

Determining the Parameters of the Intermediate Duration - Intensity Equation for Mediterranean Region, Turkey

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Abstract. The aim of this study is to determine the parameter estimates of the general intensity-duration-frequency (IDF) formula for the 20 rainfall stations in the Mediterranean Region of Turkey. The maximum annual rainfall data were statistically analyzed for the durations 1, 2, 3, 4, 5, 6, 8, 12, 18 and 24 hours. Generalized Extreme Value (GEV), Gumbel, Normal, two-parameter Lognormal, three-parameter Lognormal, Gamma, Pearson type-III and Log-Pearson type-III distributions were considered and compared using Chi square (χ^2) goodness-fit test. Parameters of these distributions were estimated using both maximum likelihood and moment methods. The parameters of the general IDF equation $i = (A * T^B) / (t^C + D)$ and coefficient of correlation (R) for different return periods (2, 5, 10, 25, 50, 75 and 100 years) were calculated by using nonlinear estimation method. Coefficient of correlation result (R = 0,875) of general IDF equation showed the suitability for intermediate durations in the Mediterranean Region.

1. Introduction

The duration and frequency of extreme rainfall rates are important climatological features to be considered in meteorology, hydrology and civil engineering. Their knowledge is essential for planning hydraulic works, roads and sewage systems, and for designing rainwater drainage network in large infrastructures. Moreover, the rainfall rate behavior could be very useful in order to detect the climatic changes and evaluate their possible effects in a given area [1].

Mediterranean region in Turkey is most likely to experience strong climatic variations for short and long time periods. This region is characterized by dry, warm-to-hot summers and mild and wet winters. A precipitation characteristic of Turkey and Mediterranean region is represented by complex pattern of spatial, seasonal and inter-annual variability. According to Kadioglu [2], inter-annual variability of precipitation in Turkey relatively large from the annual mean which is around 5-65% in the southern part of the country. Pursuant to climatic variations, natural resources of the area, such as vegetation, and the existing resources are easily damaged by changes in precipitation patterns. Therefore, this region is highly prone to climatically related environmental disasters such as drought or water supply shortage [3].

A suitable rainfall intensity-duration-frequency (IDF) relationship is commonly required for planning and designing water resource projects. There has been considerable research to identify the IDF relationship. Many procedures and formulas, mainly empirical, have been proposed in literature [4,



5, 6, 7]. On the other hand, a mathematical approach has been proposed in some studies [8, 9]. The division of the rainfall duration into three groups (short, intermediate, long durations) leads to the regionalization of the IDF relationships for different geographical areas encompassing several counties [10, 11, 12, 13, 14].

The purpose of this study is to investigate the parameter estimates of the general intensity-duration-frequency (IDF) formula for intermediate durations (1, 2, 3, 4, 5, 6, 8, 12, 18 and 24 hours) at 20 rainfall stations in the Mediterranean Region of Turkey.

2. Study area and data set

Mediterranean region takes its name from the Mediterranean Sea, and occupies 15% of the total area of Turkey with its 120.000 square kilometers of land. West and Mid-Taurus mountains run parallel to the coast line. Because of high and steep mountains, the valleys between the sea and mountain range are very narrow, the width varies between 120-180 kilometers.

The region has two different climates one of the coastal climate and the other of the inland climate. The coastal climate has a Mediterranean climate with hot, dry summers and mild, cool and sometimes wet winters. During the rainless and virtually cloudless summers the temperature can sometimes exceed 40 degrees Celsius. Snow is virtually unheard of in the coastal areas and winter lows rarely go below 5 degrees Celsius. This region's winter is November through March and receives the most rainfall.

The mountain regions tend to be cooler and wetter, but they are rarely very cold. The clear coastal winds help to ensure that the mountains also see sunshine for much of the year. The key difference is that mountain areas will typically see more rain in the fall and winter.

The maximum annual rainfall data for intermediate durations (1, 2, 3, 4, 5, 6, 8, 12, 18 and 24 hours) in 20 meteorological stations of Mediterranean Region of Turkey were obtained by Turkish State Meteorological Service (SMS). The locations, elevations and record lengths of the stations are shown in Table 1. And spatial distributions in Figure 1.

Table 1. Station locations, elevations and record lengths.

