

# Reliability Assessment of Steel-Aluminium Lattice Tower

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**Abstract.** In the subject of the present study a probabilistic approach to the analysis of steel-aluminium lattice tower was used. Structural design parameters are defined as the deterministic values and random variables. Random variables are not correlated. The criterion for structural failure is expressed the limits of functions referring to the serviceability limit state. The description of the limit state of structure implicit forms of the random variables function was used. The study presents a combination of the reliability analysis program with the MES3D external FEM program. The NUMPRESS software, created at the IFTR PAS, was used in the reliability analysis. The Hasofer-Lind reliability index, determined using an iterative procedure of Rackwitz-Fiessler, was used as a reliability measure. The values of reliability index for different cases of the vector of random variables, that is, different descriptions of mathematical model of the structure, were determined. The effect of assumed probability distribution of individual random variables on the value of the reliability index was determined. In the description of random variables, the different types of probability distribution were used and the values of the reliability index for the normal distribution and the distribution chosen according to the kind of a variable were compared. The primary research method is the FORM method. In order to verify the correctness of the calculation Monte Carlo and Importance Sampling methods are used. The relative error of the reliability index was calculated taking the simulation Monte Carlo method as a reference. The effectiveness of the primary research method was performed by comparing the number of calls of the limit state, which is connected with the calculation time. The sensitivity of reliability index to the random variables was defined.

## 1. Introduction

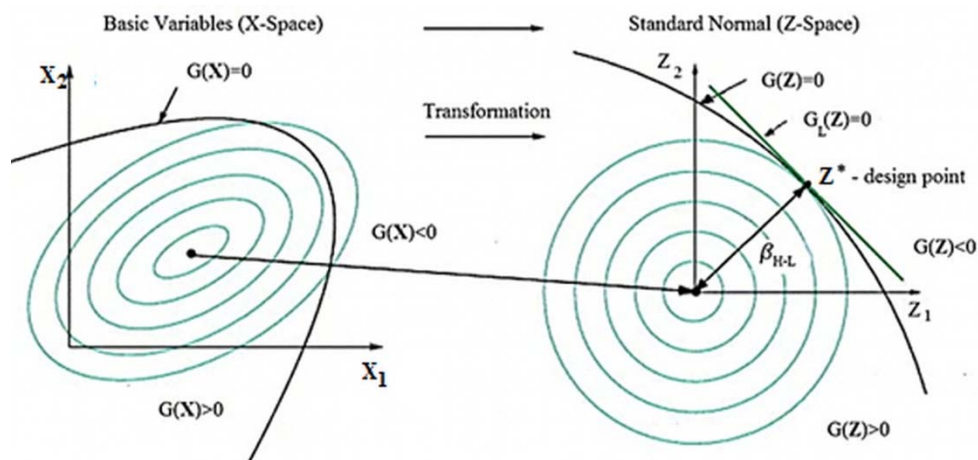
The most advanced reliability analysis methods are probabilistic methods. They allow quantitative assessment of structure reliability. The probabilistic finite element methods for analysis of structures have become of increasing interest in recent years. These are aimed at accounting for the random nature of material properties or loads. The existing methods can be placed in two distinct categories. The first group is aimed at second – moment analysis, where the objective is computing the means and variance of response quantities. One of the methods of this type is the perturbation method proposed in the works [1], [2], [3] to solve problems of linear elasticity theory. The application of the perturbation method to calculation by the finite element method is presented in the monograph [4]. In the paper [5] authors proposed take into account the random nature of material properties or loads in dynamic reliability analysis. Lately, interesting works are [6], [7]. The second group is aimed at reliability analysis, where the objective is computing probabilities associated with prescribed limit states. The reliability theory is already a well-established research area. One can mention a number of textbooks and monographs, the most well-known being [8], [9], [10], [11] [12]. Special attention should be paid



to publications by [13], [14]. They present the basic concepts of reliability theory with a particular reference to their uses in civil engineering. Interesting works are [15], [16] where numerical aspects of application of first order reliability method FORM in node snapping truss structures are considered. Different reliability assessment of structure propose work [17] about system reliability using serial and parallel systems. In order to determine the reliability of this approach it is necessary to set KAFM (kinematic admissible failure mechanism). The KAFM by spectral analysis of stiffness matrix are specified [18], [19]. Reliability issues are important not only in static analysis, but also in stability analysis or dynamic analysis [20].

