Control Method Stretches Suspensions by Measuring the Sag of Strands in Cable-Stayed Bridges

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Abstract. In the article is described the method that allows on evaluation and validation of measurement correctness of dynamometers (strain gauges, tension meters) used in systems of suspensions. Control of monitoring devices such as dynamometers is recommended in inspections of suspension bridges. Control device (dynamometer) works with an anchor, and the degree of this cooperation could have a decisive impact on the correctness of the results. Method, which determines the stress in the strand (cable), depending on the sag of stayed cable, is described. This method can be used to control the accuracy of measuring devices directly on the bridge. By measuring the strand sag, it is possible to obtain information about the strength (force) which occurred in the suspension cable. Digital camera is used for the measurement of cable sag. Control measurement should be made independently from the controlled parameter but should verify this parameter directly (it is the best situation). In many cases in practice the controlled parameter is not designation by direct measurement, but the calculations, i.e. relation measured others parameters, as in the method described in the article. In such cases occurred the problem of overlapping error of measurement of intermediate parameters (data) and the evaluation of the reliability of the results. Method of control calculations made in relation to installed in the bridge measuring devices is doubtful without procedure of uncertainty estimation. Such an assessment of the accuracy can be performed using the interval numbers. With the interval numbers are possible the analysis of parametric relationship accuracy of the designation of individual parameters and uncertainty of results. Method of measurements, relations and analytical formulas, and numerical example can be found in the text of the article.

1. Introduction

External cables are an essential structural component of suspended bridges, responsible for the distribution of internal forces throughout the system: pylon-cables-bridge, e.g. [1, 2, 3]. The tension of suspension cables has the significant influence on the dynamic characteristics of suspension bridge.

That is why it is important to control tension (capacity) of suspension cables checks whether the values of forces in the cables have not changed more than in the project ranges. Cables are sensitive for damaging, especially in the anchorage zone. Such damages as uncontrolled slip anchorage are difficult to visually inspect. The variety of structural systems, suspensions systems, ways of suspension, construction of a bridge pylon what makes difficult to formulate problem in a general way, should also have in mind that hung from bridges require advanced computational models and nonlinear analysis.

Control of the correctness of indications of measuring devices generally requires removing them from the bridge object; it is often difficult or even impossible. Control device works with an anchor, and the degree of this cooperation could have a decisive impact on the correctness of the results. Such control is recommended in reviews of suspended bridges (see. e.g. [4, 5, 6, 7, 8]). In the article is described a method enabling the validity check of tensions meters in systems of suspending directly in the bridge. Author proposes method of determining the stress in the tendon (cable) depending on its sag [5, 6, 7]. Other methods of monitoring suspended bridges are known in the literature, requiring much higher costs and earlier activities (e.g. [9, 10, 11, 12]).

The article presents a procedure enabling the assessment of the reliability of the results, that is, the accuracy of calculation of the force/stress in the stayed cables. This is important because in the proposed control method, the force/tension in the tendons is fixed not by direct measurement, but the calculations, i.e. as a relation of auxiliary parameters. In such cases appears the problem of evaluation of overlapping measurement errors and their impact on the final result [5, 7].

2. Relationship of the tendon in its sag

The fundamental problem arising in the suspension construction of all types is non-linear behaviour of the system as a result of changes of the cable sag (e.g. [1, 2, 3, 13, 14]). In this article, I analyse sag as a function of force/stress in the cable.

Precise measurement of sag in the mid-span cable (maximum sag) is difficult to carry out because of the high altitude of the pylon, the difficulty in determining where is half the span, where is the cables chord, the chord to which is measurements the sag.