Stations	Longitude (°E)	Latitude (°N)	Elevation (m)	Record Length (years)
Adana	34.34	37.00	23	64
Ceyhan	35.79	37.01	30	37
Karaisali	35.06	37.25	240	36
Karataş	35.39	36.56	22	43
Kozan	35.81	37.43	112	44
Yumurtalık	35.79	36.76	34	38
Dinar	30.15	38.05	864	37
Alanya	31.98	36.55	6	46
Antalya	30.68	36.88	47	60
Finike	30.14	36.30	2	45
Korkuteli	30.19	37.05	1017	32
Manavgat	31.44	36.78	38	45
Burdur	30.33	37.68	1241	32
Tefenni	29.77	37.31	1142	38
Bozkurt	29.61	37.81	885	44
Isparta	30.56	37.78	977	42
Anamur	32.86	36.06	2	46
Erdemli	34.33	36.62	7	43
Mersin	34.60	36.78	7	49
Silifke	33.93	36.38	10	46

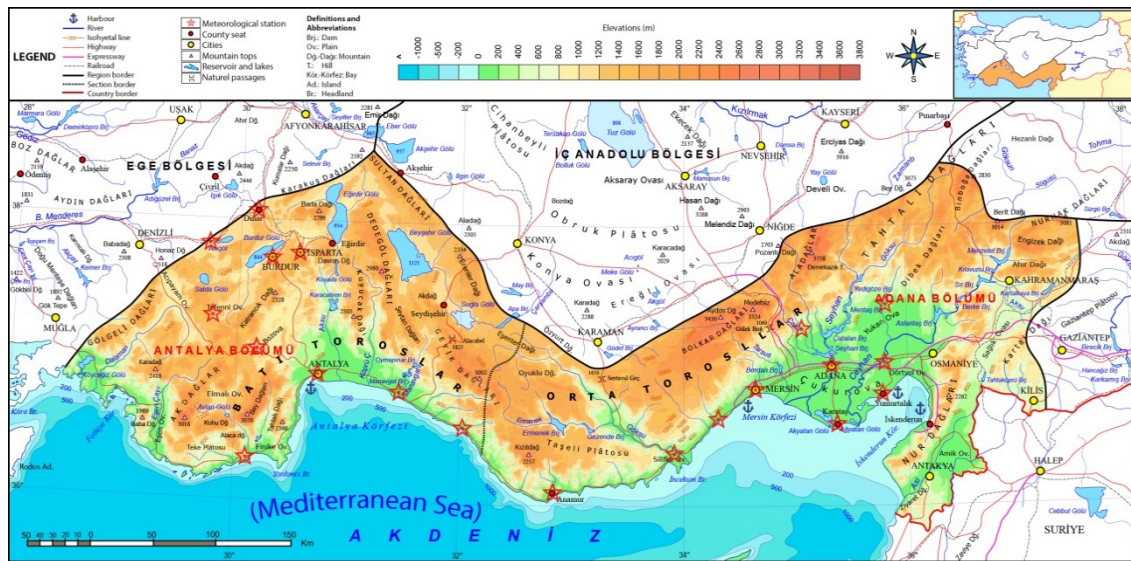


Figure 1. Mediterranean Region in Turkey [15]

3. Statistical distributions

All the parameters are estimated by the method of moments and maximum likelihood given by the Weibull formula in all calculation as [16]

$$F = k / (n + 1) \quad (1)$$

All statistical distributions and their functions that were analyzed in this study are shown in Table 2.

Table 2. Statistical distributions and their functions.

Statistical Distributions	Functions
Generalized Extreme Value (GEV) [17]	$f(x) = \left(\left[1 - k \left((x-u)/\alpha \right) \right]^{1/(k-1)} \cdot e^{-[1-k \cdot ((x-u)/\alpha)]^{1/k}} \right) / \alpha$
The Extreme Value Type I (Gumbel) [4, 18]	$f(x) = \frac{1}{\alpha} \exp \left[\left(-\frac{x-u}{\alpha} \right) - \exp \left(-\frac{x-u}{\alpha} \right) \right]$
Normal [19]	$f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2\sigma^2} (x-\mu)^2}$
Two-Parameter Lognormal (LN)	$f(x) = \frac{1}{x \cdot \sigma_y \sqrt{2\pi}} \cdot \exp \left[-\frac{(\ln x - \mu_y)^2}{2\sigma_y^2} \right]$
Three-Parameter Lognormal (LN3) [20]	$f(x) = \frac{1}{(x-a) \cdot \sigma_y \sqrt{2\pi}} \cdot \exp \left[-\frac{(\ln(x-a) - \mu_y)^2}{2\sigma_y^2} \right]$
Two-Parameter Gamma	$f(x) = \frac{1}{\alpha^\beta \Gamma(\beta)} x^{\beta-1} e^{-(x/\alpha)}$