A fundamental problem in structural reliability theory is the computation of the probability failure  $P_f$  of the structural element in the  $X$ -space must be equal to the probability defined in  $Z$ -space. The FORM method transforms random variables  $\mathbf{X}$  to the standard space  $\mathbf{Z}$ . Next surface boundary  $G(\mathbf{Z})=0$  is approximated by hyperplane tangent to it at the design point. It is the most probable point of failure out of all points on this surface. With the linearization of the limit state function at the design point, it is possible to obtain a measure of reliability which is invariant due to the equivalent formulations of the boundary condition, i.e. the so-called Hasofer-Lind reliability index  $\beta$  [21]. Finding a design point is a task for non-linear programming with limitations.

One major advantage of the FORM method is that it allows the calculation of the sensitivity of the reliability index  $\beta$  on change of parameters occurring in the task description, practically without the need for additional calculations. The sensitivity of the reliability index  $\beta$  is the first derivative of the ratio  $\beta$  on the specified variable and denote it as  $\alpha$ . In the sensitivity analysis the higher absolute value  $\beta$  is the greater the impact of the variable on the reliability index value.



**Figure 1.** Illustration of the linearization proposed by Hasofer and Lind [21] in standard normal space

## 2. Investigation methodology

In early applications of reliability analysis methods, it was accepted that the limit state function is an explicit function of random variables. Such functional dependency can be realized only for very simple examples. In practical realizations, this dependence is not explicit and it is determined using numerical procedure, e.g. the finite element methods.

This article aims to present the communication between the NUMPRESS reliability analysis program and the MES3D external FE programs. The NUMPRESS program was developed at the Institute of Fundamental Technological Research of the Polish Academy of Sciences by Kowalczyk, Rojek, Stocki, Bednarek, Tauzowski, Lasota, Lumelskyy, Wawrzyk [22]. The MES3D [23] program was developed by Szaniec.

The first step of the reliability analysis task using NUMPRESS is a definition of the stochastic model. The user must provide marginal distributions of the basic random variables, for correlated variables, their correlation matrix. In the current version of the program, the following marginal distributions are

available: exponential, Frechet, Gumbel, log-normal, normal, Rayleigh, uniform, Weibull and a general empirical distribution described by a set of experimental points. At this step, definitions of the so-called external variables, which are outputs of finite element method programs, can also be introduced. In the presented examples, basic random variables are load, member section area and elastic modulus. Basic random variables are not correlated.

After defining the variables, the limit state function is defined. In NUMPRESS, the limit state function is symbolically given in the standard math notation as a function of the basic random variables and external variables. The definition may include most of the basic mathematical functions. If the external variables appear in the definition, it is identified which basic random variables implicitly influence the limit state function by analyzing which of them are involved in modifying input data files used by programs that generate values of the considered external variables. In the paper, the condition of non-exceeding of the admissible horizontal displacement is considered as the limit state function.

The consecutive steps are dedicated to the selection, parameter setting and execution of the reliability analysis algorithm. The most computationally efficient methods for failure probability estimation are based on an approximation of the failure domain in the standard normal space. In FORM the failure domain is approximated by the half space that is defined using the limit state surface linearized in the so-called design point. In the standard normal space, the design point is the point on the limit state surface which is closest to the origin. Finding a design point is a task for non-linear programming with limitations. There are two standard, gradient based algorithms for solving this problem implemented in NUMPRESS.

