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Impact of the Mathematical Model Description on the Assessment of the Reliability of Structural Elements

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Abstract. The study presents a probabilistic approach to the problems of static analysis of structural elements. A steel and reinforced concrete elements were analysed. Structural design parameters were defined as deterministic values and random variables. The latter were not correlated. The criterion of structural failure is expressed by limit functions related to the ultimate limit state. In the performed analyses explicit form of the random variables function were used. The Hasofer-Lind index was used as a reliability measure. In the description of random variables were used the different types of probability distribution appropriate to the nature of the variable. Sensitivity of reliability index to the random variables was defined. If the reliability index sensitivity due to the random variable X_i is low when compared with other variables, it can be stated that the impact of this variable on failure probability is small. Therefore, in successive computations it can be treated as a deterministic parameter. Sensitivity analysis leads to simplify the description of the mathematical model, determine the new limit functions and values of the Hasofer-Lind reliability index. Besides the effect of the assumed level of the variation coefficient of selected random variables on value of the reliability index was determined. The primary research method is the FORM method. In order to verify the correctness of the calculation SORM, Monte Carlo and Importance Sampling methods were used. In the examples of reliability analysis the NUMPRESS program was used. In the considered issues the time was not taken into account explicitly.

1. Introduction

Uncertainties in specifying material properties, geometric parameters, boundary conditions and applied loadings are unavoidable in describing real engineering structural system. Traditionally, this has been catered for through the use of safety factors at the design stage. The studies proposed to treat building structure reliability as a random event and to analyse it with probability calculus methods. On the basis of these concepts, a semi-probabilistic method of limit states, using nowadays in Eurocodes, was developed.

The most advanced reliability analysis methods are probabilistic methods. They allow quantitative assessment of structure reliability. In these methods, information on types of distribution of design variables and distribution parameters is employed. Such formulation makes it possible to explicitly account for randomness in the design process. As a result, it is possible to construct a mathematical model which allows estimation of the probability of a certain structure performance.

The practical methods for reliability calculations can be broadly divided into approximate analytical methods like FORM (First Order Reliability Method) and SORM (Second Order Reliability Method) simulation based methods, for example, Importance Sampling and Monte Carlo. In each case the efficiency and applicability of a particular methodology largely depends on the efficient computation of the so-called design point. The design point and the 'region' around it contains the most important information regarding the probability of failure of structure. The calculation of the design point requires the solution of a constrained optimization problem. The concept of localization design point rely on realization of random variables from the failure surface which corresponds to the greatest value of the probability density function. With the linearization of the limit state function at the design point, it is possible to obtain a measure of reliability which is invariant due to the equivalent formulations of the boundary condition, i.e. the so-called Hasofer-Lind reliability index. In the reliability analysis of a complex engineering structure a very large number of the system parameters can be considered to be random variables.

Nowadays, the structural reliability theory is already a well-established research area. One can mention a number of textbooks and monographs, the most well-known being: Madsen, Krenk and Lind [1], Melchers [2], Ditlevsen and Madsen [3], Thoft-Christensen and Baker [4] Augusti, Baratta and Casciati [5]. Special attention should be paid to publications by Harr [6], Nowak and Collins [7]. They present the basic concepts of reliability theory with a particular reference to their uses in civil engineering. Interesting works are [8], [9], [10] where numerical aspects of application of first order reliability method FORM in analysis of truss and frame structures are considered. Different reliability assessment of structure propose work [11] about system reliability using serial and parallel systems. In order to determine the reliability of this approach it is necessary to set KAFM (kinematic admissible failure mechanism). The KAFM by spectral analysis of stiffness matrix are specified [12], [13]. Reliability issues are important not only in static analysis, but also in stability analysis or dynamic analysis [14].

In the present study, simulation methods were only used to validate the correctness of computations. The basic method used in the study was FORM, which is one of approximation methods. The so-called Hasofer-Lind [15] reliability index β was adopted as a reliability measure. A great advantage of the FORM method is that it allows computing the sensitivity of the reliability index to a change in arbitrary parameters that are found in the problem description practically without additional computations. The sensitivity of the reliability index is computed as a first derivative of β index with respect to a specified variable.

2. Investigation methodology

In the example presented below, the NUMPRESS software [16] was employed for the reliability analysis. The software was developed by team of scientists from the Institute of Fundamental Technological Research of the Polish Academy of Sciences. With the NUMPRESS software, reliability analysis starts with a construction of the computational model. The software user gives the parameters of boundary probability distributions of random variables. In the present software version, uniform, normal, log-normal, exponential, Rayleigh, Gumbel, Frechet and Weibull probability distributions are accepted for the description of random variables.

