

Torsional Restraint Problem of Steel Cold-Formed Beams Restrained by Planar Members

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Abstract. The effect of continuous or discrete lateral and torsional restraints of metal thin-walled members along their spans can positively influence their buckling resistance and thus contribute to more economical structural design. The prevention of displacement and rotation of the cross-section results in stabilization of the member. The restraints can practically be provided e.g. by planar members of cladding supported by metal members (purlins, girts). The rate of stabilization of a member can be quantified using values of shear and rotational stiffness provided by the adjacent planar members. While the lateral restraint effected by certain shear stiffness can be often considered as sufficient, the complete torsional restraint can be safely considered in some practical cases only. Otherwise the values of the appropriate rotational stiffness provided by adjacent planar members may not be satisfactory to ensure full torsional restraint and only incomplete restraint is available. Its verification should be performed using theoretical and experimental analyses. The paper focuses on problem of steel thin-walled cold-formed beams stabilized by planar members and investigates the effect of the magnitude of the rotational stiffness provided by the planar members on the resistance of the steel members. Cold-formed steel beams supporting planar members of cladding are considered. Full lateral restraint and incomplete torsional restraint are assumed. Numerical analyses performed using a finite element method software indicate considerable influence of the torsional restraint on the buckling resistance of a steel thin-walled member. Utilization of the torsional restraint in the frame of sizing of a stabilized beam can result in more efficient structural design. The paper quantifies this effect for some selected cases and summarizes results of numerical analysis.

1. Introduction

Utilization of thin-walled metal members in civil engineering can result in light and effective structures, on the other hand these members are prone to stability problems (global and local buckling) reducing their resistance. Metal thin-walled members (purlins, girts) often support planar members of cladding or ceiling construction. The adjacent planar members (e.g. sandwich panels, trapezoidal sheeting) can provide lateral and torsional restraint for the member along its span which prevents the displacement and rotation of the cross-section and might positively influence its buckling resistance. Proper consideration of the restraints provided by planar members might result in more economical structural design. The restraints contribute to stabilization of the thin-walled member and can be considered as full or incomplete. The lateral restraint provided by planar members is characterized using certain value of shear stiffness S , the torsional restraint using a value of the rotational stiffness C_D [1]. In the practical point of view, the lateral restraint can be in number of cases of common types



of cladding considered as sufficient. The standard [2] provides formulae for calculation of minimum values of shear and rotational stiffness provided by planar members required for full lateral and torsional restraint.

The rate of stabilization is influenced by number of factors (e.g. properties of fasteners, planar members, way of load transfer from the planar member to the thin-walled member [3]). The influence of some of them can be verified by experimental and numerical analysis. It depends also on the direction of the external load [4].