The Pearson Type III

$$f(x) = \frac{1}{\alpha \Gamma(\beta)} \left(\frac{x - \gamma}{\alpha} \right)^{\beta-1} e^{-\left(\frac{x - \gamma}{\alpha} \right)}$$

Log-Pearson III (LP)

$$f(x) = \frac{1}{\alpha x \Gamma(\beta)} \left[\frac{\ln(x) - \gamma}{\alpha} \right]^{\beta-1} e^{-\left\{ \frac{\ln(x) - \gamma}{\alpha} \right\}}$$

4. Goodness-of-fit test

It is necessary to consider a test to determine if a sample record has a specified theoretical distribution. The aim of the test is to decide how good a fit is between the frequency of occurrence observed in a sample and the expected frequencies obtained from the hypothesized distribution. A goodness-of-fit test between observed and expected frequencies is based on the Chi-square quantity, which is defined as,

$$\chi^2 = \sum_{i=1}^k (o_i - e_i)^2 / e_i \quad (2)$$

where X^2 is a random variable whose sampling distribution is approximated very closely with the Chi-square distribution. The symbols o_i and e_i represent the observed and expected frequencies, respectively, for the i -th class interval in the histogram.

If the observed frequencies are close to the corresponding expected frequencies, the X^2 value will be small, indicating a good fit; otherwise, it is a poor fit. A good fit leads to the acceptance of H_0 (null hypothesis), whereas a poor fit leads to its rejection. The critical region will, therefore, fall in the right tail of the chi-square distribution. For a level of significance equal to α , the critical value X_{α}^2 is found from readily available Chi-square tables and $X^2 > X_{\alpha}^2$ constitutes the critical region [21].

5. Intensity-duration-frequency relationship

IDF relationship can be described mathematically by means of various expressions [22]. The most common one, which groups the various intensity–duration curves for the various return periods in a single formula, is equation 3, which is applicable to locations with observatories keeping records for rainfall durations between 10 min and 24 h [23].

$$i = \frac{aT^b}{t^c + d} \quad (3)$$

where i is the rainfall intensity for the duration t (min) and the return period T (years), and a , b , c and d are parameters to be determined by fitting.

6. Results and discussions

The decision criterion described here should not be used unless each of the expected frequencies is at least equal to 5. Herein, α (the significance level) is chosen as 0.05. All statistical distributions for different durations at each station are performed by the goodness-of-fit X^2 test. The Chi-square values and probability values of the statistical distributions are compared. One can then determine the best fitting distribution of rainfall durations of 1, 2, 3, 4, 5, 6, 8, 12, 18 and 24 hours by observing which distribution yields the smallest Chi-square value. In Table 3, the Chi-square value and the corresponding probability distributions for a rainfall duration of 8 hours is shown in the Adana station. The Gamma (moment) distribution is chosen because its X^2 value is the least and the probability value (p) is the highest for 8 hours. The chosen distributions for the stations are shown in Table 4.

Table 3. Goodness of fit tests to all statistical distribution for 8 hours rainfall duration in Adana

Distribution	Chi-square	p
GEV (Max. Likelihood)	8.50	0.2037
GEV (Moment)	8.50	0.2037
EV I (Max. Likelihood)	9.44	0.2228
EV I (Moment)	8.50	0.2906
Normal (Max. Likelihood)	8.50	0.2906
Lognormal (Max. Likelihood)	17.25	0.0159
Lognormal (3 P) (Max. Likelihood)	8.19	0.3164
Lognormal (3 P) (Moment)	9.75	0.1356
Gamma (Max. Likelihood)	8.50	0.2037
Gamma (Moment)	6.63	0.4689
Pearson III (Max. Likelihood)	9.44	0.2228
Pearson III (Moment)	7.25	0.2983
Logpearson III (Moment)	7.88	0.2474

