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Forecasting of Congestion in Traffic Neural Network Modelling Using Duffing Holmes Oscillator

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Abstract. Forecasting of congestion in traffic with Neural Network is an innovative and new process of identification and detection of chaotic features in time series analysis. With the use of Duffing Holmes Oscillator, we estimate the emergence of traffic flow congestion when the traffic load on a specific section of the road and in a specific time period is close to exceeding the capacity of the road infrastructure. The orientated model is validated in six locations with a specific requirement. The paper points out the issue of importance of traffic flow forecasting and simulations for preventing or rerouting possible short term traffic flow congestions.

1. Introduction

When predicting features of short-term traffic flow, scholars have used prediction models proven in other theories. Today, there are already almost twenty of these prediction methods.

At the earliest stage three models were used the most – moving average model, autoregressive moving average model and regressive model. These models were considering traffic flow impact elements in a very simple manner. Parameters were usually derived from least square real-time estimate, which in calculations can be simply derived and it is suitable for real-time data update. But still, these prediction models could not reflect non-linear relations in traffic flow system, and impact of some other elements in the system was not sufficiently assessed. So, its disadvantages are pretty clear. That is, when prediction data intervals become shorter, the predictive precision would be largely influenced by these shortcomings.

It is exactly for this reason, that scholars have developed more complex and more precise prediction models and methods.

These new models and methods can be divided into two categories:

1. Methods based on certain mathematical models: multi□element regressive model [1], ARIMA (Autoregressive integrated moving average)[2] model, associative model of self-adaptive load sum, Kalman filter model [3], model of reference function and smooth component [4], as well as combined prediction models made out of these methods and models.

2. Non□model calculation methods: non□parameter regression, spectrogram analysis, model of space reconstructions [5], microwave network[6], multi□dimensional fractal method, and other prediction models based on micro□wave analysis and reconstruction related to neural network [7].

2. Issues with traffic flow predictions based on nonlinear dynamic systems

Determining chaotic state of traffic flow is a crucial place for short-term predictions of traffic flow chaotic phenomena, because chaotic state is very sensitive of initial state, and we cannot conduct long-term predictions, but in short terms they can be predicted with precision.

Currently, there are methods, which are based on Lyapunov exponent [8], microwave neural network, global method, partial method and other mentioned traffic flow short-term prediction methods based on resolution rate, microwave analysis and reconstruction. Researchers [9] used global and partial methods for conducting traffic flow predictions, including first order linear global method, high order polynomial global method, orthogonal polynomial global method and zero order partial method, first order partial method as well as high order partial method. Others used biggest Lyapunov exponent to predict chaotic traffic flow with flow rate sequence.

These methods, despite being proven in practice as good tools, still have a lot of obvious issues. In theory, global polynomial prediction modelling is feasible, but the process of building uses offline methods, so if embedded dimension is too high or the system is complex, this method is hard to conduct and its prediction accuracy is very low. Compared to global method, partial method is in many cases feasible.

3. Link between chaos theory and nonlinear dynamic systems

Chaotic system is a very complex and singular system. There is still not much knowledge of it, so there are many definitions of chaos. These definitions start from different angles in description of nonlinear dynamic system nature. Since there are many definitions, presentation of all would be to time consuming, but only those, which have representative chaos-mathematical definitions.

In interval, I, continuously self-mapping f(x), if it completes the following conditions

a. periodic point of period has no bounds

b. in closed interval, I, incomputable sublevel s, fulfil for random

$$x, y \in S, when\ x \neq y, limn \rightarrow \infty \sup|fn(x)\text{-}fn(y)| > 0; fx, \tag{1}$$

$$x, y \in S, when\ x \neq y, limn \rightarrow \infty sup\ /fn(x)\text{-}fn(y)\ /\ > 0; \tag{2}$$

$$x, y \in S, limn \rightarrow \infty\ inf\ /fn(x)\text{-}fn(y)\ /\ = 0; \tag{3}$$

$$\text{Any periodic point y of x}\in\text{S and f, } limn \rightarrow \infty sup\ |\ fn(x)\text{-}fn(y)\ |\ > 0 \tag{4}$$

Then we can conclude f(x) as non-linear dynamic system,

Li-Yorke theorem [10]: set f(x) as continuous self-mapping on [a,b], if f(x) has 3 periodic points, then for any positive integer n, f(x) has n periodic points.

