

# Punching Shear in Steel Fibre Reinforced Concrete Slabs without Traditional Reinforcement

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**Abstract.** The punching shear capacity of steel fibre reinforced concrete (SFRC) slabs without traditional steel bar reinforcement was investigated by conducting central point-load tests on twelve square slabs. The test parameters covered fibres with different multi-hook ends, concrete compressive strength, reinforcing index and slab thickness. The statistical performance of two existing models for the prediction of punching shear capacity of SFRC slabs without traditional reinforcement was examined. The load carrying capacity of these slabs were also assessed using the yield line theory. It is noted that the slabs failed primarily in flexure and the yield line theory predicted the load carrying capacities of the slabs most accurately. The reason for a flexural failure in SFRC slabs without steel bars is attributed to the lesser energy required in the propagation of an existing flexural cracks than in the creation of a new circumferential cracks around the column face.

## 1. Introduction

Punching shear is undesirable in structural concrete flat slab systems owing to its brittle nature, which may lead to progressive collapse of a building [1]. Several methods have been implemented to enhance the punching shear resistance of flat slabs and these include the use of traditional shear stirrups, shear studs, helix reinforcement and lattice reinforcement, apart from the provision of drop panels [2-6]. With test results affirming the success of steel fibre reinforcement in enhancing the punching shear resistance of slabs, several researchers and committees have considered its application in combination with traditional steel bar reinforcement [7-13].

Recent experimental studies reveal the possibility of constructing steel fibre reinforced concrete (SFRC) flat slabs devoid of traditional steel bars [14-17]. Researchers have successfully used the yield line theory to analyse the flexural behaviour and predict the load-carrying capacity of such flat slabs [2, 16-17]. However, recent tests showed that SFRC flat slabs without traditional reinforcement could also experience a punching shear mode of failure under highly concentrated loads [17]. Considering the results obtained on different slab panels [17], there is uncertainty as to whether a pure flexural failure or a flexural failure leading to punching could occur under a concentrated load.

In this study, a test programme consisting of central point-load tests on twelve simply supported, square SFRC slabs without traditional reinforcement was carried out to examine the effect of volume fraction of steel fibres, slab thickness, concrete compressive strength and fibre hook-end configuration



on the punching shear capacity. The performance of the yield line theory and two punching shear prediction models applicable to SFRC slabs without traditional reinforcement were evaluated and the failure mode further clarified.

## 2. Theoretical Considerations

In general, the punching shear behaviour of a slab can be examined by testing a slab simply-supported on all four edges under a concentrated load at the center. The edges simulate the lines of contra-flexure in a flat slab system. Under such a loading system, the slab would develop cracks in the radial direction at its bottom face. This could either lead directly to a punching shear failure, or to extensive cracking that might result in a flexural failure mode.

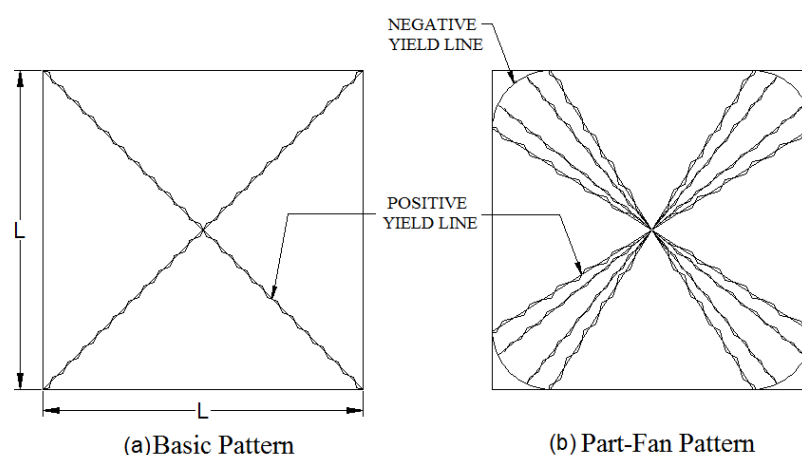
### 2.1. Existing Prediction Models for Punching Shear Capacity

The punching shear capacity of a SFRC slab,  $V_R$ , can be considered to comprise two components, that is, the contribution from “concrete”,  $V_{R,c}$ , and the contribution from “steel fibers”,  $V_{R,f}$ . Several international standards [3, 12, 18] have provided expressions to predict the “concrete” contribution. In general, the value of  $V_{R,c}$  is a function of the tensile strength of concrete (expressed in terms of cylinder compressive strength of concrete,  $f'_c$ ) and effective depth,  $d$ , to the tensile steel reinforcement. EN 1992-1-1: 2004 [18] and fib Model Code 2010 [12] also consider  $V_{R,c}$  to be governed by the reinforcing steel ratio and yield strength of steel reinforcement respectively. Also, several investigators on punching shear in SFRC slabs have proposed models to predict both the “concrete” and the “steel fiber” contribution to the punching shear capacity [7, 10, 11-13 19-21]. The “steel fiber” contribution,  $V_{R,f}$ , is primarily expressed as a function of the fiber content and thereby, the residual tensile strength of SFRC.

