IOP Conf. Series: Materials Science and Engineering **245** (2017) 042023 doi:10.1088/1757-899X/245/4/042023

Estimation Delay Variation and Probability of Occurrence of Different Level of Services Based on Random Variations of Vehicles Entering Signalized Intersections

Vahid Najafi Moghaddam Gilani[1], Meisam Ghasedi [2], Mahyar Ghorbanzadeh [3], Mehdi Jahangir Samet [4]

[1,4] School of Civil Engineering, Iran University of Science and Technology (IUST), Tehran, 16846-13114, Islamic Republic of Iran, n/a, Iran

[2,3]Master of Science of Highway & Transportation Engineering, Faculty of Engineering University of Guilan, Rasht, Iran

vahid.moghaddam90@gmail.com

Abstract. Estimation of delay in signalized intersection is a difficult and complicated process that depends on many variables. Among the factors influencing the intersection delay, variable of saturation degree (Xc) is one of the most important variables. In the present study, a new analytical method has been proposed for calculating delay variation at isolated intersections in under-saturated condition and under the influence of different number of vehicles entering to the intersections. Generally, the average delay for average number of vehicles entering intersection is currently calculated by the point to point method; then, the level of service of intersection is specified from the resulting delay. In the meantime, the influence of different number of vehicles entering to the intersections cannot be seen in such methods. But for the presented method in this paper the delay is expressed as a range of values for different values of saturation degree and different numbers of vehicles entering the intersections that it leads to achieve more real results from intersection's level of service.

1. Introduction

Quality of service in intersection is an indicator which is measured by several criteria such as delay, capacity, speed, number of stops, queue length and saturation degree. Among these, delay is the most important criterion due to displaying wasted travel time and the spent fuel. However, measuring the delay is not easy due to uncertainty in the nature of vehicles' entering and exiting models from the intersection. So, one of the important elements affecting the delay is the randomness of the number of vehicles entering to the intersection. If vehicles enter to the intersection randomly, the number of coming vehicles will constantly change and this leads to delay variation imposed on vehicles in intersection at different time intervals [1]. So it is necessary that the delay to be calculated with considering the distribution of different number of vehicles entering to the intersections, so that we can achieve more real estimation of the delay imposed on the vehicles with the help of this method.

This study aims to provide an analytical method for calculating delay variation of isolated intersection in under-saturated condition at acceptable confidence level based on random distribution of vehicles entering the intersection by using expectation value function method. After calculating the range of delay variation on intersection in the proposed method, the level of service of intersection is determined more appropriately that this results in better designing of signalized intersections in terms of timing the traffic lights. The functional index of level of service is used to show the average delay imposed on vehicles at the intersection according to Highway Capacity Manual (HCM), [2]. If the distribution type of vehicles' entering to the intersection and their variances to be considered in addition to paying attention to the average number of vehicles entering the intersection, the intersection delay can be obtained as a range of delays, thereby there is no longer a level of service for the intersection, but one can consider two or more level of services for expressing the functional level of intersection that the probability of occurring each level of service is determined in this manner.

The most famous estimation equation of vehicles' controlled average delay is the equation contained in HCM. Input parameters in this equation include geometry of intersection, traffic conditions and traffic signal timing. Depending on the type of signalized intersection and traffic conditions governing it, the average traffic flow and the average of saturation flow rate are used to estimate vehicles' controlled average delay by using the HCM equation. Therefore, there is a no position for the distribution type of vehicles entering to the intersection in the equation HCM. For this purpose, try to provide an analytical method for more accurate estimating the vehicles' delay by the HCM delay equation depending on the distribution type of vehicles entering to the intersection.

So far, three delay analytical models including uniform state stochastic model [3], deterministic queuing model [4] and time dependent stochastic model [5] have been created with different assumptions for predicting the delays on vehicles in signalized intersections. It is assumed in uniform state stochastic model that vehicles' entering to the intersection occurs randomly and discharging the headway is uniform.