After defining the computational model, the user introduces the formula of the limit function in the standard mathematical notation as a dependence on basic and external random variables. In the study, one type of condition responsible for the ultimate limit state is considered. The next stage involves selecting the method of reliability analysis and starting the computations. The problem is concluded with generating the information that provides the values of failure probability and its sensitivity to the parameters of probability distributions of random variables.

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In this paper, the author present proposals to explicitly take into account the randomness of design parameters and quantitative assessment of the probability of structural elements specific behaviour. The Hasofer-Lind reliability index was estimated. Sensitivity of reliability index to the random variables was defined. Besides the effect of the assumed level of the variation coefficient of selected random variables on value of the reliability index was determined.

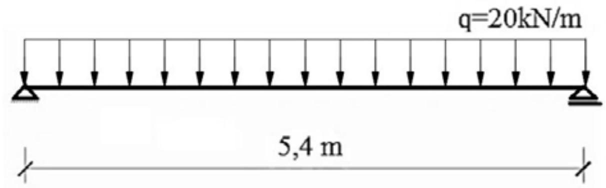
3. Aim of study

The study presents a probabilistic approach to the problems of static analysis of structural elements. The aim of study is using probabilistic methods in reliability analysis of structural elements made of different materials. A steel and reinforced concrete elements were analyzed. The criterion of structural failure is expressed by limit function related to the ultimate limit state. In the performed analyzes explicit form of the random variables function were used.

3.1. Steel beam

The reliability analysis was subjected steel beam IPE270. Structural element is simply supported beam loaded of uniform load q=20 kN/m (figure 1). The upper flange of the beam is based reinforced concrete floor slab prevents lateral displacement. So that the beam is secured against lateral-torsional buckling. Detailed of geometric parameters and material is shown in figure 1.

a)



- Material and geometrical parameters of IPE270
  Steel S235
  E=210 GPa,
  f_y=235 MPa,
  $W_{pl,y}=4.84\cdot10^{-4}\,m^3$,

b)

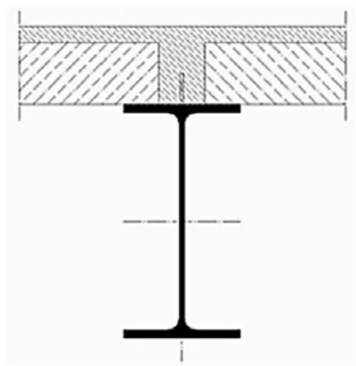


Figure 1. Geometry of steel beam a) static schema and parameters, b) cross-section

Structural design parameters are defined as the random variables. Random variables are not correlated. Below, probabilistic quantities are specified:

X_1 – uniform loading (q),

X_2 – length of beam (L),

X_3 – section modulus of beam ($W_{pl,y}$),

X_4 – yield point for S235 steel (f_y),

Description of random variables was shown in table 1.

In the analysis distribution of variables appropriate to the variable nature was proposed. The values of coefficients of variation were selected on the basis of statistical studies of the building strength and buildings materials and products from work [17]. For variables with load range coefficients of variation were adopted in accordance with recommendations of [18].

Table 1. Description of the random variables for steel beam.

Random Variable (X_i)	Probability density function	Mean value (μ_X)	Standard deviation(σ_X)	Coefficient of variation (ν_X)
X_1	Gumbel	20 kN/m	2 kNm	10%
X_2	Normal	5.4 m	0.0054 m	0.1%
X_3	Log-normal	$4.84 \cdot 10^{-4}$ m^3	$0.242 \cdot 10^{-4}$ m^3	5%
X_4	Log-normal	$235 \cdot 10^3$ kN/m^2	$11.75 \cdot 10^3$ kN/m^2	5%

The limit function as the condition of the non-exceeding of the section bearing capacity during load of bending moment was formulated. The formula for the maximum bending moment:

$$\mathrm{M_{Ed}} = \frac{X_1 {X_2}^2}{8} \tag{1}$$

as a function of random variables grouped in vector: $\mathbf{X}=\{X_1, X_2, X_3, X_4\}$ was determined.

Limit functions were formulated, which describe ultimate limit state

$$\mathrm{G_1}(\mathbf{X}) = 1 - \frac{\mathrm{M_{Ed}}}{\mathrm{M_{c,Rd}}} \quad \Rightarrow \quad \mathrm{G_1}(\mathbf{X}) = 1 - \frac{X_1 X_2^2}{8 X_3 X_4} \tag{2}$$

where: for the first class elements $\mathrm{M_{c,Rd}} = \mathrm{M_{pl,Rd}} = \frac{W_{pl,y} f_y}{\gamma_{M0}}$; $\gamma_{M0}=1$.

