

# Ground Reaction Forces Generated During Rhythmical Squats as a Dynamic Loads of the Structure

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**Abstract.** Dynamic forces generated by moving persons can lead to excessive vibration of the long span, slender and lightweight structure such as floors, stairs, stadium stands and footbridges. These dynamic forces are generated during walking, running, jumping and rhythmical body swaying in vertical or horizontal direction etc. In the paper the mathematical models of the Ground Reaction Forces (*GRFs*) generated during squats have been presented. Elaborated models was compared to the *GRFs* measured during laboratory tests carried out by author in wide range of frequency using force platform. Moreover, the *GRFs* models were evaluated during dynamic numerical analyses and dynamic field tests of the exemplary structure (steel footbridge).

## 1. Introduction

In order to check the resistance of the flexible, light-weight structures designed for human use (pedestrian bridges, long span floors etc.) to dynamic loads it is important to check the forced vibrations of the structure caused by dynamic loads generated during different type of human movement. These man-made dynamic loads may have various sources of origin e.g. forces generated during walking, running, jumps, squats or other rhythmic and choreographic activities.

Dynamic actions in form of squats occurring on building structures (excluding sports halls and gyms) take a form of partial squats characterised by knee flexion angle in a range of 0–40° (0° is for straight leg) [1, 2]. Partial squats are adjusted to the specific aim and do not reach full depth.

The partial squats can be taken into account as an exceptional load case of the structure (extreme dynamic load case of the lightweight and slender structure) in order to estimate maximal dynamic response of the structure. Dynamic loads generated during jumps or squats can be classified as the vandal dynamic loads of the structure [3].

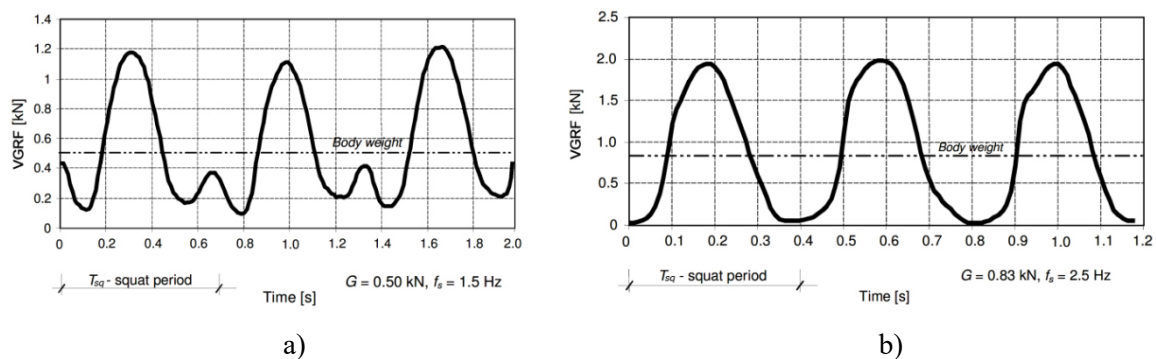
The dynamic loads in form of squats can be even more dangerous for the structure than jumps. During the numerous dynamic field tests of the footbridges it was observed that degree of synchronization between a moving person and the vibrating structure is higher for squats than for jumps. Greater synchronization leads to increased amplitude of vibrations.

The vertical component of the ground reaction forces generated during squats (*VGRF*) has a value greater than body weight of a person performing the squats and from the point of view of structural dynamics become an important dynamic load case.



## 2. The *VGRF* from partial squats

In figure 1, two different examples of the *VGRF* curves generated during successive rhythmically performed partial squats are presented. The shape of the *VGRF* curve strongly depends on the squats technique. In the case of unprofessionally performed squats (squats performed without experiences in sport exercises, without special trainings) two important differences in squat technique can occur. Namely, person performing the squats can fully straighten or partially straighten the legs at the end of the ascending phase of the squat (during body lifting). This feature leads to significant differences in the graph of the *VGRF* function (figure 2).