This model is applicable only in under-saturated conditions and when the volume of passing traffic closes to capacity; this model provides an infinite delay. In delay queuing deterministic model, when the volume of passing traffic exceeds the capacity, the super saturated state is also used; but the model has no the ability to effect of vehicles' random entry to the intersection when the volume of passing traffic is not close to capacity. As Hurdle pointed out in 1984, these two models are totally inconsistent with each other when degree of saturation is equal to one. For this reason, the delay time-dependent model has been created to fill the gap between these two models and achieve the more real results of delay on vehicles in signalized intersections [6]. This model has been obtained from a combination of two uniform state stochastic and deterministic queuing modes. Therefore, this model can be used in saturation degree of one for both under-saturated and super saturated conditions without any discontinuity. A large number of delay equations have been created based on this model in different forms that the most famous equations among them, [7] are in a book entitled "Capacity Manual (roads) of Australia", ITE 1995 [8] in the book "A Guide to Canada's capacity signalized intersections", TRB 1998 and TRB 2000 in HCM, [9]. Meanwhile, due to selecting the delay equation of HCM in this paper, mentioned short reference to this equation to calculate delay imposed on vehicles in signalized intersection.

Finally, Fambro and Rouphail [11] applied changes in model 1994 with proposing a general model to calculate delays in 1997 that these reforms have been applied in model 2000. In this regulations, vehicles' average delay is calculated by the following equations:

$$d = d_1 \times PF + d_2 + d_3 \tag{1}$$

$$d_1 = 0.5C \times \frac{\left(1-\frac{g}{C}\right)^2}{\left(1-Min(x,1)\frac{g}{C}\right)} \tag{2}$$

$$d_2 = 900T\left[(x-1) + \sqrt{(x-1)^2 + \frac{8kIx}{cT}}\right] \tag{3}$$

$$PF = \frac{(1-P)f_p}{1-\frac{g}{c}} \tag{4}$$

In the Highway Capacity Manual (HCM, 1985), in order to calculate all the effective adjustment factors on saturation flow rate at the inlet legs of signalized intersections including adjustment factor for existence of the parking lanes (f_p) and blockage effect of local buses at stops within intersection area (f_{bb}) are used. The maximum and minimum values by considering number of lanes in lane group for f_p is 0.97 and 0.7 and for f_{bb} is 0.99 and 0.83. The maximum number of parking maneuvers,N_m equals 40 maneuvers per hour and the number of stops per hour,N_B is between 0-40. In this book, there is no discussion about the exact distance from the stops near the intersection [5]. In HCM 1998 the factors f_p and f_{bb} are constant based on the related tables [6].

In which equations 1 to 4, d is control delay imposed on vehicle (second per vehicle), d_1 is uniform delay imposed on vehicle with assuming a vehicles' uniform entering to the intersection (second per vehicle), d_2 *is* random and additional delay due to changes in entering flow of vehicles to the intersection and delays resulting from inability to discharge all the entering flow in related analysis time due to flow over capacity (second per vehicle), d_3 is delays resulting from initial queuing at the start of analysis time (second per vehicle), c is capacity of lanes category (vehicle in hour), S is saturation flow rate in existing conditions (the vehicle per green hour per lane), k *is* additional delay coefficient that is state-dependent control of traffic signal; traffic signals are pre-scheduled 0.5 and this coefficient varies for intelligent traffic signal between 0.4 to 0.5, *I*: upstream filtering/metering adjustment factor is 0.1 for single intersection, *P.F* category movement adjustment factor, *fp* Supplementary adjustment factor for groups of vehicles which arrive to the intersection in green times, p is the proportion of vehicles which arrive to the intersection in time green. *P.F* category movement adjustment factor calculates the effect of coordinated traffic signals with each other and flow controller on process and vehicles' quality of traffic between them that this estimation is based on vehicles' entrance pattern. Progressive adjustment factor is raised more when the distance between the two signalized intersections to be in the extent (less than 3.2 km) that phasing and timing of upstream traffic signals to be effective on the quality of traffic and the flow process of traffic flow that reach to the studied traffic signals. Now if progressive adjustment factor is inappropriate, it indicates that a small percentage of vehicles will arrive to the intersection in green phase and the progressive adjustment factor will be more than one in this case. Likewise, if the progressive adjustment factor less than one, it indicates that vehicles will arrive to the intersection in green phase with appropriate percent (Highway Capacity Manual, 1998).

As mentioned earlier, this formula is used to calculate the delay imposed on vehicles in the study, and also because the vehicles' arrival to the intersection is assumed randomly and has a different statistical distribution, adjustment factor will be equal to 1 for all ratios of green time during the cycle based on the book HCM. It is noteworthy that the type of vehicles' arrival at investigated intersection is AT3.