Table 4. Chosen distributions for the stations

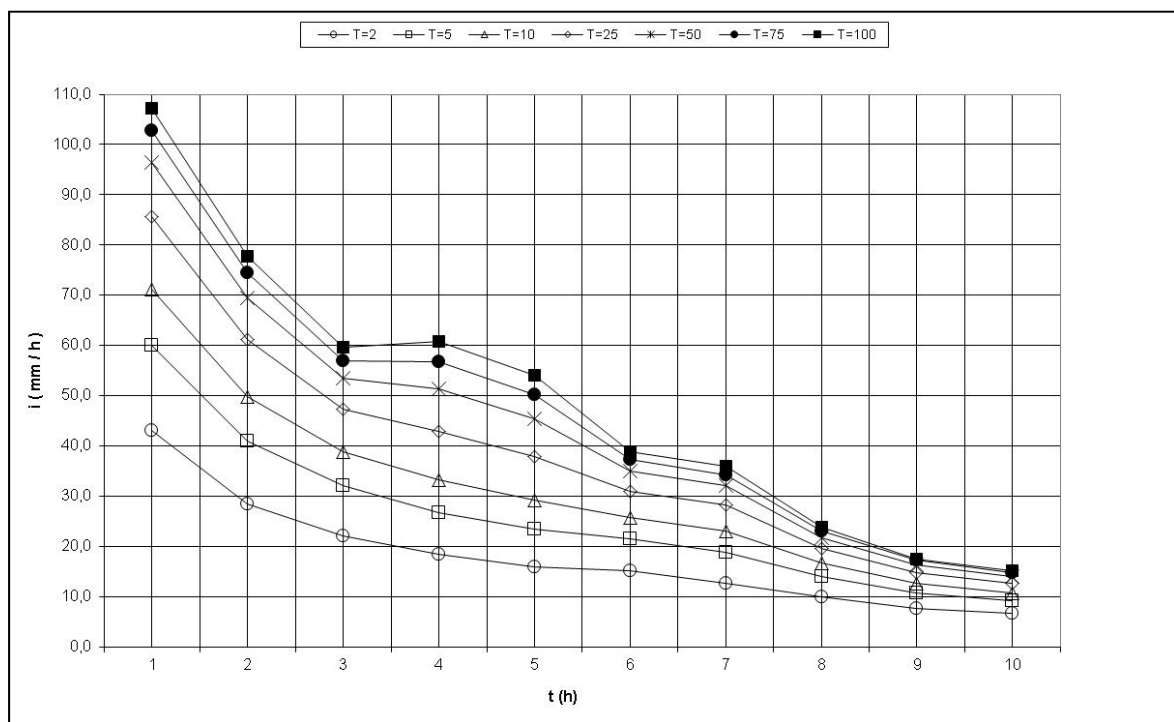
Stations	Duration (hours)									
	1	2	3	4	5	6	8	12	18	24
Adana	Gumbel	LN	Gumbel	Gamma	Gamma	Gumbel	Gamma	Gumbel	Pearson	Pearson
Ceyhan	Pearson	LN3	LN3	Gumbel	Gumbel	Gumbel	Gamma	Normal	Normal	Gumbel
Karaisali	Gumbel	Gamma	Gumbel	LN3	Gumbel	LN	Pearson	LN3	GEV	Pearson
Karataş	Gumbel	LN	LN	Gumbel	LN3	Gumbel	LN	Gumbel	Pearson	GEV
Kozan	GEV	Pearson	Gumbel	GEV	LN	Gumbel	GEV	Gumbel	LN3	Gumbel
Yumurtalık	LN	Normal	Normal	GEV	Gumbel	LN	Normal	Normal	Pearson	LN
Dinar	Gamma	Gumbel	Gumbel	Gumbel	Gumbel	LN3	LN3	Pearson	Gumbel	Pearson
Alanya	LN	Gamma	Pearson	Gumbel	LN3	Gumbel	LN3	GEV	Pearson	Pearson
Antalya	LN	Pearson	Pearson	GEV	GEV	Gumbel	Gumbel	Gamma	Gamma	Gamma
Finike	LN3	GEV	GEV	GEV	LN	Pearson	Gumbel	GEV	Pearson	Gumbel
Korkuteli	LP	LN	GEV	Gamma	Normal	Gamma	Gumbel	LN	LN	Gumbel
Manavgat	Pearson	LN	Gamma	Gamma	Gamma	Pearson	Pearson	Gumbel	Gumbel	Pearson
Burdur	LN	GEV	LP	LN3	Pearson	Gamma	GEV	GEV	Gumbel	Gumbel
Tefenni	Pearson	LN3	Gamma	Pearson	LN3	Gumbel	GEV	Gamma	Gamma	Gumbel
Bozkurt	LN3	LN	LN3	GEV	GEV	Pearson	GEV	Gamma	Gamma	Gumbel
Isparta	Gamma	Gumbel	Pearson	Gamma	Gamma	Gumbel	Gumbel	LN	Gumbel	Pearson
Anamur	GEV	Pearson	Gamma	LN	LP	LP	Pearson	Gumbel	Normal	Normal
Erdemli	Pearson	LN	Gumbel	LN	Gumbel	Gumbel	Gumbel	Gamma	Gumbel	LN
Mersin	GEV	Pearson	Gumbel	LN3	Gumbel	GEV	Pearson	GEV	Gumbel	LN3
Silifke	Gamma	LN3	Pearson	Gamma	Gumbel	LN	LP	GEV	LN3	Gamma