Among the several analytical and semi-analytical punching shear capacity prediction models, only those proposed by Choi et al. [11] and Grimaldi et al. [21] are applicable for SFRC slabs without traditional reinforcement. In other prediction models, the predicted load-carrying capacity reduces to zero when the percentage of steel reinforcement is equal to zero.

### 2.2. Flexural Capacity of SFRC Slabs Under Concentrated Load

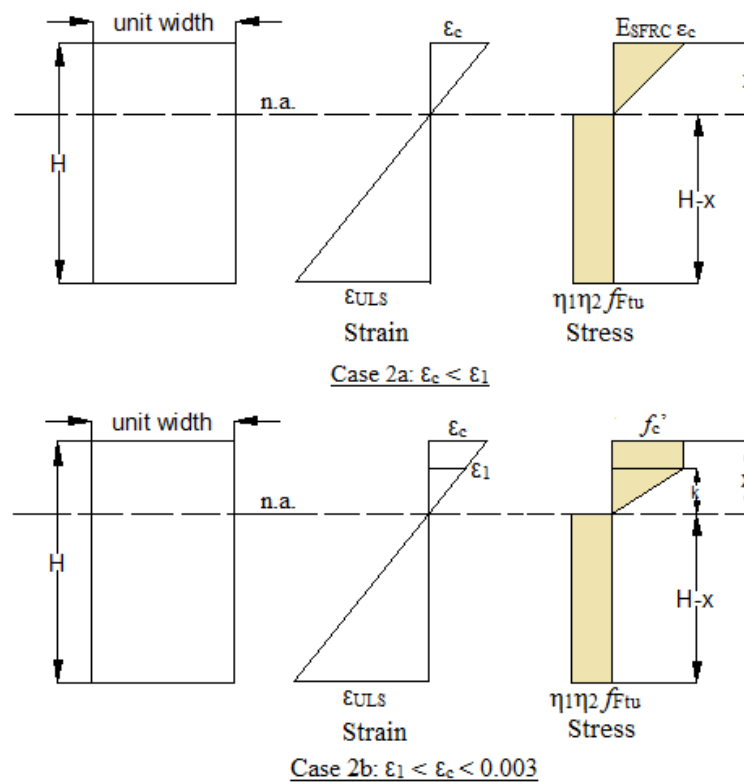
The yield line theory has been used with reasonable accuracy to predict the flexural capacity of steel fiber reinforced concrete slabs [16-17]. Figure 1 indicates two possible yield line patterns for a square simply supported slab under a central point load.



**Figure 1.** Yield line patterns

The basic yield line pattern (Figure 1a) gives a failure load,  $P_B$ , equal to  $8M_B$ , where  $M_B$  is the moment of resistance of the slab section in kNm/m while the part-fan yield line pattern (Figure 1b) predicts a failure load equal to  $2\pi M_B$ . The actual yield line pattern observed in tests is however a mix of these two patterns. Since the yield line theory gives an upper bound solution, the lower of the two failure loads, that is,  $2\pi M_B$  shall be used to predict the ultimate load carrying capacity in flexure of centrally loaded square slabs.

Based on the idealized stress-strain relations for SFRC, the strain and stress distribution across a SFRC section without traditional steel bars at ultimate limit state can be obtained as shown in Figure 2. A rigid-plastic stress block in tension is assumed with residual tensile strength equal to  $\eta_1\eta_2f_{Ftu}$ , where  $f_{Ftu}$  = residual tensile strength of SFRC corresponding to the Model Code 2010 rigid-plastic model [12] ( $=f_{R3}/3$ ) (of which test values are given in Table 1); and  $f_{R3}$  = residual flexural strength of SFRC determined from three-point bend tests on notched prisms in accordance with the BS EN 14651 [18]. Also,  $\eta_1$  = reduction factor accounting for the effect of slab thickness on the fiber orientation (taken as  $e^{0.0024(125-H)}$  [22]) with  $H$  being the thickness of the slab; and  $\eta_2$  = reduction factor to account for the low fiber content along the lines of crack propagation in flexure ( $= 0.82$  [23]).