According to the above theory and theorem, when for continuous function f(x) on closed interval I, there are 3 periodic points for one period, there is for certain some positive integer of periodic point, of existing nonlinear dynamic system phenomena.

4. Dynamical characteristics of Duffing's system

Duffing's oscillator holds an important place in non□linear dynamics research. Form of equation for Duffing's oscillator is simple, but it can derive many characteristics of a nonlinear state. This is because Duffing has added on the right side of equation an item of additional force, which for result has system's intrinsic frequency and frequency of additional force frequency interacting.

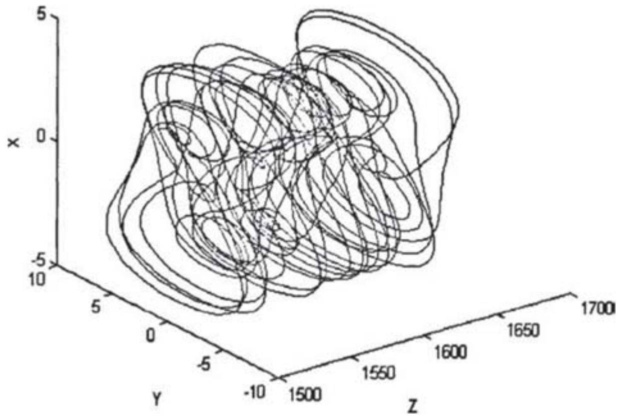

Figure 1: Duffing attractor

We have chosen Holmes model of Duffing's oscillator as model for identifying the chaotic patterns. Its equation is as follows:

Set fd as a critical value of the system, fd/k analytical value as a constant。 Experiments have proven that damping ratio k ranges from 0.2 □ 0.5. Here, we have selected k=0.5 and equation for the state above is as follows:

$$\ddot{x} + k\dot{x} - x^3 + x^5 = \gamma\cos(\omega t) \tag{5}$$

In equation, γ is the range of periodic driving force, while k is damping ratio and –x3+x5 is non□ linear resilience. We can establish virtual model of Duffing's dynamic system.

$$\begin{cases} \dot{x} = \omega y \\ \dot{y} = \ \omega - [-ky + x^3 + x^5 + rcos(\omega t)] \end{cases} \tag{6}$$

γcos(t) is the driving force. As figure 4.2 shows, to simply explain its operating principle, it takes system's frequency. In figure 4.2 system, take damping ratio as k=0.5. When k is fixed, state of the system will regularly change according to the change of γ – state of homoclinic orbit, bifurcation orbit, chaotic orbit, critical period orbit, orbits of huge periods. Time-domain waveform and phase plane orbits of various states of this system are shown in figures 4.2 – 4.8. Analysing above systems' time□domain waveform and planary orbits we can see:

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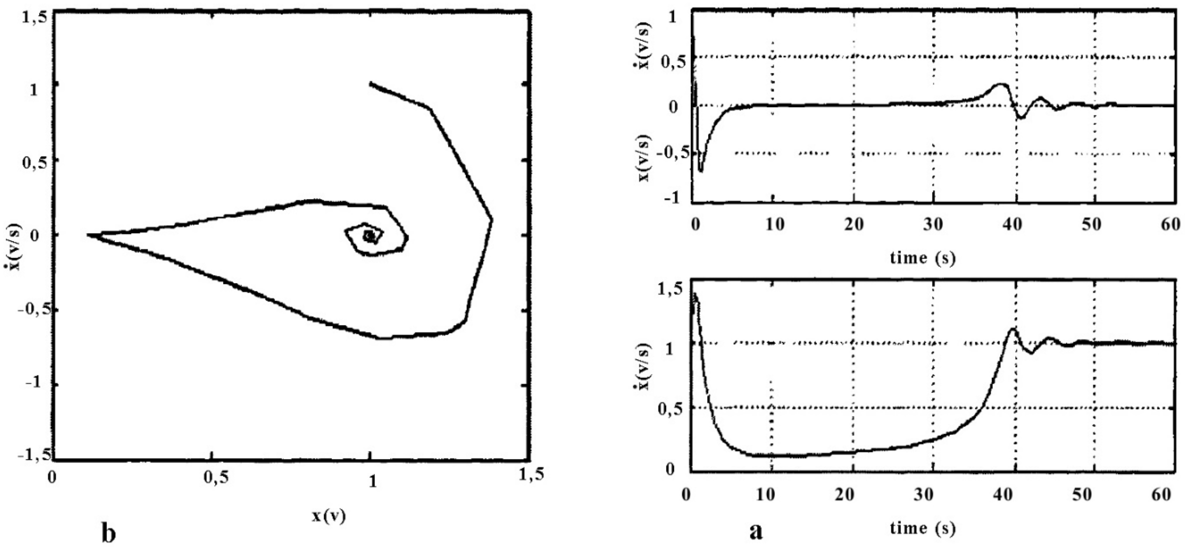

Figure 2: When γ=0, saddle point for system's phase plane is (0,0) and focal point is (±1,0). Point (x, x) should in the end stop between two focal points (as shown in figure 2).