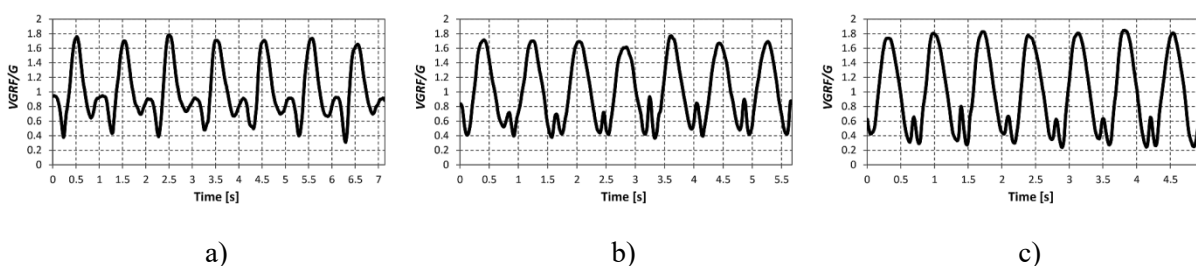
**Figure 1.** Examples of the *VGRF* generated during a) squats with fully straightened legs at the end of ascending phase of the squat, b) squats with partially straightened legs during ascending phase of the squat

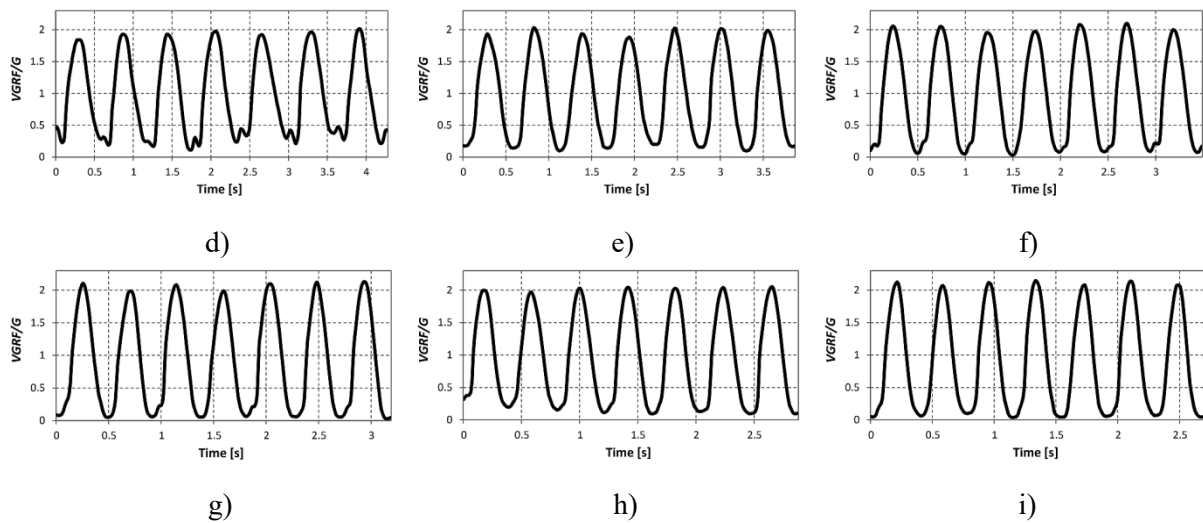
Complete legs straightening at the end of ascending phase of the squat is possible during squats performed with slow and medium speed (squats with frequency  $f_{sq} < 1.60$  Hz, this limit frequency can be an individual feature of persons performing the squats). In the case of squats performed with the fully straightened legs the *VGRF* curve is not an ideal sinusoid (figure 1a). The ideal sine curve is disturbed by occurrence of small peak at the end of the squat period ( $T_{sq}$ ) between two successive squats. It can be seen that the magnitude of the small peak does not achieve the value of body weight.

In the case of squats performed with only partially straightened legs during the ascending phase (legs are slightly bent, body motion is smooth and similar to motion of the mass on the spring) the *VGRF* curve is more or less a sine curve (figure 1b).

The *VGRFs* generated during partial squats performed with different pace (frequency of the squats) were measured during series of laboratory tests carried out by author using the force platform for human movement and stance analysis. The force platform Zebris FDM-1.5 with sensor area 149 x 54.2 cm (L x W) and signal sampling rate 100 Hz was used.