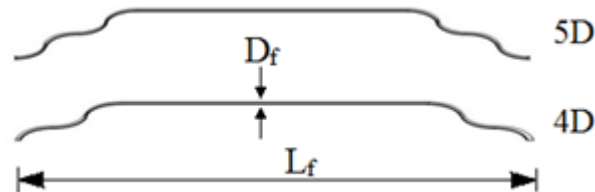
**Figure 2.** Strain and stress distributions at an SFRC section without traditional reinforcement at ultimate limit state.

In SFRC slabs without traditional bar reinforcement, the ultimate limit state for flexure occurs when the extreme tensile fibre strain,  $\epsilon_t$ , reaches 0.02 or  $2.5/H$ , whichever is earlier [12]. The neutral axis,  $x$ , is determined by considering force equilibrium on the section, and the moment of resistance,  $M_B$ , is then calculated by taking moments of internal forces about the neutral axis.

### 3. SFRC Slabs Without Traditional Reinforcement

#### 3.1. Specimen Details and Test Setup

Twelve square SFRC slabs without traditional reinforcement were tested under central point-loading. The slabs measured 700 mm by 700 mm in plan and had thickness varying from 90 mm to 150 mm. Details of the specimens are given in Table 1. Two types of hooked-end fibers, namely, 4D- and 5D-fibers having 1.5- and double-hook ends (Figure 3), respectively, were used in the concrete mixture.



**Figure 3.** Steel fibers with multiple hook-ends.

**Table 1.** Details of tested SFRC specimens without traditional reinforcement.

Specimen Label*	Concrete Class	$f'_c$ (MPa)	$f_{ftu}$ (MPa)	Fiber Type	Fiber volume (kg/m <sup>3</sup> )	Slab thickness (mm)	Remarks
M42	C30/37	36.0	1.66	4D	20	120	Variation of fiber volume
M44		36.0	2.21		40		
M46		31.4	2.70		60		
M48		36.1	3.30		80		
M52		37.4	1.67	5D	20	90	Variation of fiber volume
M54		36.5	2.21		40		
M56		40.8	2.80		60		
M58		42.2	3.36		80		
M54A	C16/20	34.9	2.19	5D	40	90	Variation of slab thickness
M54B		34.9	2.19		40	150	Variation of slab thickness
L54	C16/20	21.1	2.03		40	120	Variation of concrete strength
H54	C50/60	51.8	2.36		40		Variation of concrete strength

Note: All fibres were 60 mm long and 0.9 mm in diameter.

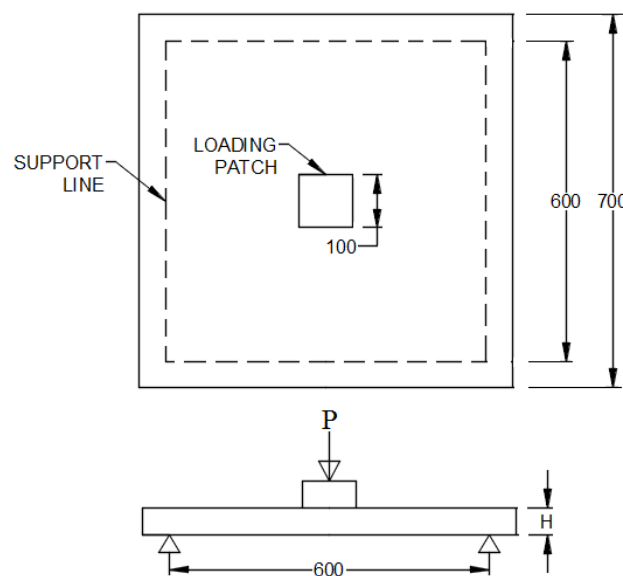
\* Prefix L, M and H indicate low, medium and high strength concrete respectively. Suffix A and B indicate slab thickness of 90 and 150 mm, respectively.

Aggregates, sand, cement and water were added into an 80-litre pan mixer in sequence. To avoid balling, the steel fibers were separated by hand and added into the mixture in the last step to simulate the addition of fibers into a mix truck as highlighted in the guidelines by ACI 544.3R-93 [24].