2. Methodology

The method proposed in this paper is an analytical method based on expectation function method. The vehicles' delay equation in the book "HCM" that was mentioned in the previous section is used to delay calculation imposed on vehicles in signalized intersections. Among the input variables in delay equation HCM (Eq. (1)), three variables of effective green time, saturation flow rate and degree of saturation are identified as random variables. Effective green time is calculated based on Eq. (5). At an intersection with fixed time traffic lights, the waste time in Eq. (5) varies from driver to driver and therefore causes random variations in the effective green time. However, the magnitude and rate of changing the waste time compared with changes in input volumes of vehicles to the intersection is negligible. Saturation flow rate is a random variable which results in the presence of heavy vehicles as well as timed and temporary changes in this variable. However, as the effective green time changes were ignored compared with changes in vehicles' input volumes to the intersection, the changes of flow rate saturation against changes in the vehicles' input volume to the intersection will be also ignored. So, only the random variations in vehicles' input volumes to the intersection are considered in this paper to calculate the delay variation at the intersection. In other words, the only random variable in this paper

IOP Conf. Series: Materials Science and Engineering **245** (2017) 042023 doi:10.1088/1757-899X/245/4/042023

is the vehicles' input volumes to the intersection (V) with assuming the constant green times related to each phase and saturation flow rate.

$$g_i = G_i + Y_i - t_{li} \tag{5}$$

gi is the effective green time related to phase i (second), *Gi* is the actual green time related to phase i (second), *Yi* is the whole yellow time and all red related to phase i (second), t_{li} is the wasted time related to the phase I (second).

Saturation degree of lines' categories entering to the intersection is related to the volume entering to the intersection based on the Eq. (6). Accordingly with assuming knowledge of the distribution, mean and variance of vehicles' entering in lanes' categories leading to the intersection, it can be said that the distribution of saturation degree of in lines' categories entering to the intersection is equal to the distribution of vehicles' volumes entering to the intersection in that path and the average saturation degree in lines' categories entering to the intersection is equal to the average volume entering to the intersection divided by the amount of capacity in that line category which is a constant value with assumptions conducted in this paper. As well as, the variance of saturation degree is obtained by dividing the variance of vehicles' volume entering the intersection divided by the square of capacity. The mentioned issues can be summarized in the following equation:

$$X_i = \frac{V_i}{c_i} \tag{6}$$

$$Average(X_i) = \frac{Average(V_i)}{C_i} \tag{7}$$

$$Var(X_i) = \frac{Var\ (V_i)}{C_i} \tag{8}$$

Where *Xi* is the random variable of saturation degree in lane category i, *Vi* the random variable of vehicles' volume entering to the intersection in lane category i, *ci* the capacity of lane category i.

After estimating the mean and variance and type of frequency distribution of saturation degree random variable (X), the expectation function method can be used to estimate the average intersection delay and also its variations.

Expectation method is an analytical method to overcome the defects of sampling methods. This method involves the expectation function, where each input random variable is known as a random variable which follows specific frequency distribution with certain mean and variance. Since the outputs of expectation function depend on input variables to this function, the output is a random variable that its high order torques are calculated based on the input variable changes. The expectation method can be used to calculate the first torque or higher degrees torques of a variable which is a function of several independent random variables that are in different components together.

Knowing the mean, variance and distribution of a random variable, the expected values of the input variables for the various powers can be calculated of the moment generating function related to it.

Moment generating function of the random variable X which is represented with the symbol M (t) is defined as follows for real values t:

$$\boldsymbol{M(t) = E[e^{tX}]} = \begin{cases} \int_{-\infty}^{+\infty} \boldsymbol{e^{tX} f(X)\, dx} \\ \sum_{x} \boldsymbol{e^{tX} P(X)} \end{cases} \tag{9}$$

That in the above equation, the first equation is related to the continuous random variables and the second equation is for discrete random variables. Since the random variable of saturation degree is discrete random variable, the second equation is used.

M(t) is called the moment generating function, because all the torque X can be obtained through continuous differentiation of M(t) and calculating their amount in t=0. In general, the n-th order derivative of M (t) is as follows:

$$M^{(n)}(t) = E[X^n e^{tX}] \tag{10}$$

That is obtained with assuming t=0 in Eq. (11).

$$M^{(n)}(0) = E[X^n] \tag{11}$$

In which E $[X^n]$ is the expected value of n-th power for random variable X or n-th torque.