It is convenient to measure the sag at a fixed point marked on the strand by marker (Figure 1). Marker should be located closer to cable anchorage.
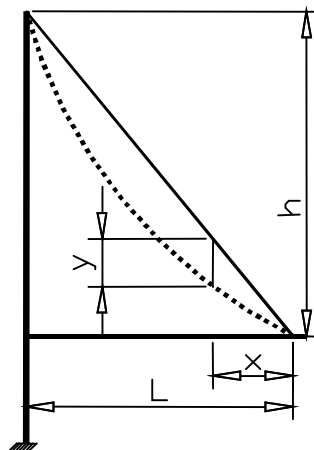

Figure 1. Sag of a cable at the point x; L and h: horizontal and vertical length of the cable's chord [5]

Changes of marker position can be measured using a digital camera locates in one point outside of the object (to eliminate vibration of bridge). (The digital camera so far has been used in suspension bridges for detection of cable damages, e.g. [15].)

It is necessary to measure the horizontal distance of the marker from the cable anchorage x (Figure 1).

Vibration strands can be filtered out. Tilts of the vibrating strands are symmetrical, so I take the average value of several successive measurements (camera records the displacements of the marker as a continuous process)

Displacements of other structural elements do not affect the results, therefore, are not recorded.

Location chord of strand is not the same in space and time as a result of large deflection of the pylon and the bridge span, changing the positions of the attachment points of cables. The reference line can be virtual chord applies during computer processing on the image captured on camera.

Author proposes also the following method: the "so-called" reference line (the cable chord) should be firmly stretched (to eliminate the sag) thin as possible (weightless) steel wire (string) attached to anchorage in the axis of the cable.

The proposed method of measurement, determines the string cable, regardless of the deflection bridge and the pylon, because the strings of attachment points vary with the change of the position of the nodes (anchors) cable.

Suitable formulas permit the calculation of the maximum sag "f" in the middle of the cable on the basis of sag "y" measured at any point of the cable (vertical displacement of the measured marker) are shown below.

The cable sag y at the point x is determined from the formula (1) according to [2]. Marking symbols according to Figure 1:

$$y = \frac{4 \cdot f_y}{L^2} \cdot \left(L \cdot x - x^2\right) \qquad (1)$$

where: f_y - cable sag measures vertically in the middle of its span,
$\quad$ y - cable sag measured vertically at marker point distant of x from the anchorage (Figure 1),
$\quad$ L - horizontal cable chord according to Figure 1,
$\quad$ w - the distance of the marker to anchor measured horizontally according to Figure 1.

After measuring the cable sag at the place of marker (Figure 1) (e.g. on the basis of computer processing images from the camera) the maximum sag f at mid-span tendon is determined by formula (2):

$$f = \frac{y \cdot L^2}{4 \cdot \left(L \cdot x - x^2\right)} \qquad (2)$$

Then, knowing the maximum cable sag f is determined analytically axial force in the tendon T (3) [2]:

$$T = \frac{q_w \cdot L^2}{8 \cdot f} \cdot \left[1 + \left(\frac{h}{L} + \frac{4f}{L}\right)^2\right]^{1/2} \qquad (3)$$

where: $\quad$ q_w - horizontal project weight per unit length (4).

$$q_w = \frac{q_k}{\cos\alpha} \qquad (4)$$

where: $\quad$ q_k - cable weight per unit length,
$\quad$ α - suspension cable angle of inclination (to the horizontal axis).

In the formula (3) to determine the force in the cable are the same parameter in the denominator and numerator: f, L.

To evaluate the accuracy of the determination of the tension in the cable is used interval algebra [16] - in the interval algebra we should avoid division the same parameters by themselves, because when it comes to an uncorrected extension of the interval [5, 16].

Formula (3) is converted in such a way that the same parameter is not present in both the numerator and the denominator (5) [5].

$$T_k = \frac{q_w \cdot L}{8} \cdot \left[\frac{L^2}{f_y^2} + \frac{h^2}{f_y^2} + \frac{8 \cdot h}{f_y} + 16 \right]^{1/2} \tag{5}$$

Axial tensions in suspension cables are calculated according to the formula (6) [5].

$$\sigma = \frac{T}{A} = \frac{q_k \cdot L}{8 \cdot A \cdot \cos\alpha} \cdot \left[\frac{L^2}{f_y^2} + \frac{h^2}{f_y^2} + \frac{8 \cdot h}{f_y} + 16 \right]^{1/2} \tag{6}$$

3. Estimation accuracy of the results

3.1. The method of describing uncertainty. Interval numbers

During the inspection/validation dynamometer/tension meter in suspension cables we should measure directly controlled parameter by another independent method.