The specimens were consolidated via internal vibration. After the concrete was allowed to harden for a day, the wooden moulds were removed and the slabs were left to cure under wet gunny sacks for

the next 28 days. Concrete cubes were cured under the same conditions so as to ensure strength readings are representative of the slab.

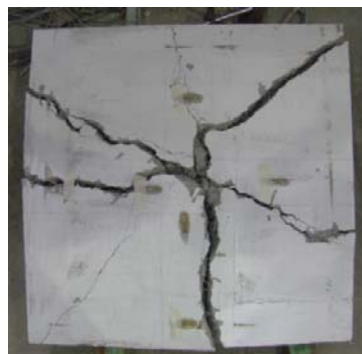
The tests were conducted using the 100 ton MTS Actuator System. As shown in Figure 4, the load was applied over a patch measuring 100 mm by 100 mm at the centre of the slab, at a displacement rate of 0.1 mm/min. The slab was simply supported on a square steel frame with supporting lines measuring 600 mm on each side of the slab. At the bottom of the slab, a displacement transducer was placed at the centre to measure the slab deflection. The slabs were loaded monotonically to failure. After the peak load was reached, the loading rate was gradually increased to 0.3 mm/min and the slab was further loaded to obtain the descending portion of the load-deflection curve. The test was terminated when the load dropped to about 50 – 60% of the peak load, to ensure that no further load rebound would occur.



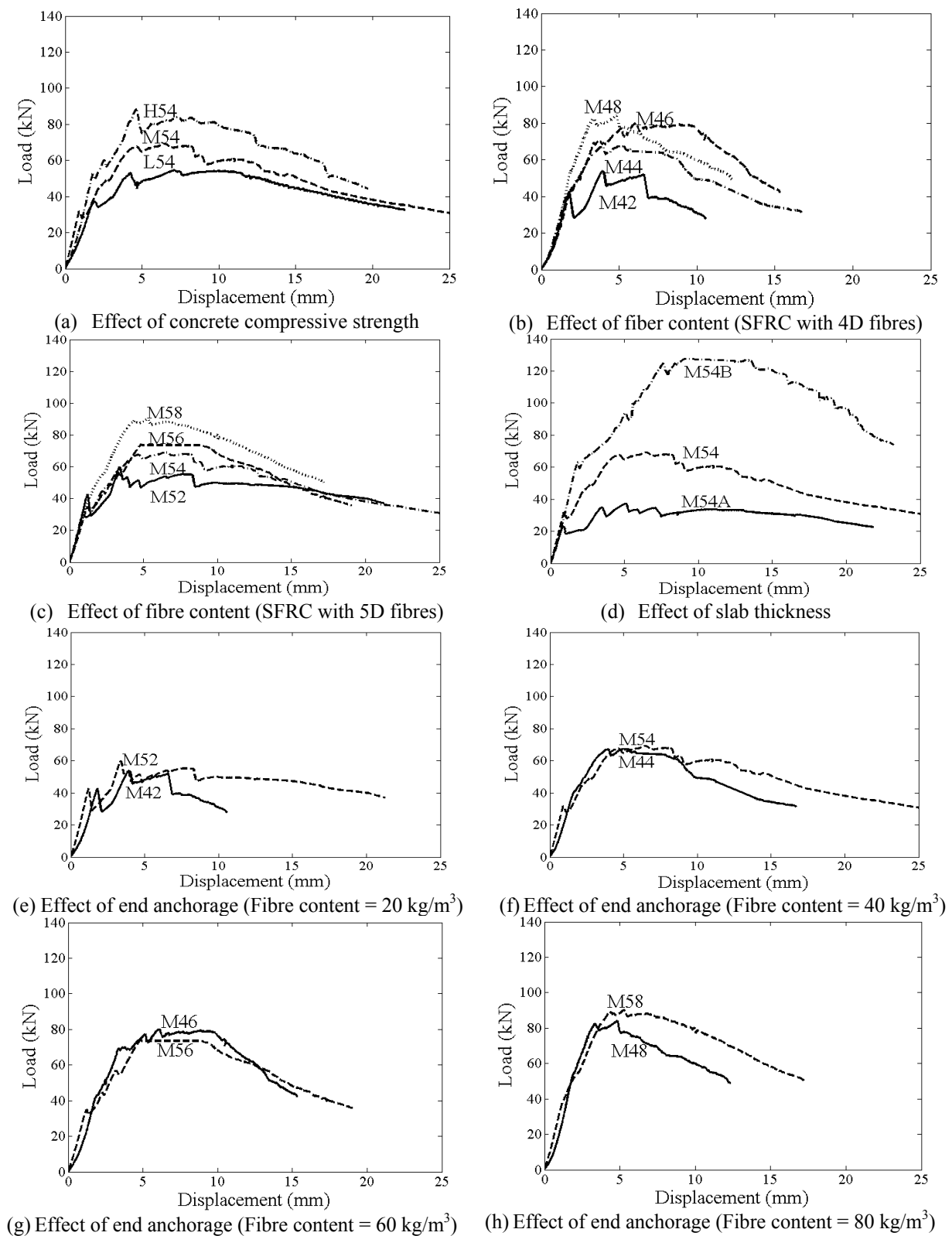
**Figure 4.** Central point load tests on SFRC slabs without traditional reinforcement.