It is understood that if other variables assumed constant in HCM delay equation except the random variable of saturation degree (X), a single-variable function of vehicles' entering volumes to the intersection will be obtained. However, it can be noted that the expected values obtained from expectation value method were calculated for using in power functions, so these values couldn't be placed in HCM delay equation directly.

For this purpose, HCM delay equation should be changed to a power equation of random variable with saturation degree X. This can be achieved by using delay estimation via a polynomial that is a function of variable with saturation degree X. For this reason, the Taylor series is used to estimate the delay imposed on the vehicles derived from the HCM delay equation.

Generally, Taylor series for each function such as F(x) is as follows:

$$F(\mathrm{X}) = \mathrm{F}(\mathrm{X}_0) + \sum_{\mathrm{n}=1}^{\mathrm{j}} \left(\frac{1}{\mathrm{n}!} \times \frac{\mathrm{d}^{\mathrm{n}}\mathrm{F}(\mathrm{X}_0)}{\mathrm{d}\mathrm{X}^{\mathrm{n}}} \times (\mathrm{X}-\mathrm{X}_0)^{\mathrm{n}}\right) \tag{12}$$

In which F(x) is a function that we want to estimate it at a certain point and X_0 is the point where we want to estimate the function around it.

If we use the Taylor series (Eq. (12)) in order to estimate the delay of vehicles, using the value X_0 is equal to the average value of saturation degree, which is obtained from average value of the entering volumes entering to the intersection that it seems logical. This leads to be the best estimated value. It should be noted that the delay estimation equation will also change through changing the value X_0.

Calculation of the expected values for X^n depends on the frequency distribution, mean and variance of the entering vehicles entering to the intersection. Expected values for the first and second torques of vehicles' delays at the hypothetical intersection i.e. E(D) and E(D^2), are obtained from the expected values for X^n. The following equation can be expressed by using Eq. (12) and equations related to expected values for the variable X^n.

$$E(D_i) = \sum_{n=0}^{j} a_n\, E({X_i}^n) \tag{13}$$

So, we can calculate the vehicles' delay variance at intersection with following equation.

$$\sigma_{D_i}^2 = E({D^2}_i) - [E(D_i)]^2 \tag{14}$$

In which E (D_i) is the average delay or the first torque of random variable delays in lanes category i-th (seconds per vehicle), E (${D_i}^2$) is the second torque of a random variable delays in line category i-th. After calculating the delay of lanes' category entering the intersection in the proposed method, the average delay of intersection can be obtained by using the Eq. (15) with the help of expectation value function method, [12].

$$E(DI) = \frac{\sum_{i=1}^{r} E(D_i)\times(x_i\times c_i)}{\sum_{i=1}^{r}(x_i\times c_i)} \tag{15}$$

As well as, variance of intersection's delay is obtained as follows by using the Eq. (16).

$$Var(DI) = \frac{\sum_{i=1}^{r} E(D_i)\times(x_i\times c_i)^2}{\left(\sum_{i=1}^{r}(x_i\times c_i)\right)^2} \tag{16}$$

In which E (DI) is average of intersection delay or first torque of random variable in intersection delay (second per vehicle) and Var(DI) is variance of intersection delay.

Knowing the distribution of intersection delays, delay confidence interval leading to the specifying the level of service at the intersection is calculated. The delay confidence interval (C.I) can be calculated from the following equation by using the standard deviation of delay (σ) which is calculated from the square of equation (16), and the average delay (μ) which is obtained from Eq. (15).

$$C.I(DI) = \mu_{DI} \pm \sigma_{DI} \times (p.v) \tag{17}$$

Where in the above equation, p.v is the numeric value which is obtained from the statistical tables depending on the frequency and the desired confidence level.

IOP Conf. Series: Materials Science and Engineering **245** (2017) 042023 doi:10.1088/1757-899X/245/4/042023

3. Results and discussions

After presenting the proposed method, this method is examined by two cycles length: 120 seconds cycle length with green times of 62 and 50 seconds in South-North and East-West inlets, also 110 seconds cycle length with green times of 42 and 40 seconds in South-North and East-West inlets. Normal, Poisson and uniform distributions of entering vehicles to the legs of signalized intersection with two phases were investigated which can be seen in Figure 1. It should be noted that in this study it is assumed that vehicles' entering to the intersection in one of the inlets follows the Poisson distribution. Figure 2 show the process of intersection delay variation considering the cycle length of 120 and 110 seconds, respectively. Table 1 shows the traffic specifications of inlet legs of the investigated intersections.