In practice often the value of cable force is estimates not in direct measure method but by the indirect measurement and calculations. In such cases, there is the problem of overlapping errors of intermediate parameters (data) and the evaluation of the reliability of the results [5].

The accuracy can be performed using interval numbers [16]. With these numbers are possible the analysis of relationship of accuracy of measured parameters and the uncertainty of results.

Interval method gives great analytical capabilities of the uncertain (interval) parameters, e.g. the use of a set containing all the possible outcomes.

Interval arithmetic allows you to perform arithmetic operations on that specific sets - internal numbers [16].

Interval number allows direct description of the uncertainty of the individual parameters [5, 6, 7] .

In this way, you can determine the intervals at which change the results obtained through taking to the calculation uncertain (interval) parameters

Interval analysis provides tools for processing uncertain [5].

Interval arithmetic [16] provides tools for processing uncertain data in a way that guaranteed reliable results. Such guarantee is being a part of the exact solution to the resultant interval.

Interval number is called the set of the form (formula (7) [5]:

$$[x] = [\underline{x}; \overline{x}] = \left\{ x \in \Re \mid \underline{x} \le x \le \overline{x} \right\} \tag{7}$$

where: $\underline{x}, \overline{x} \in \Re, \ \underline{x} \le \overline{x}$.

The real numbers are identified with the interval numbers with the range of zero width, i.e. if $a \in \Re$, then [a]=(a; a).

Any traditional number can be described as interval number, e.g.: [4] = (4; 4).

For such a defined interval number are provided basic arithmetic operation called interval algebra. Moor [16] proved that arithmetic on intervals can be expressed by the ends of the intervals as follows:

$$[x] + [y] = [\underline{x} + \underline{y}; \overline{x} + \overline{y}] \tag{8}$$

$$[x] - [y] = [\underline{x} - \overline{y}; \overline{x} - \underline{y}] \tag{9}$$

$$[x] \cdot [y] = [\min\left\{ \underline{x} \cdot \underline{y}, \underline{x} \cdot \overline{y}, \overline{x} \cdot \underline{y}, \overline{x} \cdot \overline{y} \right\}; \max\left\{ \underline{x} \cdot \underline{y}, \underline{x} \cdot \overline{y}, \overline{x} \cdot \underline{y}, \overline{x} \cdot \overline{y} \right\}] \tag{10}$$

$$[x]/[y] = [\underline{x}; \overline{x}] \cdot [1/\overline{y}; 1/\underline{y}], \ 0 \notin [y] \tag{11}$$

The measurement is made with certain accuracy, resulting from the characteristics of the measuring device; inaccuracy is usually described as $x \pm \Delta x$ and interval: $(x-\Delta x; \ x+\Delta x)$.

The primary disadvantage in the use of interval numbers is their property which consists in the fact that subtraction and division are inverse operations for addition and multiplication [5].

In equation *Ax±B=0* the serious analytic problem is isolate interval *[x]* because:

$$x+A+(-A)\neq x \tag{12}$$

$$x*A*A^{-1}\neq x \tag{13}$$

Example No. 1.:

The consequence of lack of inverse operations for addition and multiplication is illogical widening of intervals, e.g. for the number interval: [a] = (1; 2):

[a] − [a] = (−1; 1), and therefore contrary to expectations [a] − [a] ≠ (0; 0),

[a]/[a] = (0.5; 2), and therefore contrary to expectations [a]/[a] ≠ (1; 1).

Mathematical formulas should be converted in such way, which eliminates the subtraction and division of the same parameters.