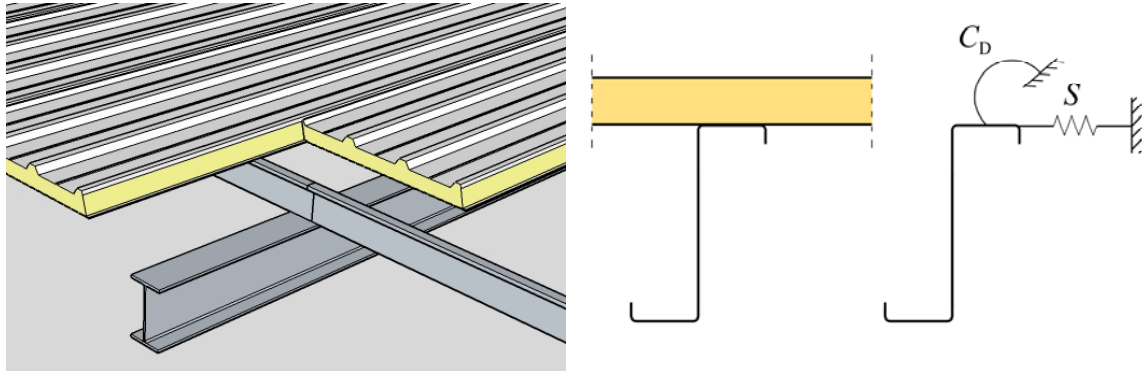


Figure 1. Lateral and torsional restraint provided by adjacent planar member

The problem of stabilization of a thin-walled member has been a subject of many studies. In [1] provisions for minimum required shear and rotational stiffness for full stabilization are provided including formula for elastic critical moment of a continuously laterally and torsionally restrained thin-walled beam. Paper [5] deals with theoretical analysis of determination of elastic critical moment of a beam with full lateral and incomplete torsional restraint along the span. Other solutions can be found in [6]. Results of numerical analyses of beams with torsional restraint provided by trapezoidal sheeting can be found in [7]. Although general approaches for consideration of the restraint in the frame of calculation are available, the determination of the values of the shear and rotational restraint provided by various planar members still can be questionable. The availability of the restraint can also be dependent on direction of the external load. This problem was particularly studied for sandwich panels supported by metal members (purlins, girts). Sandwich panels can provide lateral restraint for the metal member both for downward and uplift load. For uplift load, no torsional restraint is assumed to be available due to gap between the panels and metal member [8, 9]. In case of members of thin-walled cold-formed cross-section, certain torsional restraint may be available, but has to be verified by testing. As members of cold-formed cross-sections exhibit these notable specifics, they will be focused on in this paper. The attention will be paid specially to steel thin-walled beams stabilized by planar members providing full lateral and incomplete torsional restraint under uplift load which results in compressed free flange of the beam (prone to buckling). The buckling resistance of the free flange in compression will be investigated.

2. Torsional restraint provided to thin-walled beams by planar members

2.1. Definition of the torsional restraint

The rate of torsional restraint is characterized using a magnitude of rotational stiffness C_D provided by adjacent planar members assumed to be placed on a flange of a beam (figure 1). It is defined as torsional moment resulting in unit rotation of the flange of the beam cross-section. Its value is influenced by stiffness of connection, bending stiffness of the adjacent planar member and stiffness corresponding to the distortional stiffness of the thin-walled member, which is relevant especially for the members of cold-formed cross-sections. According to [2], the total rotational stiffness is defined

using (1), where $C_{D,A}$ indicates rotational stiffness of connection between the beam and planar member, $C_{D,B}$ the rotational stiffness corresponding to distortion of the cross-section of the beam and $C_{D,C}$ the rotational stiffness corresponding to the bending stiffness of the planar member. The components $C_{D,B}$ and $C_{D,C}$ depend on geometrical characteristics of the beam or the planar member, respectively. The value of $C_{D,A}$ can be determined using testing. The component $C_{D,C}$ can be in common cases neglected [2].

$$\frac{1}{C_D} = \frac{1}{C_{D,A}} + \frac{1}{C_{D,B}} + \frac{1}{C_{D,C}} \quad (1)$$

2.2. Analysis of cold-formed beams with torsional restraint

According to the standard [2], two approaches to determine the resistance of a steel cold-formed beam of a span L under external load of a design magnitude of q_{Ed} stabilized by adjacent planar members are possible. The first one is a simplified method which modifies the problem of a beam with torsional restraint of rotational stiffness C_D along its flange supporting the planar members to a problem of a beam with laterally restrained free flange with equivalent lateral spring stiffness K (figure 2). The free flange is loaded by equivalent load $q_{h,Ed}$ resulting from torsion and transversal bending of the cross-section.

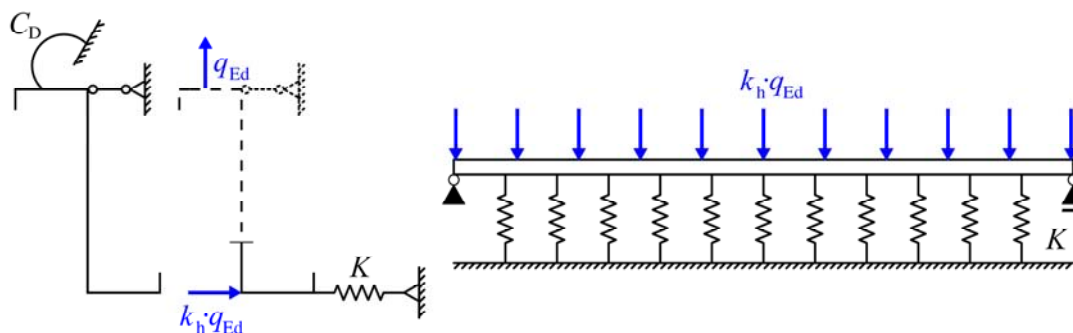


Figure 2. Modelling of the torsional restraint and equivalent load of the free flange [2]