3.2. Reinforced concrete beam

The reliability analysis was subjected reinforced concrete beam. Structural element is simply supported beam loaded of uniform load q=20 kN/m (figure 2). Detailed of geometric parameters and material is shown in figure 2.

a) b)

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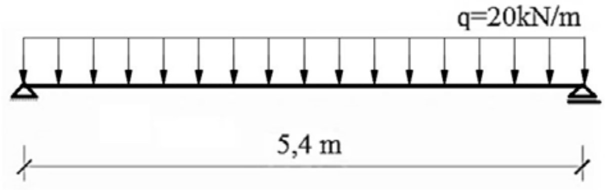

Material and geometrical parameters

- Concrete C16/20
  f_{cd}=11.4 MPa,
- Steel – B Class
  f_{yd}=347.8 MPa,
  E_s=200 GPa,

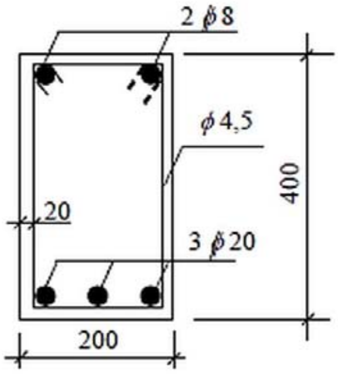

Figure 2. Geometry of reinforced concrete beam a) static schema and parameters, b) cross-section

Structural design parameters are defined as the random variables. Random variables are not correlated. Below, probabilistic quantities are specified:

X_1 – uniform loading (q),

X_2 – length of beam (L),

X_3 – section width (b),

X_4 – section hight (h),

X_5 – cross section of steel in tension (A_{s1}-3ø20),

X_6 – compressive strength of concrete (f_{cd}),

X_7 – yield point for steel (f_{yd})

Description of random variables was shown in table 2.

In the analysis distribution of variables appropriate to the variable nature was proposed. The values of coefficients of variation were selected on the basis of statistical studies of the building strength and buildings materials and products from work of [25]. For variables with load range coefficients of variation were adopted in accordance with recommendations of [26].

Table 2. Description of the random variables for reinforced concrete beam

Random Variable (X_i)	Probability density function	Mean value (μ_X)	Standard deviation(σ_X)	Coefficient of variation (ν_X)
X_1	Gumbel	26 kN/m	2.6 kN/m	10%
X_2	Normal	5.4 m	0.027 m	0.5%
X_3	Normal	0.2 m	0.002 m	1%
X_4	Normal	0.4 m	0.004 m	1%
X_5	Normal	$9.43\cdot10^{-4}$ m^2	$9.43\cdot10^{-6}$ m^2	1%
X_6	Log-normal	$11.4\cdot10^3$ kN/m^2	$0.57\cdot10^3$ kN/m^2	5%
X_7	Log-normal	$347.8\cdot10^3$ kN/m^2	$17.39\cdot10^3$ kN/m^2	5%

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Limit function as the condition of the non-exceeding of the section bearing capacity during load of bending moment was formulated. The formula for the maximum bending moment:

$$\mathrm{M_{Ed}} = \frac{X_1 X_2^{\;2}}{8} \tag{3}$$

as a function of random variables grouped in vector : $\mathbf{X}=\{X_1, X_2, X_3, X_4, X_5, X_6, X_7\}$ was determined. Limit functions were formulated, which describe ultimate limit state

$$\mathrm{G}_2(\mathbf{X}) = 1 - \frac{\mathrm{M_{Ed}}}{\mathrm{M_{Rd}}} \quad \Rightarrow \quad \mathrm{G}_2(\mathbf{X}) = 1 - \frac{X_1 X_2^2}{8 X_5 X_7 \left[(X_4 - 0.0345) - \dfrac{X_5 X_7}{2 X_3 X_6} \right]} \tag{4}$$

where: load capacity of rectangular section without taking into account the reinforcement in the compressed zone is describe by $\mathrm{M_{Rd}} = b x_{eff,\lim} \eta f_{cd} \left(d - \frac{x_{eff,\lim}}{2} \right)$ and $x_{eff,\lim} = \frac{A_{s1} f_y}{b \eta f_{cd}}$; $d = h - d_1$.

4. Results and discussion.

The value of the Hasofer-Lind reliability index was determined with the FORM method, and for the sake of comparison, with other methods, i.e. SORM, Monte Carlo and Importance Sampling. The results for steel beam and reinforced concrete beam are presented in table 3.