3.2. Accuracy assessment of the results. The degree of uncertainty

To evaluate the accuracy of the results (for comparison) is used the degree of uncertainty Δx which is calculated according formula (14), (15) (for interval number: $x = (\underline{x}; \overline{x})$ [5]:

$$\Delta x = \frac{(\overline{x} - \underline{x})}{d} \cdot 100\% \tag{14}$$

where: d - the average (*d*>0, all parameters in described method are positive).

$$d = \frac{\underline{x} + \overline{x}}{2} \tag{15}$$

At the calculation example, which is placed in this article, is shown that the accuracy of the measurement read from the measuring device does not have a relationship with an accuracy of calculated results - this is an important conclusion, because often in practice, an uncertainty of the input data and determined analytically results describes approximately as the same.

Interval evaluation of the relationship between precision measurements of input parameters (data) to the inaccuracy end result allows to evaluate which parameters can be measured less accurately or even take on the basis of estimates (what is cheaper and faster), since its do not affect the degree of uncertainty of results; but also which parameters are measured "too little" exactly or in the wrong place of the structure.

For the theoretical evaluation of method, it is properly to take a constant uncertainty of input parameters (uncertainty of estimation), for example: ± 1%.

A major problem, having a huge impact on the uncertainty of the final results, is a proper assessment of the "real" uncertainty of the individual parameters (appropriate for measurements perform on the bridge).

It is important that the interval to which belongs the measure value of the parameter is so narrow as possible from the point of view of quality (uncertainty) results, and wide enough to include (take into account) all errors and inaccuracies in the setting of the parameter (these are such errors, which cannot be controlled, e.g. due to the accuracy of a measuring device, not the simple mistakes).

Such interval, in article describes by interval number, should contain the actual "real" value of the parameter.

4. Bridge calculation example

4.1. Assumptions for the calculation

Sag of the cable, at the stage of testing described analytical method, is based on the calculations in FEM program.

In article calculated tension in the suspension (on the basis of the determined value of cable sag).

By interval numbers are calculated the impact of the accuracy of input parameters on the final result.

Number of suspensions has no influence on the results of such analysis. Therefore, in this example, I assume a model with two tendons (Figure 1).

It is established that the bridge span is supported on a pylon using the rocker.
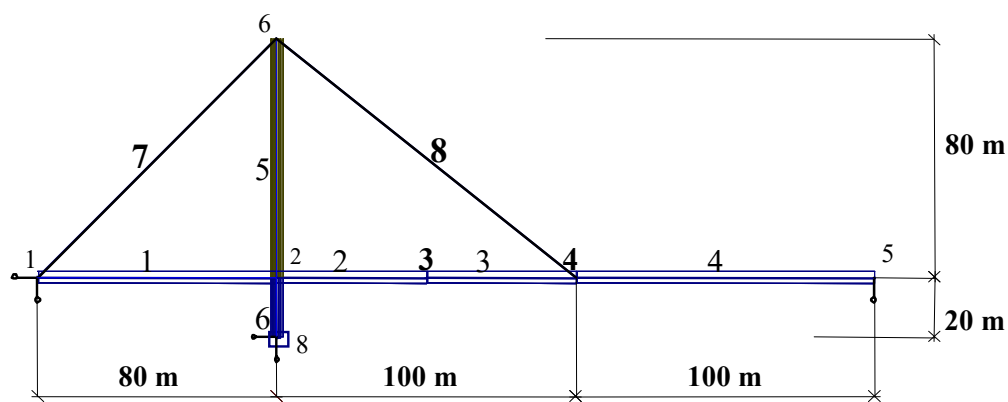
Figure 2. Simplified model of suspension bridge in the FEM program

The FEM model takes into account the weight of all bridge elements and exploitation load on the span q_{exp} = 100 kN/m.

Bridge span is in simpler modelled as I-beam with a height of 4 m, a width of 18 m and cross sectional area F = 1.012m^2, which corresponds to parameters of real bridges.