More accurate investigation can be performed using numerical analysis according to the theory of 2nd order considering a value of the rotational stiffness C_D . The imperfections should be taken into account using initial bow imperfection of the free flange. It should conform to the appropriate eigenmode obtained from linear buckling analysis [2]. According to the standard, the amplitude of the initial bow imperfection e_0 for elastic analysis should be considered as $L/600$.

3. Numerical analysis

3.1. Subject of the study

To investigate the effect of the torsional restraint of a steel cold-formed beam (e.g. purlin) on its buckling resistance, a numerical analysis of a selected practical case was performed. A steel cold-formed simply supported beam of a span L of 6 m was considered. The depth of the beam was 300 mm. The thickness of the beam was considered as 3.0 mm.

The load was considered as uplift uniformly distributed along its span causing the free flange to be in compression and prone to lateral buckling. The considered material is steel of grade S350 GD+Z (yield strength $f_y = 350$ MPa).

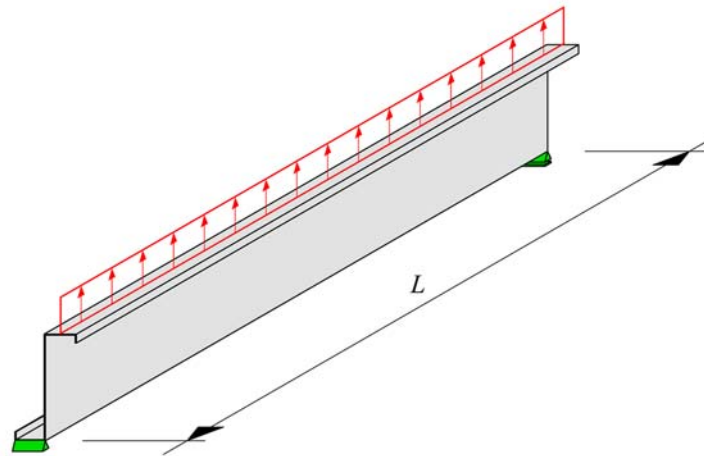


Figure 3. Uplift load applied on the upper flange of the beam

3.2. Description of the model and process of the analysis

The numerical analysis was performed using ANSYS 14.0 [10] code based on finite element method. The model of the beam was created using SHELL181 finite elements. The boundary conditions of the beam complied with simple supports at the ends. The nodes forming the intersection of the web and upper flanges were restrained laterally to model the full lateral restraint. It is assumed that the distortion of the web at supports is constructionally prevented and no special investigations of local effects at the supports resulting from reactions were needed. In finite element model it was taken into account using prevention of the lateral displacement of the web of the beam at the supports. The material was considered as ideal elastic (modulus of elasticity of 210 GPa and Poisson's ratio of 0.3).

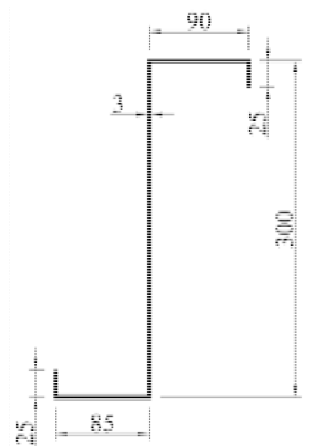


Figure 4. Cross-section considered for the analysis

It was started with a model with full lateral and no torsional restraint ($C_D = 0$ Nmm/mm/rad) along the upper flange. The linear buckling analysis was performed which provided eigenvalues and appropriate buckling modes. First positive eigenvalue is of great importance. It can be used for implementation of the initial bow imperfection of the beam into the model. For the investigated case, its amplitude e_0 is equal to 10 mm. The finite element model was modified according to the first positive eigenmode. After completion of this step, the geometrically nonlinear analysis (with 20 substeps) with imperfections was performed and used to find the level of load resulting in equivalent stress in the free flange equal to yield strength. It resulted in a magnitude of -6.60 kN/m. The load was applied on the

restrained flange of the beam. Then the procedure was repeated but with increased value of the rotational stiffness C_D which was expected to result in decrease of the equivalent stress.