### 3.2. Test Results

The typical crack pattern on the underside of the slabs is shown in Figure 5. The formation of yield lines is clearly observed indicating a flexural mode of failure. The load-deflection curves for the tested slabs are shown in Figure 6.



**Figure 5.** Crack pattern on the underside of slab M44.



**Figure 6.** Load – Displacement curves for SFRC slabs without traditional reinforcement.

The curves display ductile behaviour from a deflection of about 3 to 5 mm to a deflection of around 8 to 10 mm in many slabs after which, the load drops drastically. The slab thickness affected the ultimate load-carrying capacity most [Figure 6(d)], followed by the reinforcing index [Figures 6(b) and (c)] and concrete compressive strength [Figure 6(a)]. The end anchorage of hooked fibers has a marginal effect on the peak load. Figures 6(e) to (h) indicate that specimens with 5D fibres displayed a slightly higher peak load compared to those with 4D fibres with the exception of M56. Specimens with 5D fibres also exhibited more ductile behaviour than those with 4D fibres and sustained larger deflections [Figures 6(e) to (h)].

### 3.3. Predictions by Yield Line Theory and Punching Shear Prediction Models

Table 2 shows the statistical performance of the yield line theory and the two punching shear prediction models by Grimaldi et al. [21] and Choi et al. [11], comparing with the test results.

**Table 2.** Statistical Performance of Prediction Models.

Test/Predicted Values	Yield Line Theory	Grimaldi et al. [21]			Choi et al. [11]
		Assuming Truncated Cone Failure Surface, with		Assuming Truncated Pyramid Failure Surface, with	
		$V_{R,c}$ by EN 1992, 2004	$V_{R,c}$ by ACI 318-14	$V_{R,c}$ by ACI 318-14	
AVERAGE	0.99	0.19	0.43	0.44	1.26
STDEV	0.19	0.03	0.06	0.07	0.58
COV	0.19	0.16	0.15	0.15	0.46
MIN	0.81	0.16	0.32	0.32	0.72
QUARTILE 1	0.83	0.17	0.40	0.41	0.93
MEDIAN	0.96	0.17	0.42	0.43	1.15
QUARTILE 3	1.06	0.19	0.47	0.49	1.33
MAX	1.43	0.26	0.53	0.55	2.82

Since the model by Grimaldi et al. [21] considers  $V_{R,c}$  to be constant for a given concrete compressive strength, it consequently over predicts the load-carrying capacity of the SFRC slabs without traditional reinforcement by a big margin (Table 2). Choi et al.'s model [11] considers the neutral axis depth in the prediction of punching shear capacity of SFRC slabs. However, owing to the conservative predictions of contribution by steel fibres,  $V_{R,f}$  to the punching shear capacity, the model under predicts the mean load-carrying capacity of the test slabs by 26% with a large standard deviation of 0.58.

The yield line theory accurately predicts the average load-carrying capacity of SFRC slabs without traditional reinforcement within 1% of the observed values with a standard deviation of 0.19. Thus, it is deduced that a flexural failure mode occurs in SFRC slabs under a concentrated load. This is probably due to the lesser energy required to propagate existing yield lines than otherwise needed to form new circumferential cracks around the column face typical of punching failure.

## 4. Conclusion

In this study, an investigation on the punching shear capacity of SFRC slabs without traditional reinforcement was carried out. Predictions from two punching shear prediction models and the yield line theory were compared with results from twelve SFRC slabs. It was observed that the yield line theory provides accurate predictions of the load carrying capacity of the SFRC slabs while the punching shear prediction models fail to predict the observed values accurately. It is noted that all tested slabs experienced a flexural mode of failure rather than a punching shear failure.



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