Pylon is modelled as tube, diameter d = 4.0 m, tube wall thickness is t=2.5 cm above the span and t=3.2 cm below the deck.
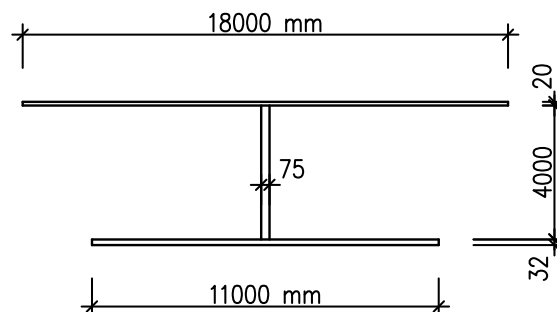
The cross section of the bridge span is shown on Figure 3.

Figure 3. The cross section of suspension bridge

A weight of steel: q_{steel} = 80 kN/m^3, modulus of elasticity: E = 205 GPa, cross-area of the tendon (suspension cable): A=380 cm^2. Suspension is modelled by means of cable element in FEM, nonlinear analysis is made.

Initial pre-tension of the cables is taken in order to optimize the dispersion of banding moments in the span: cable 8: 13.87MN cable 7: 15.2MN (Figure 2).

Geometric parameters and load are similar to the parameters of real cable-stayed bridges.

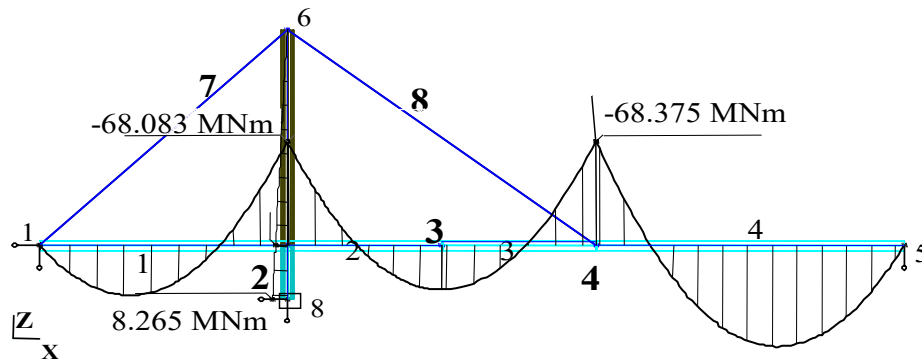Figure 4 shows a graph of bending moments from the permanent loads.

Figure 4. Bending moments from the weight of bridge elements

4.2. Two cases of uncertainty of input data

At the stage of writing (test) algorithm is advantageous (because of the ease of interpretation) accept some constant, the same accuracy of the input data for all parameters, for example ±1%.

The first case of uncertainty can also be used to analyse the relationship accuracy of the designation of the input parameter and final results. Constant uncertainty/accuracy of ±1% of the input parameters is given in Table 1.

Table 1. Accuracy of input parameters

parameter		measured value of parameter	constant accuracy ±1%	constant accuracy determine parameters	"real" accuracy of measuring device	"real" accuracy determine parameters
L [m]	the horizontal projection of the chord strand	100	±1%	(99; 101)	±0.01	(99.99; 100.01)
h [m]	the vertical projection of the chord strand	80	±1%	(79.2; 80.8)	±0.01	(79.99; 80.01)
x [m]	the distance of the marker to the anchorage	30	±1%	(29.7; 30.3)	±0.01	(29.99; 30.01)
A [m^2]	cross sectional area of the strand	0.380	±1%	(0.3762; 0.3838)	±2% (by [4])	(0.3724; 0.3876)
q [MN/m^3]	weight of strand/cable by volume	0.080	±1%	(0.0792; 0.0808)	±2% (by [4])	(0.0784; 0.0816)
y [m]	sag of strand/cable	by measuring	±1%	(0.99y; 1.01y)	±0.002	(y−0.002; y+0.002)

In the case of field investigation appears uncertainty, what is the exact "real" value of the measured parameters. This uncertainty is associated generally with a precision measuring device.