The torsional restraint was modelled using COMBIN14 finite elements that enable to model a spring or rotational element of given magnitude of the lateral or rotational stiffness. The elements are defined using two nodes, the degree of freedom to be restrained and the magnitude of the relevant stiffness. In figure 5, numerical model can be seen. The uplift uniformly distributed load was applied at the half width of the upper flange of the beam. Several values of the rotational stiffness were assigned to the COMBIN14 finite elements to parametrically investigate the influence of its magnitude on the buckling resistance starting with 0 Nmm/mm/rad (no torsional restraint) and ending with 5000 Nmm/mm/rad with step of 250 Nmm/mm/rad (from 0 to 1000 Nmm/mm/rad) or 500 Nmm/mm/rad (from 1000 to 5000 Nmm/mm/rad), respectively. For each investigated level of the rotational stiffness, first the linear static analysis was performed. Its results are illustrated in figure 6 (for rotational stiffness $C_D = 0$ Nmm/mm/rad and, for comparison, for rotational stiffness $C_D = 2000$ Nmm/mm/rad).

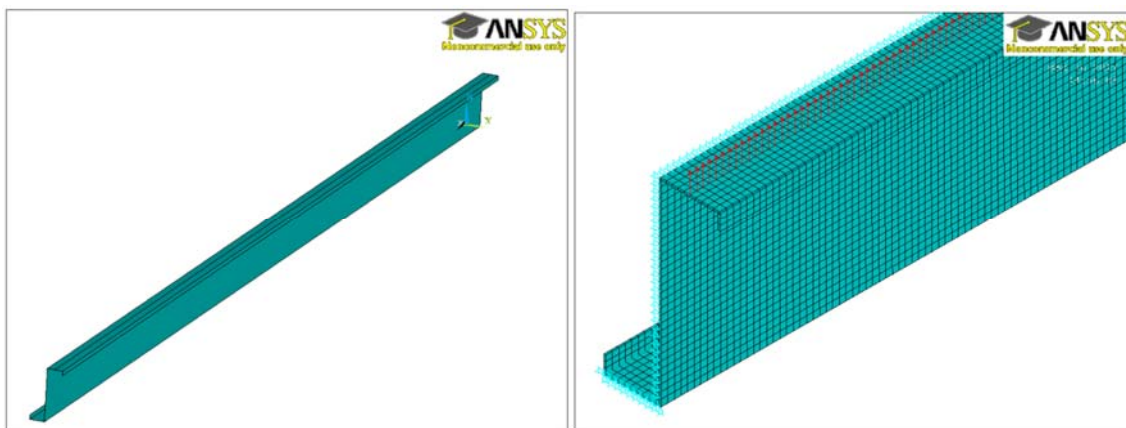


Figure 5. Numerical model

When the linear static analysis was completed, the linear buckling analysis to obtain the eigenvalues and eigenmodes (buckling modes) of the beam was performed. Thereafter, the geometry was modified according to the first positive eigenmode with given amplitude of the initial bow imperfection and geometrically nonlinear static analysis with imperfections was performed. After completion of the nonlinear analysis, the maximum equivalent normal stress could be checked. This analysis confirmed the midspan of the free flange as decisive section of the beam (with the highest value of the equivalent normal stress). Its distribution over the cross-section (for the rotational stiffness of 0 Nmm/mm/rad and 2000 Nmm/mm/rad) is in figure 7. For the rotational stiffness 2000 Nmm/mm/rad, significant reduction of the equivalent stress occurs.