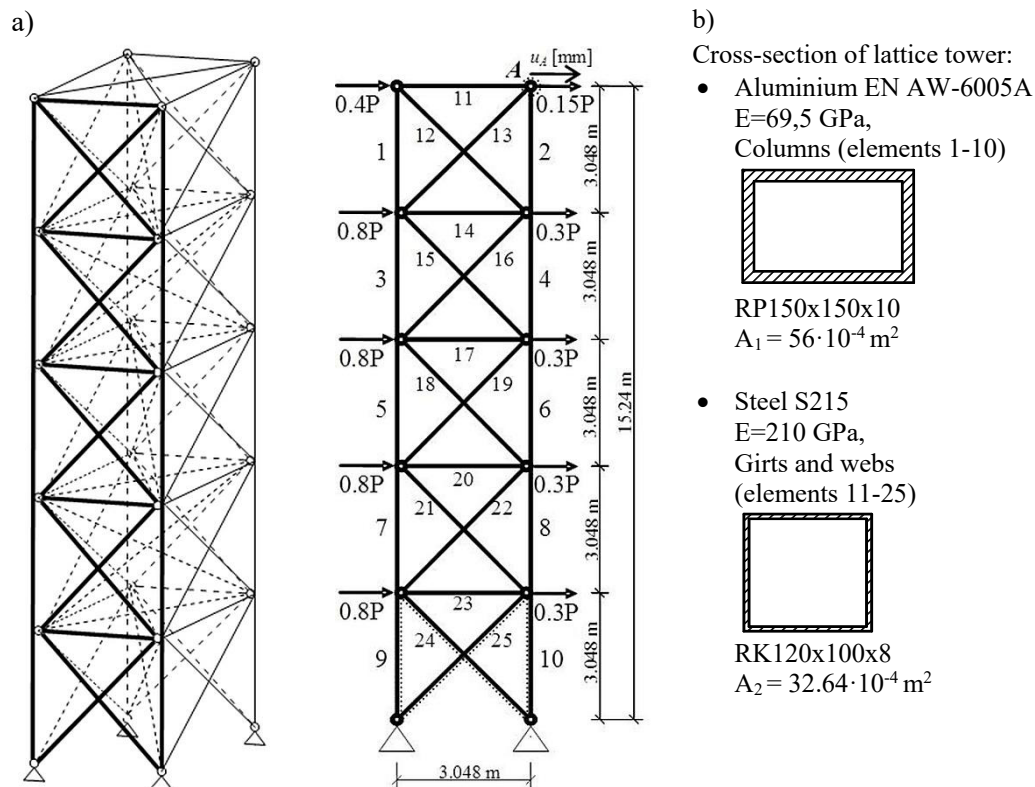
### 3. Aim of study

Analyzed bar structure is the shorter side of the lattice tower base dimensions 3.048 m x 6.069 m and a height of 15.24 m (figure 2a). The structure was designed by truss elements and charged by the wind. Effect of wind by the nodal concentrated forces was replaced. Static scheme of the truss and values  $P=34.125$  kN was taken from the work [24].

In the study geometry of the cross-section elements and chosen material referring to European standards was designed. Columns were designed of aluminium EN AW-6005, used in building and construction elements, which require high strength. Girts and webs were made of S215 steel. Detailed geometric parameters and material is shown in figure 2b.

Structural design parameters are defined as the deterministic values and random variables. Random variables are not correlated. The limit functions imposed on the load bearing structure are displacement constraints related to the serviceability limit state. The initial analysis of the displacement state, performed with the Robot software, made it possible to locate the sites at which displacement extreme values occur. Then, with the MES3D program, using the Finite Element Method, the maximum horizontal displacement  $u_A$  was determined. Below, probabilistic quantities are specified:

- $X_1$  – nodal concentrated force replacing the influence of wind (0.4P),
- $X_2$  – nodal concentrated force replacing the influence of wind (0.8P),
- $X_3$  – nodal concentrated force replacing the influence of wind (0.15P),
- $X_4$  – nodal concentrated force replacing the influence of wind (0.3P),
- $X_5$  – member section area of RP150x150x10 columns,
- $X_6$  – member section area of RK120x100x8 girts and webs,
- $X_7$  – member inertia moments of RP150x150x10 columns,
- $X_8$  – member inertia moments of RK120x100x8 girts and webs,
- $X_9$  – elastic modulus for EN AW-6005A,
- $X_{10}$  – elastic modulus for S215 steel.



**Figure 2.** Geometry of lattice tower

Limit functions were defined as functions of random variables grouped in five different vectors. The following cases of random variables vector were analyzed:

- Case A :  $X = \{X_1, X_2, X_3, X_4\}$ ,
- Case B :  $X = \{X_1, X_2, X_3, X_4, X_5, X_6\}$ ,
- Case C :  $X = \{X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8\}$ ,
- Case D :  $X = \{X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9, X_{10}\}$
- Case E :  $X = \{X_5, X_6, X_7, X_8, X_9, X_{10}\}$ .