Inaccuracy determines the force/tension in the stayed-cable is estimated as the sum of several influences.:

- Readability camera (−0.0005; 0.0005) [m]. Measurement error for standard resolution camera (12 Megapixel) is approx. 0.2%, i.e. ±0.0005 for sag smaller than 0.5 m.
- Vibration of a strand (−0.0005; +0.0005) [m]. Vibration of the strand can be filtered assuming that the tilts are symmetrical. A resolution of the digital camera is affected to the accuracy of measuring/reading this tilt.
- Virtual chord of strand (−0.001; +0.001) [m]. Virtual chord is drawn during the computer processing on the image captured by the camera, this chord connects the anchorage points by a straight line. The accuracy of such designated chord is related to the accuracy of the draw/application of virtual line on the image.

"Real" uncertainty of input parameters is given in Table 1.

4.3. Uncertainty of cable tension for the uncertainty of input parameters ±1% (Figure 5)
All input parameters have the same uncertainty/accuracy ±1%, but despite the degree of uncertainty of results has different values, e.g. in the case of parameter L almost 11%.

The degree of uncertainty is determined according to formula (14).

The degree of uncertainty is a dimensionless quantity expressed as a percentage, is the ratio of the width of interval number describing the uncertainty of the parameter to determine the mean value. This assumption allows comparisons of different parameters according to the degree of uncertainty.

Figure 5 demonstrates how change the degree of uncertainty describing the accuracy of the designation of tension in the cable, if only one of the parameters is accepted as interval with uncertainty ±1%, and the remaining parameters are accepted without any uncertainty, i.e. as accurate values.

"Sum" in Figure 5 is a degree of uncertainty of cable tension for all input parameters (data) witch the uncertainty ±1%.

It is shown, that uncertainty of input parameters (uncertainty ±1%) aren't being directly transferred into the uncertainty of results. This conclusion is essential, because in practice accuracy is often incorrectly accepted as the same with reference to input data and final results appointed on the attitude of this data.
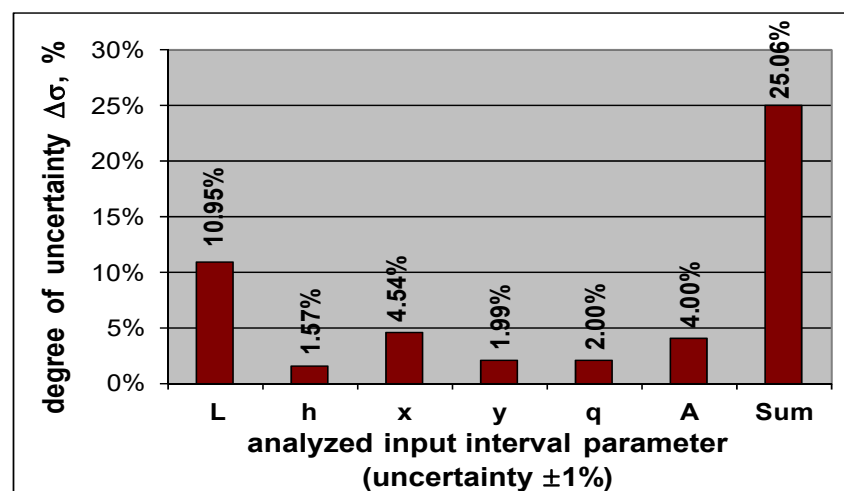

Figure 5. Degree of uncertainty of cable tension Δσ for input data witch uncertainty ±1%

4.4. Uncertainty of cable tension for the "real" uncertainty of input parameters (Figure 6)
Using the interval number, it is possible to calculate how the accuracy of the designation of input data affects at the uncertainty in the final results (it is the element of parametric analysis). This allows on finding those critical of the test method parameters which affect the uncertainty of results in the significant way. This allows also on finding those parameters which do not have an effect on the uncertainty of final result, e.g. A - cable cross area.