3.3. Influence of rounded edges of the cross-section

For comparison, two different types of model were created. The first one with sharp edges and the second one (more accurate) with rounded edges of the cross-sections with a radius of 5 mm to compare the results of both types of models and to get closer to the actual shape of the cross-section of the cold-formed beam. Details of both models can be seen in figure 8. The numerical analysis consisted of several steps. After creating the model, meshing, definition of the boundary conditions and lateral and torsional restraints and loads the linear analysis and linear buckling analysis were performed. The magnitude of the uplift load was equal for both types of models. Then the model was modified according to the first positive eigenmode and geometrically nonlinear static analysis with imperfections was performed to obtain the equivalent stress in the decisive section.

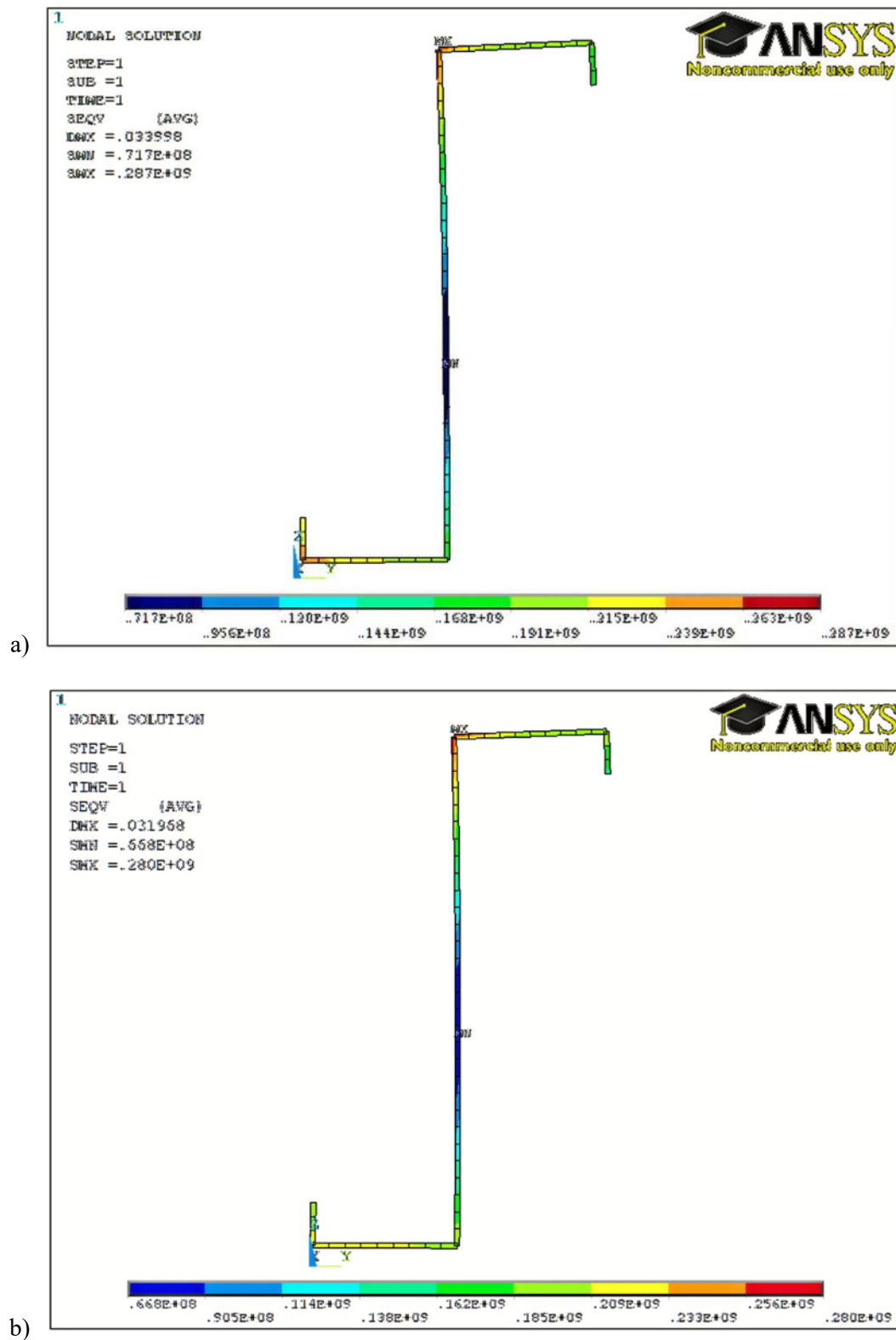


Figure 6. a), b) Equivalent stress ($C_D = 0$ Nmm/mm/rad, $C_D = 2000$ Nmm/mm/rad) resulting from geometrically linear static analysis

