if} \quad w_r < w \le w_f \\ \sigma = 0 & \text{if} \quad w > w_f \end{cases} \tag{4}$$

$$\begin{cases} \sigma = f_{ct} + \left(\frac{\sigma_k - f_{ct}}{w_k}\right) w & \text{if} \quad 0 < w \le w_k \\ \sigma = \sigma_k + \left(\frac{\sigma_r - \sigma_k}{w_r - w_k}\right)(w - w_k) & \text{if} \quad w_k < w \le w_r \\ \sigma = \sigma_r + \left(\frac{-\sigma_r}{w_f - w_r}\right)(w - w_r) & \text{if} \quad w_r < w \le w_f \\ \sigma = 0 & \text{if} \quad w > w_f \end{cases} \tag{5}$$

3. Discussion

Several experimental results of fracture tests performed in cementitious materials were reproduced using the aforementioned implementation. The test reproduced in the case of the PFRC tests had been conducted in 100x100x430mm³ specimens. The span of the three-point bending tests was 3*D* and the notch 1/3*D*, being *D* the height of the square cross section (100mm). Such specimens had been manufactured adding 3, 4.5, 6 and 10kg/m³ of 60mm-long polyolefin fibres to a self-compacting concrete. The details of the manufacturing process as well as the testing setup and procedure followed can be found in [20]. The recommendation followed was RILEM-TOC 187 [12]. The curves shown are the average of at least three successful tests. Although it could be argued that a larger amount of test would be required to provide sound conclusions it has to be highlighted that the careful handling and preparation of the test setup provided results with a limited degree of scatter. Regarding the simulations, if Figure 2 is observed it can be seen how the softening function implemented was capable of reproducing with a remarkable accuracy the behaviour of PFRC. This implementation was found both robust and numerically efficient besides of the great adaptability that it boasts. This can be clearly perceived in Figure 2. By changing the points that define each stretch of the softening functions the several implementations were able to reproduce all the features of the experimental results. The latter is of high relevance because the minimum post-peak registered changes noticeably between the formulations. Similarly, the values of the maximum post peak loads of the simulations appear in similar values in the simulated curves. Not only these values were reproduced but also the slopes of the unloading and reloading branches of the experimental curves are accurately obtained.

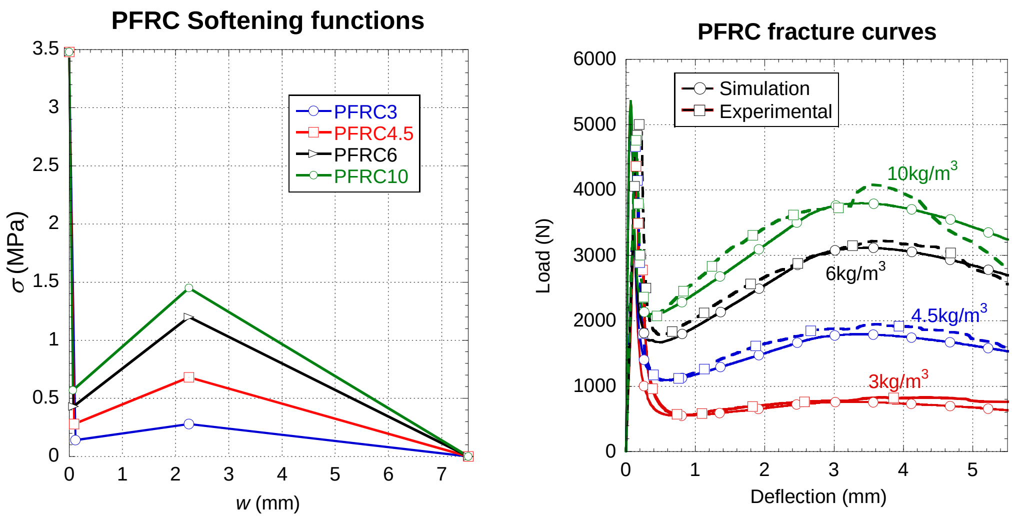

Figure 2: Softening functions (left) and comparison between simulated and experimental results (right).

In order to check the adaptability of the multilinear softening curves another fibre reinforced cementitious material of a totally different nature was simulated. In this case the material chosen was a glass fibre reinforced cement (GRC). This material is manufactured by merging cement mortar and randomly distributed short glass fibres. Similarly to the case previously explained of PFRC the multilinear softening functions were implemented in this case for reproducing the fracture test results obtained in previous research [21]. Three GRC formulations were manufactured, being among them the use of some chemical additives the only difference. Such additives seek to prevent the change of mechanical properties that undergo the traditional GRC formulations as times passes. Such formulations were named GRC, GRC-M and GRC-P which correspond to traditional GRC, GRC with a Metaver additive and a GRC with Powerpozz additive. In contrast to PFRC the fracture tests conducted with the several formulations of GRC could not be performed following any recommendation as at the time of such research there was no one available. Nevertheless, the closest adaptation of the previously mentioned RILEM-TOC 187 recommendation was followed. Similarly to PFRC the curves shown in Figure 3 are the average of at least three successful tests and a remarkably low degree of scatter was registered.