Table 3. Values of the Hasofer-Lind reliability index.

Type of element	FORM	SORM	Importance Sampling	Monte Carlo
Steel beam	3.06	3.06	3.04	3.05
Reinforced concrete beam	2.27	2.26	2.20	2.28

The relative error of reliability index was estimated with the assumption that the reference is the Monte Carlo method (table 4).

Table 4. Values of relative error computing the Hasofer-Lind reliability index

Type of element	FORM	SORM	Importance Sampling
Steel beam	0.3%	0.3%	0.3%
Reinforced concrete beam	0.4%	0.9%	3.5%

In addition, graphs were provided that show the sensitivity of the reliability index to random variables for a steel beam (figure 3a) and for a reinforced concrete beam (figure 3b).

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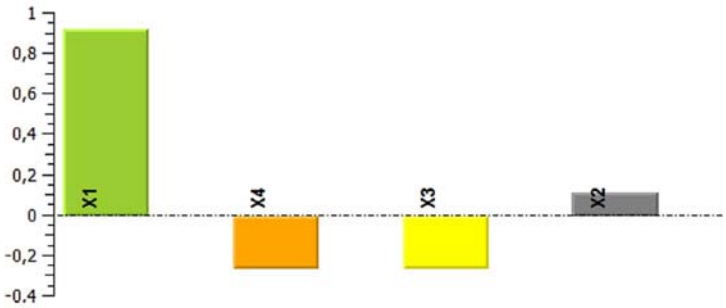
a)



b)

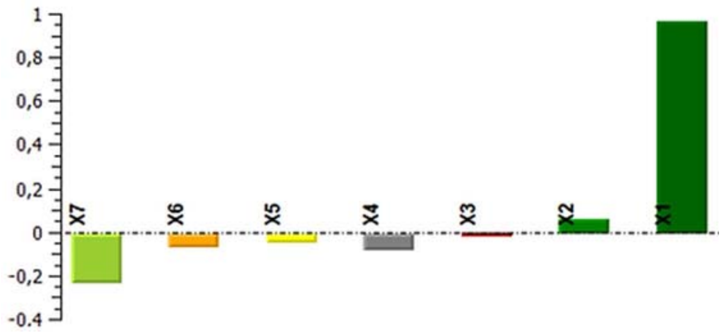


Figure 3 Sensitivity of the reliability index β to random variables for a) steel beam, b) reinforced concrete beam

The graphs in figures 3 define the impact of individual random variables on value of reliability index. The sensitivity analysis allows for identification of random variables which have the smallest impact on the value of reliability index. The next stage of the analysis relied on defining of limit function for the ultimate limit state again. The random variables characterized by the lowest sensitivity for value of reliability index was treated as deterministic parameter. For the steel beam the lowest sensitivity has variable X_2 and for the reinforced concrete beam variables: X_2, X_3, X_5. After reducing the description of the mathematical model the limit functions have the form:

- for steel beam

$$\mathrm{G}_1^{\mathrm{red}}(\mathbf{X}) = 1 - \frac{729X_1}{200X_3X_4} \tag{5}$$

- for reinforced concrete beam

$$\mathrm{G}_2^{\mathrm{red}}(\mathbf{X}) = 1 - 3865.32\frac{X_1}{X_7\left[(X_4 - 0.0345) - 2.3575\cdot10^{-3}\dfrac{X_7}{X_6}\right]} \tag{6}$$

For reduced limit functions the values of reliability index by the FORM method was estimated. The resulting values were compared with the results for the initial description of the mathematical model. The results in table 5 are shown.

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Table 5. Values of the reliability index estimated by the FORM method for limit functions before and after reduction

Type of element	G_i (initial description)	G_i^{red} (after reduction)
Steel beam	3.06	3.08
Reinforced concrete beam	2.27	2.28

Besides the graphs (figure 3) show which random variables have the highest impact on the value of reliability index. In both cases the highest influence on values of reliability index has random variable X_1 which describes uniform loading. In the next part of analysis the effect of the assumed level of the variation coefficient of random variable X_1 on value of the reliability index was determined. For this purpose, six cases were further analysed. The description of the random variables from table 1 (steel beam) and from table 2 (reinforced concrete beam) was taken, except the level of the random variable of X_1. For these random variable the following values were assumed: σ_x={4%, 6%, 8%, 12%, 14%, 16%}. The results of the analysis in figure 4 are shown.