Two variants of random variables description were considered in the analysis. In Variant I (table 1) only normal distribution for each random variable was used, whereas in Variant II (table 2) distribution of variables appropriate to the variable nature was proposed. The transformation of normal parameters on the parameters of another distribution was estimated by collocation method at a central point. The values of coefficients of variation were selected on the basis of statistical studies of the building strength and buildings materials and products from work of [25]. For variables with load range coefficients of variation were adopted in accordance with recommendations of [26].

The lengths of columns, girts and webs and coordinates of nodes were assumed to be deterministic quantities. Limit functions as the condition of the non-exceeding of the admissible horizontal displacement of node A (figure 2) was formulated:

$$G(X) = 1 - \frac{|u_A(X)|}{u_{dop}} \quad (1)$$

where  $u_{dop}$  denotes permissible displacement in accordance with PN-EN 1999-1-1:2011 standard. For the structure under consideration, value  $u_{dop}=120$  mm.

**Table 1.** Description of the random variables (Variant I)

Random Variable ( $X_i$ )	Probability density function	Mean value ( $\mu_X$ )	Standard deviation ( $\sigma_X$ )	Coefficient of variation ( $v_X$ )
$X_1$	Normal	13.65 kN	1.365 kN	10%
$X_2$	Normal	27.3 kN	2.73 kN	10%
$X_3$	Normal	5.119 kN	0.5119 kN	10%
$X_4$	Normal	10.238 kN	1.0238 kN	10%
$X_5$	Normal	$56 \cdot 10^{-4} \text{ m}^2$	$2.8 \cdot 10^{-4} \text{ m}^2$	5%
$X_6$	Normal	$1838.667 \cdot 10^{-8} \text{ m}^4$	$91.933 \cdot 10^{-8} \text{ m}^4$	5%
$X_7$	Normal	$32.64 \cdot 10^{-4} \text{ m}^2$	$1.632 \cdot 10^{-4} \text{ m}^2$	5%
$X_8$	Normal	$486.323 \cdot 10^{-8} \text{ m}^4$	$24.316 \cdot 10^{-8} \text{ m}^4$	5%
$X_9$	Normal	$69.5 \cdot 10^6 \text{ kN/m}^2$	$2.76 \cdot 10^6 \text{ kN/m}^2$	4%
$X_{10}$	Normal	$210 \cdot 10^6 \text{ kN/m}^2$	$8.4 \cdot 10^6 \text{ kN/m}^2$	4%

**Table 2.** Description of the random variables (Variant II)

Random Variable ( $X_i$ )	Probability density function	Mean value ( $\mu_X$ )	Standard deviation ( $\sigma_X$ )	Coefficient of variation ( $v_X$ )
$X_1$	Gumbel	13.036 kN	1.065 kN	8.2%
$X_2$	Gumbel	26.072 kN	2.129 kN	8.2%
$X_3$	Gumbel	4.888 kN	0.399 kN	8.2%
$X_4$	Gumbel	9.777 kN	0.799 kN	8.2%
$X_5$	Log-normal	$55.93 \cdot 10^{-4} \text{ m}^2$	$2.795 \cdot 10^{-4} \text{ m}^2$	5%
$X_6$	Log-normal	$1836.373 \cdot 10^{-8} \text{ m}^4$	$91.761 \cdot 10^{-8} \text{ m}^4$	5%
$X_7$	Log-normal	$32.599 \cdot 10^{-4} \text{ m}^2$	$1.629 \cdot 10^{-4} \text{ m}^2$	5%
$X_8$	Log-normal	$485.716 \cdot 10^{-8} \text{ m}^4$	$24.271 \cdot 10^{-8} \text{ m}^4$	5%
$X_9$	Log-normal	$69.444 \cdot 10^6 \text{ kN/m}^2$	$2.777 \cdot 10^6 \text{ kN/m}^2$	4%
$X_{10}$	Log-normal	$209.832 \cdot 10^6 \text{ kN/m}^2$	$8.39 \cdot 10^6 \text{ kN/m}^2$	4%

#### 4. Results and discussion.

The value of the Hasofer-Lind reliability index was determined with the FORM method, and for the sake of comparison, with other methods, i.e. Monte Carlo and Importance Sampling. The results for two variants (Variant I and Variant II) of random variables description and five different cases of random variables vector (from A to E Cases) are presented in table 3 and table 4.