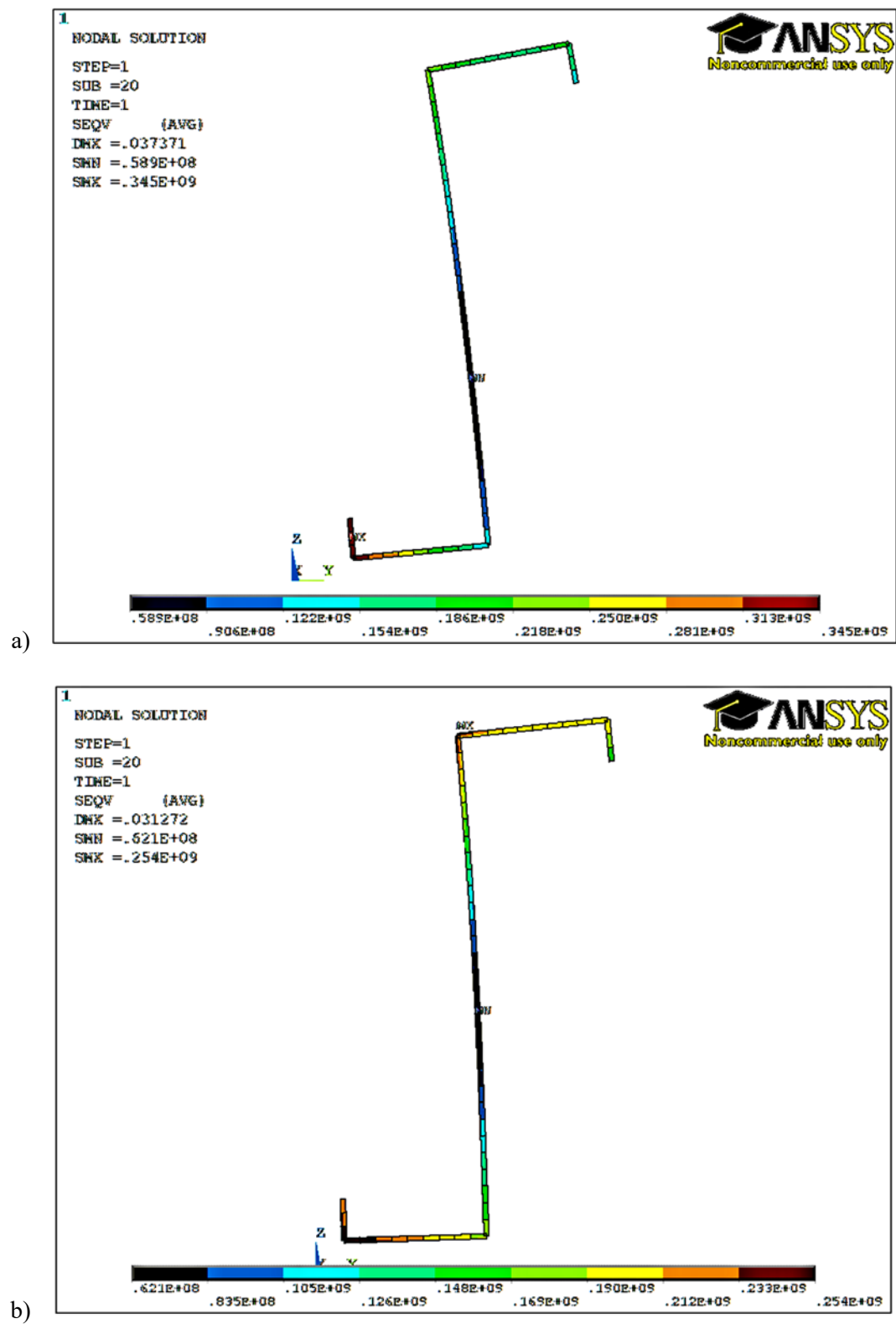


Figure 7. a), b) Equivalent stress ($C_D = 0$ Nmm/mm/rad, $C_D = 2000$ Nmm/mm/rad) resulting from geometrically nonlinear static analysis with imperfections