Figure 6 shows the influence of the "real" uncertainty of the designation of individual parameters (analysed separately) the uncertainty of cable tension.

Degree of uncertainty of results (i.e. cable tension) due to all parameters with "real" uncertainty, taking into account possible to get the measuring accuracy of individual parameters, does not exceed 6% ("Total" in Figure 6), or simplifying the inaccuracy determine the result does not exceed 3% (±3%), what is the value acceptable, qualifying the above method to practical applications.

Knowing the degree of uncertainty (accuracy) the results can be assessed several methods of control cable tension and choose the one in which the degree of uncertainty of final result (i.e. cable tension) is the smallest (in the case of the method described in the article degree of uncertainty is 5.4%). This allows the conscious design of research in terms of accuracy (reliability) results.
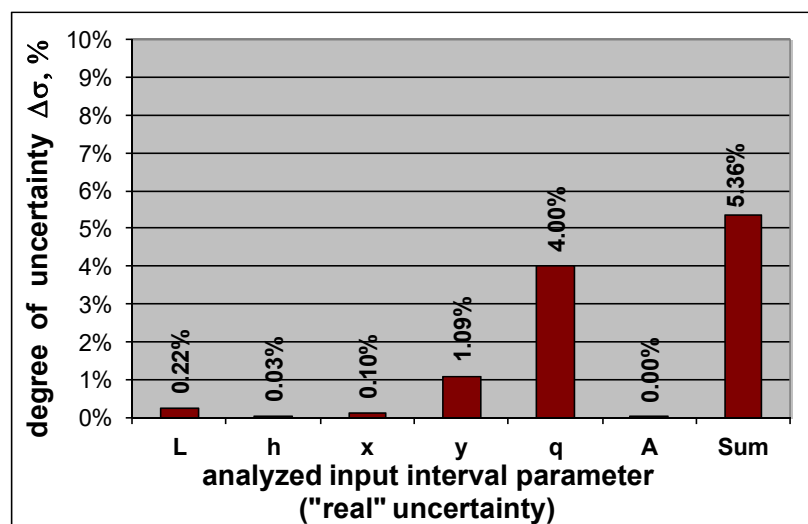

Figure 6. Degree of uncertainty of cable tension $\Delta\sigma$ for input data with "real" uncertainty

5. Conclusions

Suspension cables are an important structural element of cable-stayed bridges. However, external cables are very sensitive to damage. Therefore, different methods are used to control suspension cables. Other methods of monitoring cable-stayed bridges requiring much higher costs and earlier activities, then method describes in article.

Presented in the article author's method can be applied in practice to control dynamometer (strain gauges) embedded in tendons in suspension bridges This control is realized on the basis of indirect measurements and analytical formulas, i.e. cable forces are calculated, not directly measured. With this approach, it is necessary to take into account the impact of overlapping errors determine/measure the intermediate parameters (data) on the accuracy of the final result.

Estimation of accuracy is made using the interval numbers. An important advantage of this study is possibility of verifying the "real accuracy" of research. Interval numbers also allows on evaluation (and improving) the test methods due to the accuracy (uncertainty) results.

Interval numbers allows researching which parameters can be measured less accurately (and thus faster and cheaper) or even take on the basis of a preliminary estimate without measurement, which is certainly convenient, since these parameters have no effect on the uncertainty of the final results.

In the case of several proposals researches/methods for devices monitoring force (tensions) in the cables (bands), such analysis allows to choose the method that gives the best quality (accurate) results.

The estimation of uncertainty is useful when you want to get reliable results, but the accuracy of the data is of questionable.

It is important that the degree of complexity of the proposed method does not exceed the typical engineering application, and in return it is possible to receive a practical solution to assess the correctness (accuracy) of results.

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