**Table 3.** Values of the Hasofer-Lind reliability index for Variant I.

Case of random variables vector	FORM	Importance Sampling	Monte Carlo
A	2.22	2.21	2.21
B	2.20	2.12	2.13
C	2.20	2.18	2.19
D	1.82	1.85	1.85
E	3.24	3.20	3.18

**Table 4.** Values of the Hasofer-Lind reliability index for Variant II.

Case of random variables vector	FORM	Importance Sampling	Monte Carlo
A	2.70	2.63	2.62
B	2.69	2.60	2.63
C	2.69	2.59	2.60
D	2.48	2.39	2.42
E	4.51	4.48	4.49

The relative error of reliability index was estimated with the assumption that the reference is the Monte Carlo method (table 5 and table 6). In addition, the difference in the value of reliability index determined by the FORM method for different descriptions of the mathematical model (Variant I and Variant II) and for different cases of random variables vector (from A to E Cases) was estimated (table 7). The relative error of reliability index in table 7 was estimated with the assumption that the reference is the mathematical description in Variant II which precisely model the real work of structure and the nature of loads.

**Table 5.** Values of relative error computing the Hasofer-Lind reliability index for Variant I

Case of random variables vector	FORM	Importance Sampling
A	0.5%	0.0%
B	3.7%	0.5%
C	0.5%	0.5%
D	1.6%	0.0%
E	1.9%	0.6%

**Table 6.** Values of relative error computing the Hasofer-Lind reliability index for Variant II

Case of random variables vector	FORM	Importance Sampling
A	3.0%	0.3%
B	2.2%	1.1%
C	3.3%	0.4%
D	2.4%	1.2%
E	0.4%	0.2%

**Table 7.** Relative error of computing the reliability index for FORM method in accordance with Variant I and Variant II

Case of random variables vector	Relative error
A	17.8%
B	18.2%
C	18.2%
D	26.6%
E	28.2%



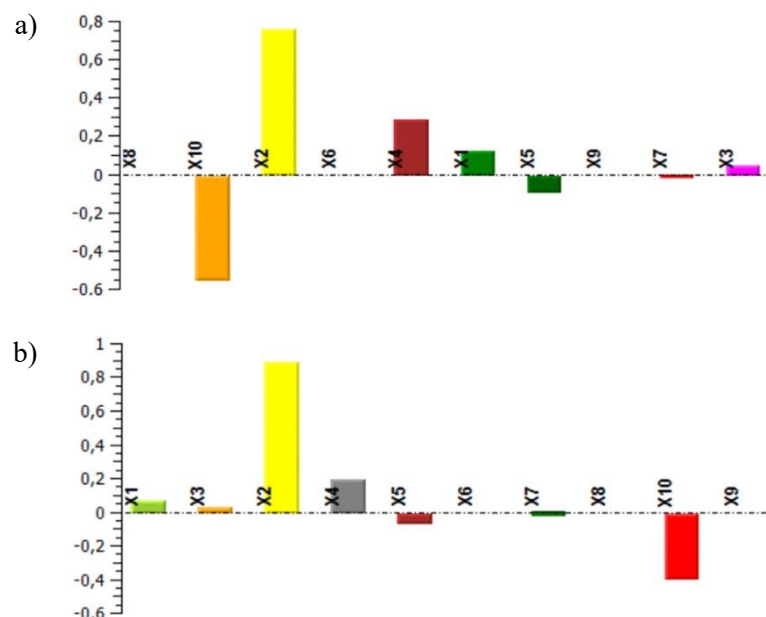
The reliability analysis performed with the NUMPRESS software also provides information on the number of calls of the limit function, and thus on the time necessary to estimate the reliability index (table 8 and table 9). In addition, graphs were provided that show the sensitivity of the reliability index to random variables for Variant I and Variant II. The sensitivity of reliability index to the random variables for Case D of random variables vector only was shown.