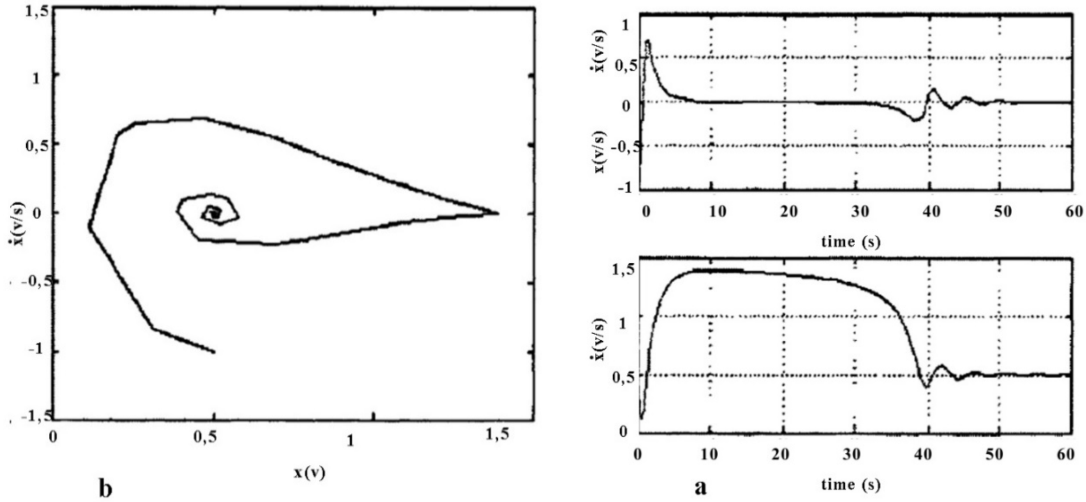

Figure 3: When γ is less than zero, system is encountering complicated dynamical states. They can be divided into several cases. (a) Time□domain waveform (b) phase plenary.

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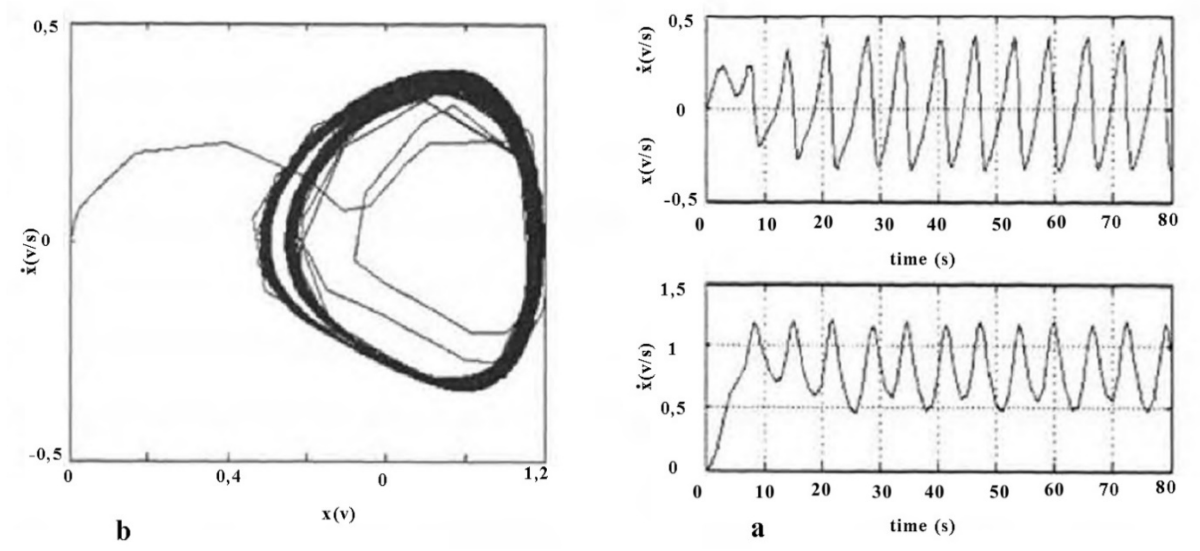

Figure 4: When γ is relatively small, phase orbit behaves as attractors in Poincare mapping.