The analysis of the results demonstrates that the FORM method is good enough and much simpler to apply. The maximum relative error compare to Monte Carlo method amounted to 0.3% for the steel beam and to 3.5% for the reinforced concrete beam.

Another important component of the study was to investigate the sensitivity of the reliability index to changes in probabilistic characteristics of the random variables under consideration. If the reliability index sensitivity due to the random variable X_i is low when compared with other variables, it can be stated that the impact of this variable on failure probability is small, and in successive computations it can be treated as a deterministic parameter. In our consideration the random variables characterized by the lowest sensitivity for value of reliability index was treated as deterministic parameter. Maximum relative error between values of the reliability index β for the description of mathematical model before and after sensitivity analysis is 0.7% (table 5). So sensitivity analysis show that not all parameters describing the mathematical model of the structure should be considered as random variables.

Important part of analysis is the effect of the assumed level of the variation coefficient of random variable X_i on value of the reliability index. Based on the tests, we can see that a change of the variation coefficient compared to the output values (σ_X=10%) generates significant changes in the reliability index (figure 4). For example, for the steel beam and the coefficient of variation σ_X=10%, the reliability index is β=3.06, and for σ_X=4% the value of the index increases by 52% (β=4.64), whereas for σ_X=16% value of the index decreases by 23% (β=2.36). For the reinforced concrete beam and the coefficient of variation σ_X=10%, the reliability index is β=2.27, and for σ_X=4% the value of the index increases by 65% (β=3.74), whereas for σ_X=16% value of the index decreases by 26% (β=1.68). These analysis show how important in reliability analysis is appropriate level of the variation coefficient especially those random variables which impact on the value of reliability index is greatest.

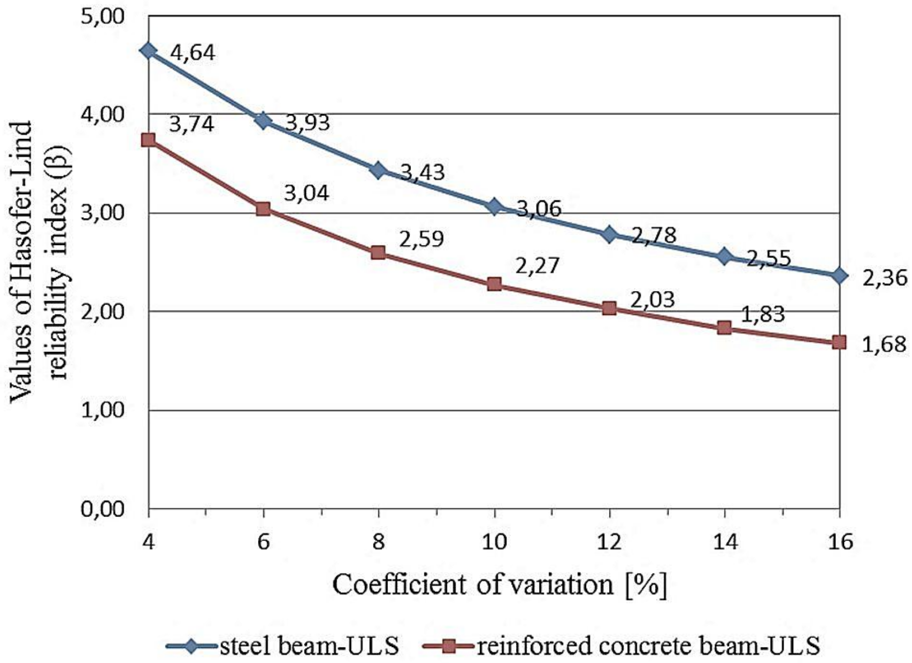

Figure 4 Effect of the assumed level of the coefficient of variation for random variable X_1 on value of the reliability index

5. Conclusions

The analysis of the results demonstrates that the FORM method is sufficiently precise and authoritative research method. The FORM method allows obtaining a quick response, which makes it possible to use the method in engineering practice as one of the modules of computational software that support structure design. In order to precisely modelling of the real work of structure should therefore apply probability distributions of random variables appropriate to their nature. Sensitivity analysis is an important element in the assessment of the impact of random variables on the reliability index value and thus on the factors which determine the safety of the structure. If the reliability index sensitivity due to the random variable X_i is low when compared with other variables, it can be stated that the impact of this variable on failure probability is small, and in successive computations it can be treated as a deterministic parameter. Besides we should carefully to take level of the coefficient of variation especially those random variables which impact on the value of reliability index is greatest. So level of the coefficient of variation should be taken on the basis of statistical studies of the building strength and buildings materials and products.

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