LN: two-parameter Lognormal, LN3: three-parameter Lognormal, LP: Log-Pearson III

The rainfall estimates are obtained for intermediate durations and different return periods for 20 stations in the Mediterranean Region. As an example, Table 5 presents the rainfall intensity estimates in the Antalya station. Intensity-duration-frequency (IDF) curves are shown for Antalya station in Figure 2.

Table 5. Rainfall intensity (mm/h) estimates in the Antalya station

T \ t (hours)	Rainfall intensity (mm/h)									
	1	2	3	4	5	6	8	12	18	24
T = 2	43.2	28.4	22.1	18.5	16.0	15.1	12.6	9.9	7.6	6.6
T = 5	60.0	41.0	32.1	26.7	23.4	21.5	18.9	14.1	10.7	9.2
T = 10	71.2	49.8	38.8	33.2	29.3	25.7	23.0	16.6	12.6	10.8
T = 25	85.6	61.2	47.3	42.9	37.9	31.0	28.2	19.6	14.8	12.7
T = 50	96.4	69.5	53.5	51.3	45.5	35.0	32.0	21.8	16.4	14.0
T = 75	102.7	74.4	57.0	56.7	50.3	37.3	34.3	23.0	17.3	14.7
T = 100	107.2	77.8	59.6	60.8	54.0	38.9	35.9	23.8	17.4	15.2

**Figure 2.** Intensity-duration-frequency (IDF) curves for Antalya station

7. Conclusions

The chi-square test was used on one hand to examine the combinations or contingency of the observed and theoretical frequencies, and on the other hand, to decide about the type of distribution which the available data set follows. Rainfall intensities for intermediate durations (1, 2, 3, 4, 5, 6, 8, 12, 18 and 24 hours) are determined at 20 rainfall stations in the Mediterranean Region of Turkey. Intensity-duration-frequency (IDF) curves were plotted for each station.

According to the IDF curves, rainfall intensities decreased with rainfall duration in all return periods. Rainfall intensities rise in parallel with the rainfall return periods. When long return periods are compared with short return periods, the rainfall intensity ratio in duration becomes less. For example in Mersin, for 1 hour rainfall (shortest intermediate duration) $i = 23$ mm/h for $T = 2$ year and $i = 69,7$ mm/h for $T = 100$ years. For 24 hours rainfall (longest intermediate duration) $i = 62,8$ mm/h for $T = 2$ years and $i = 9,4$ mm/h for $T = 100$ years.

The parameters of the IDF equation (equation 3) and coefficients of correlation for all Mediterranean Region are analyzed by using nonlinear estimation method (Table 6).

Table 6. Parameters and R in the Mediterranean Region.

Equation	a	b	c	d	R
$i = (a \cdot T^b) / (t^c + d)$	26.21	0.219	0.722	-0.015	0.875

This IDF equation parameters will be useful for developing intermediate-duration design storms needed to evaluate hydraulic structures when more accurate rainfall intensity-duration-frequency relations are not available.

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