Phase points are circling around focal points, or another focal point and vibrating. When y exceeds fixed closing value γc (size of γc can be derived from Melnikov's method)[11]. Simultaneously with the rise of γ, the system will go from homoclinic orbit periodical bifurcation all the way to chaotic state). This process is very fast. If γ is present for a long time, the system will always be in chaotic state. Only after, when higher closing value appears γd, the system will enter the periodic state). At that moment, phase orbit will encircle focal and saddle point, and in corresponding Poincare map it will also be a static point.

5. Chaotic criteria in Duffing system

We take –x3+x5 as equation for Duffing oscillator restoring force item x+kx x3+x5=γcos(ωt) Adjusting the range value of the driving force, which makes the system alternate between chaotic state and periodic state, we can use four step Runge-Kutta calculation method to derive new equation. On a derived x(t), we should apply Wigner transform, which is written as W(t, ω). Its amplitude frequency is shown in figure 4.10.

In figure 5, the horizontal coordinate represents the frequency and the vertical one represents the width. It clearly shows differences between chaotic state and periodic state. Its Wigner distribution domain is flat, but it still has certain regularity, and it is exactly that intrinsic regularity we have mentioned above when talking about chaotic nature. This explains that at that moment Duffing system has been entering from one into another periodic state. Still, these frequency spectrograms cannot describe or identify these two types of states. This requires usage of Lebesgue measure in order to further explore the system.

In this way, due to the change of selected value the observed situation where we use Lebesgue measure (L(k0)) there is a change in size of chaotic and periodic states. Here we will take $L(k_0) = (\omega$ *of Lebesgue measure* $\|W(t,w)\| \geq k_0)$.

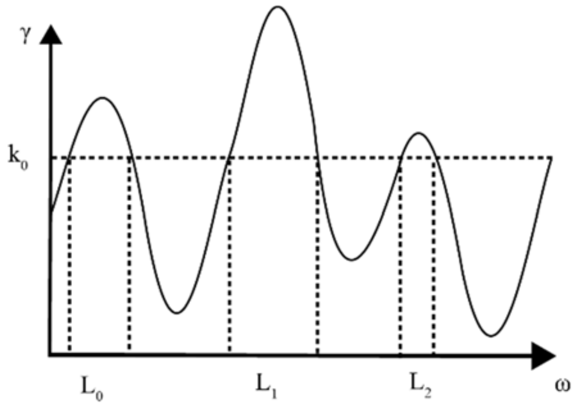

Figure 5: Visual explanation of L(k0) size.

Figure 5 has visually and in a simple way explained $L(k_0)$ size. According to the section above where Lebesgue measure and additivity of union of all non-overlapping countable interval lengths, it must be pointed out the following:

$$L(k_0)=L_0+L_1+L_2 \tag{7}$$

There are many methods to select k_0, but whether the selection of k_0 is suitable immediately decides the accuracy of chaotic pattern identification. Golden section point was used to, that is

$$k_0 = max\ \|W(t,w)\|\ x\ 0.618^n \tag{8}$$

In this way, thanks to the change of selected value observe situation with Lebesgue measure $L(k_0)$ when there is a change in size of chaotic and periodic states.

Table 1: Changes according to N value change in the size of $L(k_0)$ in different states.

N value	Chaotic state $L(k_0)$	Large period $L(k_0)$
3.4	0.8548	0.02232
38	0.9324	0.03587
…		
7.2	1.4325	0.14326
…		
11.2	1.4328	1.3256
…		
12.1	1.81067	1.837079
12.1	1.81067	1.837079