Table 1. Traffic specifications of inlet legs of the investigated intersections

Variance of saturation degree in East-West route	Saturation degree in East-West route	Variance of saturation degree in South-North route	Saturation degree in South-North route
0.08	0.33	0.05	0.35
0.08	0.47	0.05	0.52
0.08	0.61	0.05	0.64
0.08	0.70	0.05	0.70
0.08	0.74	0.05	0.76
0.08	0.87	0.05	0.87
0.08	0.99	0.05	0.99

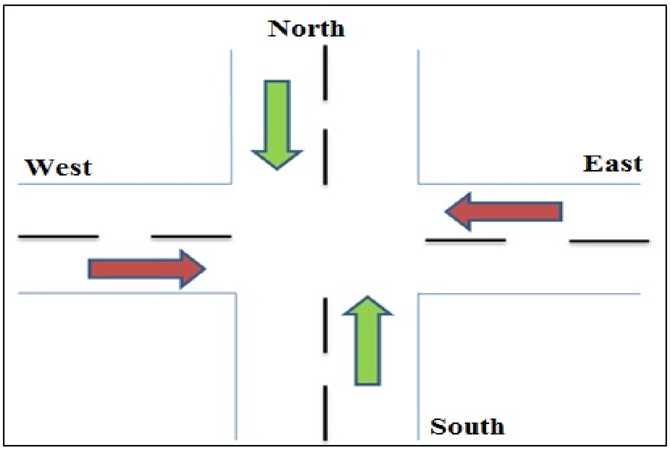
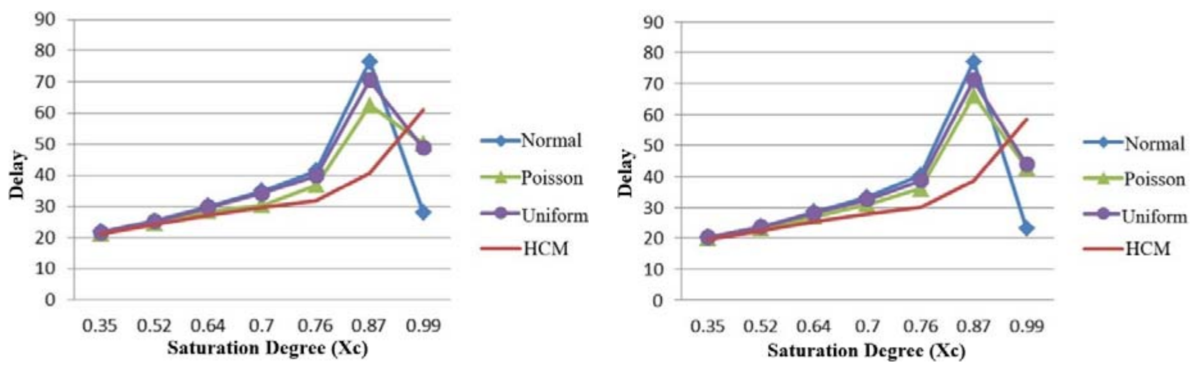


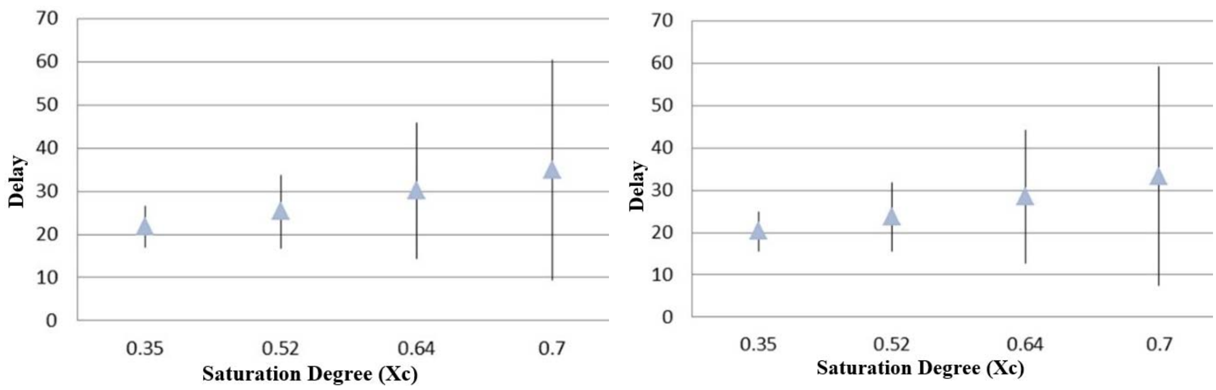
Figure 1. The investigated hypothetical intersection.