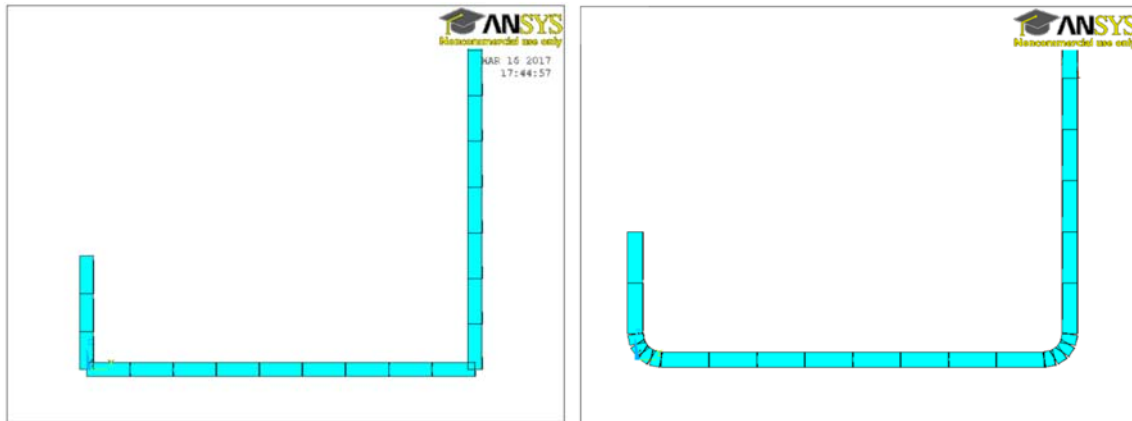


Figure 8. Details of numerical models

4. Results and discussions

4.1. Influence of the torsional restraint on the buckling resistance

The results of the parametrical study are summarized in the chart in figure 8 with maximum equivalent stress (in the free flange) on the vertical axis and investigated magnitude of the rotational stiffness on the horizontal axis. Both types of models (with sharp and with rounded edges) are included. As the nonlinear effect are included in the model, the check of the beam can be directly performed using comparison of the maximum equivalent stress σ_{eq} with yield strength f_y of steel, i.e. equation (2) should be fulfilled otherwise the cross-section is not satisfactory.

$$\eta = \frac{\sigma_{eq}}{f_y} \leq 1 \quad (2)$$

This ratio η (unit check) is plotted in the chart in figure 9. The chart indicates considerable influence of the rotational stiffness on the equivalent stress and shows significant difference between a beam with and without torsional restraint. The differences between the model with sharp edges and with rounded edges are not significant (up to approx. 2%).