**Table 8.** Effectiveness of the FORM method when compared with other methods – for Variant I

Case of random variables vector	Number of the limit state function calls/Calculation time		
	FORM	Importance Sampling	Monte Carlo
A	26/9sec	526/3min22sec	75000/48min56sec
B	34/13sec	534/3min26sec	95000/58min16sec
C	52/19sec	552/3min33sec	100000/1h23min4sec
D	62/23sec	562/3min38sec	100000/1h40min17sec
E	65/23sec	565/3min39sec	100000/52min49sec

**Table 9.** Effectiveness of the FORM method when compared with other methods – for Variant II

Case of random variables vector	Number of the limit state function calls/Calculation time		
	FORM	Importance Sampling	Monte Carlo
A	42/20sec	542/4min47sec	95000/58min50sec
B	54/27sec	554/4min36sec	95000/49min3sec
C	66/32sec	566/4min41sec	100000/1h27min9sec
D	63/31sec	563/4min43sec	90000/1h49min11sec
E	65/30sec	565/4min42sec	100000/51min12sec



**Figure 3.** Sensitivity of the reliability index  $\beta$  to random variables for a) Variant I, b) Variant II

The analysis of the results demonstrates that the FORM method is good enough and much simpler to apply. The maximum relative error compare to Monte Carlo method amounted to 3.7% for the Variant I (Case B) and to 3.3% for Variant II (Case C). In the analysis two Variants of random variables description were considered. We observed that using distribution of variables appropriate to the variable nature influences on the value of reliability index. The differences in the value of reliability index determined by the FORM method for different descriptions of the mathematical model were shown in table 7. The maximum relative error is 28.2% for Case E of random variables vector. Accounting for a larger number of random variables considerably reduces the reliability index value (table 3 and table 4). We observed that the lowest values of reliability index are for Case D of random variables vector where we have the highest number of variables. The timescale of computations, which is related to number of calls of the limit function, should also be taken into account. Calculation time for FORM method equals amount to 30 seconds but for Monte Carlo method is almost 2 hours.

Another important component of the study was to investigate the sensitivity of the reliability index to changes in probabilistic characteristics of the random variables under consideration. After analysing the results obtained for Variant I and Variant II (figure 3) it can be seen that the sensitivity of the reliability index is the highest for the random variable  $X_2$  which describes the nodal concentrated force replacing the influence of wind on the windward side and  $X_{10}$  elastic modulus for S215 steel. The sensitivity of the reliability index is the lowest for the random variable  $X_6$  (member section area of RK120x100x8 girts and webs),  $X_8$  (member inertia moments of RK120x100x8 girts and webs) and  $X_9$  (elastic modulus for EN AW-6005A). Knowing this sensitivity is crucial for better understanding the structure performance. If the reliability index sensitivity due to the random variable  $X_i$  is low when compared with other variables, it can be stated that the impact of this variable on failure probability is small, and in successive computations it can be treated as a deterministic parameter. Sensitivity analysis is an important element in the assessment of the impact of random variables on the reliability index value and thus on the factors which determine the safety of the structure.

## 5. Conclusions

The analysis of the results demonstrates that the FORM method is sufficiently precise and authoritative research method. The timescale of computations, which is related to number of calls of the limit function, should also be taken into account. The results indicate that the FORM method allows obtaining a quick response, which makes it possible to use the method in engineering practice as one of the modules of computational software that support structure design. We observed significant differences in the values of reliability index determined by the FORM method for different descriptions of the mathematical model Therefore in order to precisely model the real work of structure the probability distributions of random variables appropriate to their nature should be applied. So we assume Monte Carlo method is not applicable to huge tasks of reliability structure. That results from a considerably longer computation time when compared with FORM method. Due to their high accuracy, however, those methods are well suited to determine an error made by the first and second order methods.

However, it has to be remembered at all times that the FORM method yields the best results when only one design point exists, the limit state function is not strongly linear and it is differentiable. The most commonly encountered disadvantage, also observed by other users, involves problems with calculating the gradients of the limit state function. Consequently, before taking a decision on applying the method to other problems in the structure analysis, it is necessary to run a number of functionality tests.

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