As seen in Figure 2, in the average of low saturation degree in signalized intersection the proposed method provides responses close to the common method of calculating the intersection delay which is used in HCM book and this difference is more visible in saturation degree over 0.8. The amount of intersection delay varies by changing the model of vehicles' entering to the intersection, in a way that the distribution of normal entering in saturation degree less than 0.9 comes to the maximum delay at the intersection and then uniform and Poisson entering distributions have the next ranks. Another clear point is that the increase of average delay at the intersection is due to increasing the cycle length. Some interval changes of the studied intersection delay at confidence level of 95% in average condition of less than 0.7 saturation degree in both cycle lengths of 120 and 110 seconds are given in Figure 3.



a) The 120-seconds cycle *b) The 110-seconds cycle*

Figure 2.*Changes of intersection delay corresponding to vehicles' different distributions entry to intersection during cycle time*



a) The 120-seconds cycle *b) The 110-seconds cycle*

Figure 3. *The interval variation of intersection delay for normal-Poisson entering distribution*

As it is clear in Figure 3, the interval variation of intersection delay will increase by increasing the average saturation degree in intersection. The level of service at the intersection can be obtained by specifying the interval variation of intersection delay. For example, the delay varies in the range of 10 seconds to 60 seconds in the average saturation degree of 0.7 during 120 seconds cycle length (Figure 3a); as a result, this intersection acts in three level of services B, C and D.

Six different intervals based on vehicles' average delay at the intersection have been defined in the HCM to determine 6 level of services at the intersection according to Table 2.

Table 2. Level of services corresponding to delay at the intersection (Highway Capacity Manual, 2010)

Level of service	The average delay (seconds per vehicle)
A	Less than 10
B	10-20
C	20-35
D	35-55
E	55-85
F	More than 80

However, delay at intersection is a random variable and this emphasizes that the delay has values which varies around its average value based on the type of frequency distribution and its variance. Thus, intersection should act on several levels of services; the probability of specific level of service at intersection can be measured through having the delay average value and variance. For example, during 120-seconds cycle length, if the value of average delay [E (DI)] is equal to 34.988 seconds per vehicle in average saturation degree of 0.7 and at normal-Poisson distribution for entering legs to the intersection and intervals variation of intersection delay is between 9.49 to 60.48 seconds per vehicle, the graph of the delay probability function is seen in Figure 4 with assuming the normal distribution for the delay.

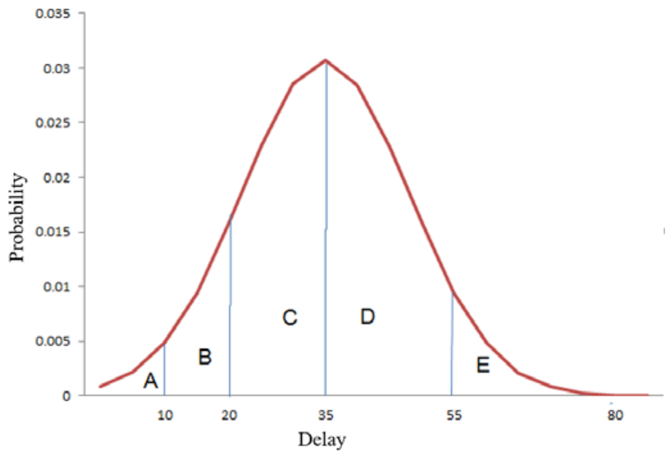


Figure 4. Changes interval in level of services

The interval delay variation appears that this intersection acts in the given specifications at level of services A, B, C, D and E. But as it is clear in table 2, the probability of occurring these level of services is not equal to each other in such a way that the probability of occurring each level of services is as Table 3.