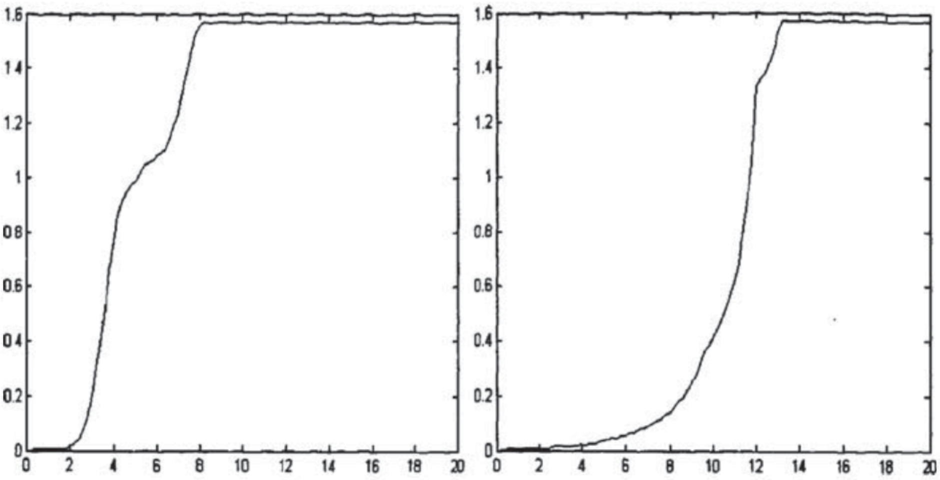

Figure 6: Change of Lebesgue measure following the change in n. Left chaotic state, right periodic state.

System is in chaotic and periodic state, while ω of Lebesgue measure rises along with the rise of *n* value.

Despite L(k0)'s n□value increases follow the increase of n, their curves are still different. Rise of the curve of periodic state is smooth, stable, and that is because of the shape of spectrogram after curve goes through Wigner transform. In chaotic state, its shape is not smooth and stable, it shows irregular ups and downs, and the curve's speed of ascending is faster than the one in the periodic state. When n value is the same, L(k0) value in the large periodic state has to be lower than the same values in the chaotic state. Using the above simulation experiments and conclusions, we can see that the method of chaotic pattern identification which is based on Wigner transform and Lebesgue measure is actually based on assessing trends of speed changes in L(k0) value when it follows the changes of n value in order to observe chaotic state. At the same time Duffing system was used to prove this method's effectiveness. It can be concluded as follows:

1. Different states of system. System is in chaotic and periodic state, while ω of Lebesgue measure rises along with the rise of n value.

2. Despite L(k0)'s n□value increase follows the increase of n, their curves are still different. Rise of the curve of periodic state is smooth, stable, and that is because of the shape of spectrogram after curve goes through Wigner transform. In chaotic state, its shape is not smooth and stable, it shows irregular ups and downs, and the curve's speed of ascending is faster than the one in periodic state.

3. When n value is the same, L(k0) value in the large periodic state has to be lower than the same values in chaotic state. Using the above simulation experiments and conclusions, we can see that the method of chaotic pattern identification which is based on Wigner transform and Lebesgue measure is actually based on assessing trends of speed changes in L(k0) value when it follows the changes of n value in order to observe chaotic state. At the same time, we have used Duffing system to prove this method's effectiveness.

6. Conclusions identification of nonlinear dynamic system in traffic flow□time sequence

We have discussed uncertainty of traffic flow. General effects of manmade subjective elements and non-manmade objective elements make phenomena of regular traffic flow and chaotic traffic flow coexist. When there is a low number of cars in the street, cars are in state of free movement. Along with the increase of cars and other external disturbances or characteristics of drivers and their cars, which affect the element of uncertainty, there is a greater chance for occurrence of chaotic state phenomenon.

As to the observation done in time-sequence, we have to say that although there is a certain correlation between time and traffic flow, there might be many other elements (such as noise), which make chaotic state less visible. So, in order to make its chaotic state more visible, we have to make some preparations regarding time-sequence, because in that way we can reach better chaotic pattern identification and more precise predictions.

We can see from the figure above that the curve which n value of L(k0) produces when n starts to increase is clearly different from the one in sequence 1. The growth is smooth and stable. According to before mentioned conclusion we can establish that this traffic flow time-sequence does not have chaotic features. This time, traffic flow time sequence pattern had features similar to those of periodic states found in Duffing system. But, this state is not the periodic state for traffic flow. Instead it is a state of regular movement. The reason for this state is a really small number of cars, that is, cars are moving freely, and there are no accidental elements (car accident, manmade traffic control etc.). During this periodic state of traffic flow system, movement should be described as a regular movement.

The figures and numbers in tables presents new model possible model for traffic flow prediction. With the use of Duffing system, calculation with Lebesgue measure and Wigner-Ville distribution for search of traffic flow patterns has been presented. We have now estimated when and where the possible congestion will occur. Further research will incorporate the presented model in a new model of chaotic neural network. It can be assumed the model will be faster and reliable than the ordinary neural network.

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