As in the previous case of PFRC, the multilinear softening functions implemented were divided into three stretches. However, in the case of GRC the slope of the stretches were in all cases negative. The changes in the values that define the stretches of the softening functions were able to reproduce with remarkable accuracy the fracture behaviour of GRC including the noticeable changes in the maximum load sustained, ductility and in the unloading rate that the material boasted. In the first case it can be seen that there is no clear connection between the maximum load obtained in the test and the tensile strength of the material. It seems that the slope of the material in the first unloading branch is also of high importance to determine the maximum load of the test. This can be seen comparing the behaviour of GRC-P and GRC. Both softening functions have the same tensile strength but on the contrary the slope of the first unloading branch of the GRC-P provides the material a higher loading capacity.

Regarding the changes in the ductility, the crack width of the softening functions is the major factor that define it. However there is also influence of the slope of the third stretch of the softening function because as can be seen in Figure 3, GRC-M and GRC have the same maximum crack width but the maximum crack mouth opening displacement (CMOD) is different in both formulations.

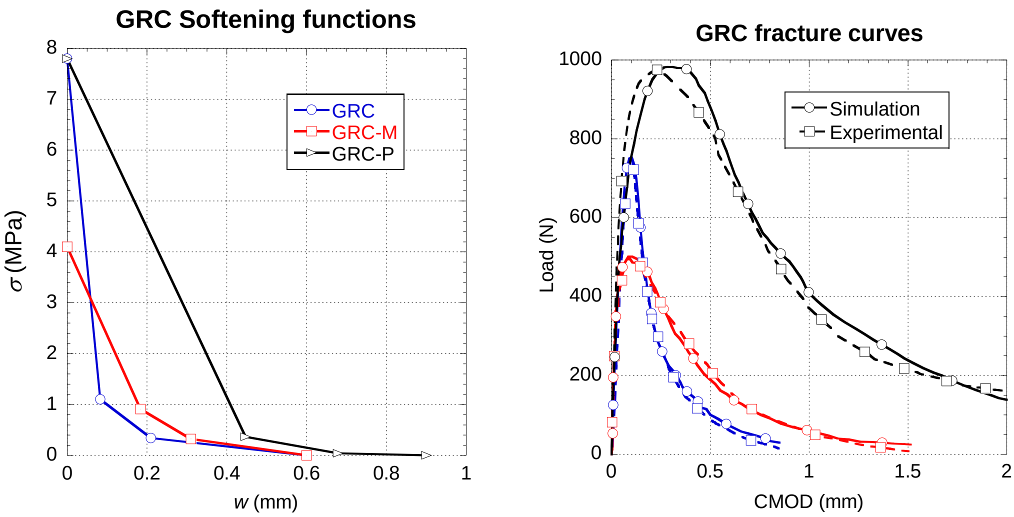

Figure 3: Softening functions (left) and comparison between simulated and experimental results (right).

Comparing the features of the softening functions implemented it can be stated that when the first unloading branch has a great negative slope the maximum load that the material can sustain in the fracture tests is mainly determined by the tensile strength of the material as can be seen in all the cases of PFRC and also in GRC formulation. Nevertheless, as in the case of GRC-P when there is a more gradual loss of stiffness the material is still able to increase its capacity of sustaining load although the closer parts of the tip of the notch are already damaged and the stiffness of the sample decreased noticeably before reaching the maximum load. This phenomenon was also perceived in the numerical models performed where several elements closer to the notch tip were already damaged when the maximum load was reached.

4. Conclusions

The implementation of multilinear softening functions has been performed successfully in a material user subroutine in a commercial finite element code. Using this implementations, fracture tests of PFRC and GRC have been reproduced with a remarkable degree of accuracy. The implementation has been proven as versatile, robust and efficient from a numerical point of view.

The changes in the tensile strength of the material and the points that define the stretches of the softening functions have been shown suitable to simulate the progressive unloading that appear in the GRC formulations fracture tests and the reload events that take place in the PFRC ones.

The variations of the length and slope of the parts of the softening functions enable to analyse the changes of the material behaviour that are introduced by several the amount of fibres in PFRC and the effects of the chemical additives in the case of GRC. It has to be underlined that this approach was able to simulate a ductile unloading behaviour, such the one that appears in GRC, as well as the rapid unloading followed by a reloading and afterwards a slow unloading as the one that appears in PFRC.

When there is a first unloading branch with a high slope the peak load registered in the tests is mainly determined by the tensile strength of the material. Nevertheless, if such slope is more gradual the combination of the slope and tensile strength of the material determine the peak load registered experimentally. In what regards the material ductility, it is mainly influenced by the maximum crack width that the material sustains.

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