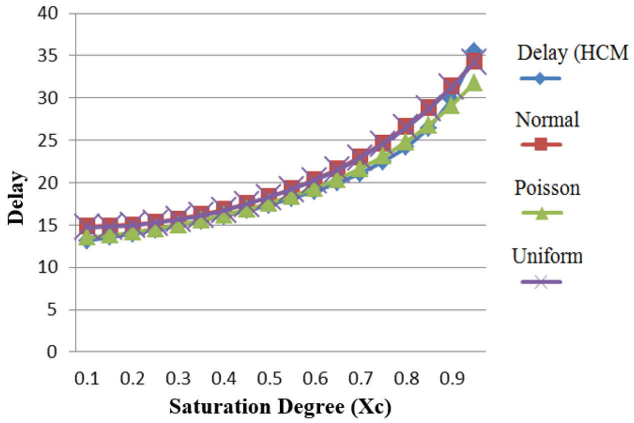
Table 3. Probabilities of occurrence in different level of services

Level of services	The average delay (seconds per vehicle)	Probability of occurrence
A	Less than 10	0.062
B	10-20	0.159
C	20-35	0.429
D	35-55	0.320
E	55-85	0.03

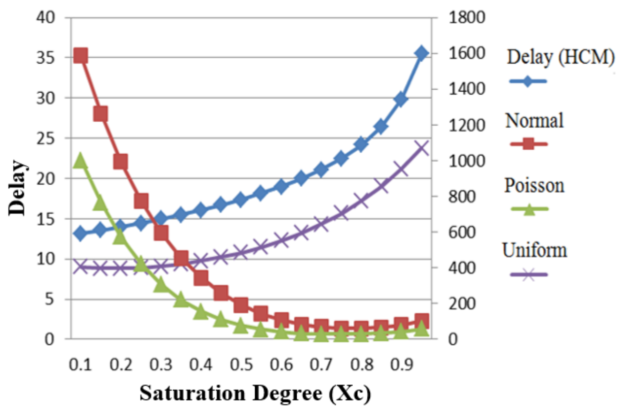
In order to analyze the effect of various distribution of the vehicles at nearside legs of intersection on the delay estimation function, three different input distribution of normal, Poisson and uniform are considered for the vehicles. In addition, delay estimation function for three different input distributions were analyzed. For instance, the diagram of three average saturation s of 0.5, 0.85 and 0.9 are presented to evaluate their effect on delay estimation function. The results for the average saturation of 0.5, 0.85, 0.9, and various delay distributions are as follows figure 5.

After lots of analyses and with regard to Figure 5, the delay estimation function of vehicle properly predicts vehicles' delay low degree of saturation (less than 0.8), but as the value of degree of saturation increases this function predicts values higher than the value obtained by HCM delay function. As previously stated and it is clear from the figures at low degree of saturation the delay estimation function predicts values close to values estimated by HCM function and as the value becomes greater the difference becomes wider.

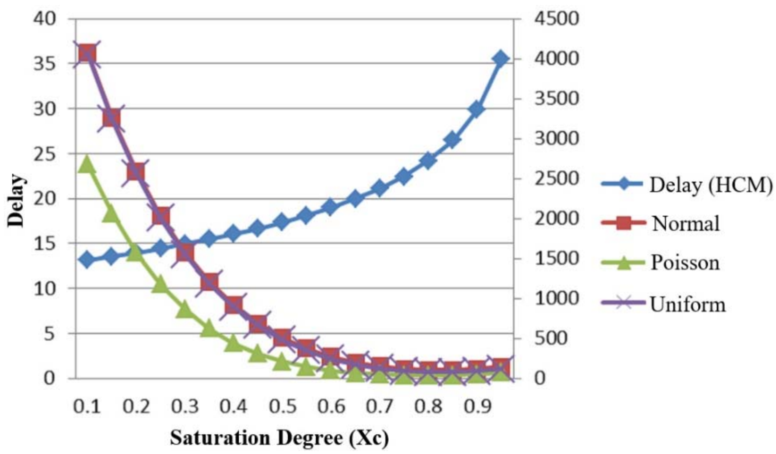
At average saturation of less than 0.8, there is significant difference between different input types the amount of estimated delay but the delay caused by the entry of Poisson of the nearside legs vehicle in to the estimated delay, it is closer than the HCM equation and then the normal and uniform distribution predict values closer than the values obtained by HCM. In the average high saturation levels (more than 0.8 and less than 0.9), the difference between the estimated values by HCM is higher than the delays estimated in low average saturations. In this average levels of saturations, by increasing the saturation level, the difference between the estimated delay and obtained delay by HCM is reduced in low degrees of saturation. At this average saturation, the least difference between the estimated and calculated values by HCM equation is achieved by uniform distribution and then it is obtained by Poisson and the normal finally.