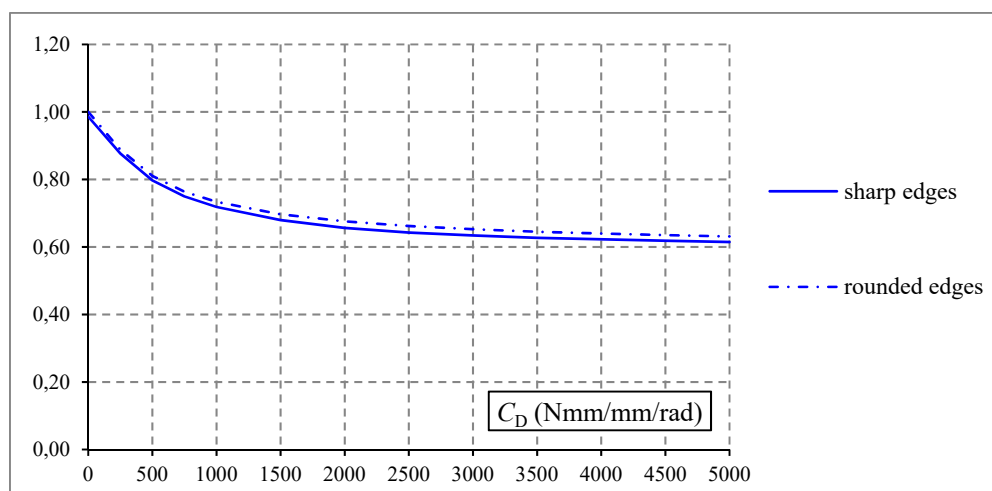


Figure 9. Comparison of results (unit checks for both types of models considering various magnitudes of the rotational stiffness)

5. Conclusions

The performed parametrical study indicates the significant influence of the torsional restraint on the buckling resistance of the beam. The maximum equivalent stress considerably decreases with increasing of the rotational stiffness (provided by a planar member). The nonlinear numerical analysis indicates the free flange of the beam as decisive for structural check due to lateral buckling. As the vector of the vertical load does not cross the shear centre of the cross-section, also torsion of the cross-section occurs. For the investigated cases, the most distinctive reduction of the equivalent stress occurs after reaching approximately 1000 Nmm/mm/rad (reduction of the equivalent stress to approximately 70% of the value within no torsional restraint), then the reduction stabilizes and further increasing of the rotational stiffness leads to only slight reduction of the equivalent stress. With increasing of the rotational stiffness of the torsional restraint, the buckling resistance of the beam significantly increases.

Although the specific rotational stiffness provided by particular planar members should be verified by experiments, the numerical study indicate that even relatively low values of the rotational stiffness may result in notable rate of stabilization of the beam and thereby to possible more economical and efficient structural design of the beam considering the torsional restraint.

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References

- [1] J. Lindner, "Zur Aussteifung von Biegeträgern durch Drehbettung und Schubsteifigkeit (Beams in bending restraint by rotational spring stiffness and shear stiffness of adjacent members)," *Stahlbau*, 77 (6), pp. 427–435, 2008, ISSN 0038-9145.
- [2] EN 1993-1-3: Eurocode 3: Design of steel structures – Part 1–3: General rules – Supplementary rules for cold-formed members and sheeting, 2008.
- [3] J. Melcher, "Ohyb, kroucení a stabilita ocelových nosníků (Bending, torsion and stability of steel members)", Brno, Brno University of Technology, 1975.
- [4] T. Vraný, "Effect of loading on the rotational restraint of cold-formed purlins," *Thin-Walled Structures*, 44 (12), pp. 1287–1292, 2006, ISSN 0263-8231.
- [5] R. Kindmann and R. Muszkiewicz, "Verzweigungslasten und Eigenformen seitlich gestützter Biegeträger unter Berücksichtigung der Drehbettung (Critical loads and modal shapes of lateral supported beams taking into account torsional restraints)," *Stahlbau*, 71 (10), pp. 748–759, 2002, ISSN 0038-9145.
- [6] T. Misiak, S. Kämpfe, H. Saal and T. Ummenhofer, "Stabilisation of beams by sandwich panels: Lateral and torsional restraint," *EUROSTEEL 2011: 6th European Conference on Steel and Composite Structures*, Budapest, European Convention for Constructional Steelwork, pp. 615–620, 2011, ISBN 978-92-9147-103-4.
- [7] C. Zhao, J. Yang, F. Wang and A. H. C. Chan, "Rotational stiffness of cold-formed steel roof purlin-sheeting connections," *Engineering Structures*, 59, pp. 284–297, 2014, ISSN 0141-0296.
- [8] European Recommendations on the Stabilization of Steel Structures by Sandwich Panels, International Council for Research and Innovation in Building and Construction, European Convention for Constructional Steelwork, 2013.
- [9] M. Dürr, T. Misiak and H. Saal, "The torsional restraint of sandwich panels to resist the lateral torsional buckling of beams," *Steel Construction*, 4 (4), pp. 251–258, 2011, ISSN 1867-0539.
- [10] ANSYS® Academic Research, Release 14.0.