a) Average Saturation X=0.5



b) Average Saturation X=0.85



c) Average Saturation X=0.90

Figure 5. Delay estimation under different input distribution C = 93 and g = 45 s and average saturation

At high average saturation (more than 0.9), the difference between the estimated values of delay resulted by various frequency distributions and the estimated value by HCM is much more than the estimated delay at average saturation lower than 0.8 and 0.9. In this average levels of saturations by increasing the saturation level, the difference between the estimated delay and obtained delay by HCM is reduced as in low degrees of saturation. At this average saturation, the least difference between the estimated and calculated values by HCM equation is achieved by uniform distribution and then it is obtained by Poisson and the normal distribution finally.

IOP Conf. Series: Materials Science and Engineering **245** (2017) 042023 doi:10.1088/1757-899X/245/4/042023

4. Conclusions

The proposed method in this paper provides more appropriate responses that close to the intersection delay calculation methods such as HCM method in average saturation degree less than 0.8 that calculates the delay only by considering average number of vehicles' volumes entering to the intersection. In addition to considering the average values of saturation degree in entering routes to the intersection by this method, the more real delay values imposed on vehicles are obtained at the intersection considering the distribution and variance of vehicles' entering to the intersection; in a way that the old simplified assumptions based on uniform distribution of entering vehicles to the intersection can be modified and there will be the exclusive delay function related to the it's specific type of entering vehicles distribution.

Also it is seen, there is no certain level of service for intersection with the help of this method, however, it results in several level of services with probability of occurring each one for intersection that this will help engineers a lot to better and more operational understanding of the signalized intersections and the better design of traffic lights' timing on them.

References

[1] I. Bargegol, M. Nikookar, R. N. Vatani, E. Jafarpour, A. M. Roshandeh, "Timing Optimization of Signalized Intersections Using Shockwave Theory by Genetic Algorithm," Computational Research Progress in Applied Science & Engineering. Pearl Publication 1(4): 160-167, 2015.

[2] Transportation Research Board, Highway Capacity Manual, National Research Council, Washington D.C., 2010.

[3] G. F. Newell, "Statistical Analysis of the Flow of Highway Traffic through a Signalized Intersection," Quarterly of Applied Mathematics, 13, 353-368, 1956.

[4] F. V. Webster, "Traffic Signal Settings," Road Research, Technical Paper 39. Road Research Laboratory, Her Majesty's Stationery Office, Berkshire, England, 1958.

[5] A. J. Miller, A.J., "Australian Road Capacity Guide Provisional Introduction and Signalized Intersections," Australian Road Research Board, Bulletin 4, Vermont South, Australia, 1968.

[6] V. R. Hurdle, "Signalized Intersection Delay Models – A Primer for the Uninitiated," *Transportation Research Record., TRB, National Research Council,* Washington, D.C., 96-105, 1984.

[7] R. Akcelik, "Time-dependent Expressions for Delay, Stop Rate and Queue Length at Traffic Signals," *Internal Report AIR 367-1. Australian Road Research Board*, Vermont South, 1980.

[8] ITE, Canadian Capacity Guide for Signalized Intersections, 2th ed. Institute of Transportation Engineers (ITE), District 7, (ed. Teply, S.) Canada, 1995.

[9] TRB, Highway Capacity Manual, TRB Special Report 209, 3th ed. (1997 update), Transportation Research Board, National Research Council, Washington D.C., USA, 1998.

[10] Transportation Research Board, Highway Capacity Manual, National Research Council, Washington D.C., 2000.

[11] D. B., Fambro, N. M, Rouphail, Generalized Delay Model for Signalized Intersections and Arterial Streets. Highway Capacity Issues and Analysis, *Transportation Research Record 1572, Transportation Research Board*, 112–121, Washington, D.C, 1997.

[12] I. Bargegol, F. Jamshidpour, V.N.M. Gilani, "Estimating Delay of Vehicles in Nearside Legs of the Signalized Intersections under Expectation Method in Under Saturation Conditions for Isolated Intersection," *Journal UMP Social Sciences and Technology Management.* Vol. 3(3): 910-916, 2015.