During the tests the squats were performed rhythmically at the pace determined by the electronic metronome. The frequency of the squats (the pace of the squats) ranged from 0.60 Hz to 2.80 Hz and was increased by 0.20 Hz. For each frequency, three series of squats lasting 20-25 seconds with 5-10 minutes rest between series were performed. The data were acquired on the portable personal computer and analysed in WinFDM software dedicated to FDM platform. Sample test results of the *VGRFs* generated during continuous rhythmical partial squats are presented in figure 2.





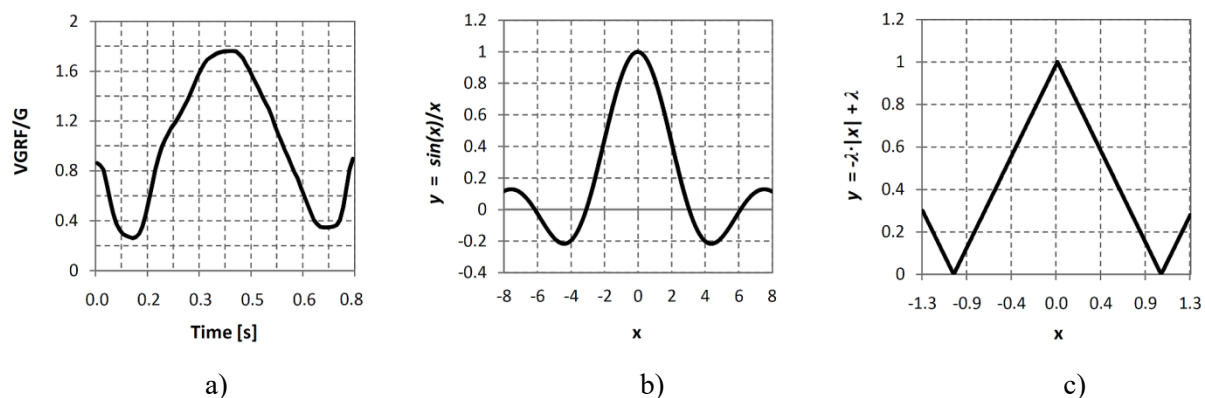
**Figure 2.** The  $VGRF/G$  curves for partial squats performed with different frequencies a) 1.00 Hz, b) 1.20 Hz, c) 1.40 Hz, d) 1.60 Hz, e) 1.80 Hz, f) 2.00 Hz, g) 2.20 Hz, h) 2.40 Hz, i) 2.60 Hz ( $G$  – the body weight of a person performing squats)

The peak values of the  $VGRFs$  generated during partial squats  $F_{max,sq}$  in frequency range  $f_{sq} = 1.00 \div 2.60$  Hz were about  $F_{max,sq} \approx (1.60 \div 2.15) \cdot G$ . The minimum values of the  $VGRFs$   $F_{min,sq}$  achieved  $F_{min,sq} \approx (0.25 \div 0.50) \cdot G$  in squat frequency range  $f_{sq} = 1.00 \div 1.80$  Hz,  $F_{min,sq} \approx (0.15 \div 0.25) \cdot G$  in squat frequency range  $f_{sq} = 1.8 \div 2.2$  Hz and  $F_{min,sq} \approx (0.05 \div 0.15) \cdot G$  in squat frequency range  $f_{sq} > 2.5$  Hz. During correctly performed squats when both feet have the continuous contact with the ground value of the  $VGRFs$  do not reach zero.

### 3. Mathematical models of the $VGRF$

Considering the above conclusions the mathematical models of the  $VGRF$  generated during partial squats were elaborated for two frequency ranges  $f_{sq} \leq 1.60$  Hz and  $f_{sq} > 1.60$  Hz.

For frequencies  $f_{sq} \leq 1.60$  Hz the similarity of the graph presented in figure 1a and figure 2a–d to the selected part of the function  $y = sinc(t) = \sin(x)/x$  plotted for negative and positive values of  $x$  [rad] for  $x \neq 0$  or selected part of the triangle function  $y = -\lambda \cdot |t| + \lambda$  can be noted (figure 3).



**Figure 3.** Comparison of the  $VGRF/G$  curve with basic mathematical functions a)  $VGRF/G$  generated during single squat (single force impulse), b) graph of the function  $y = sinc(x) = \sin(x)/x$ , c) graph of the function  $y = -\lambda \cdot |t| + \lambda$

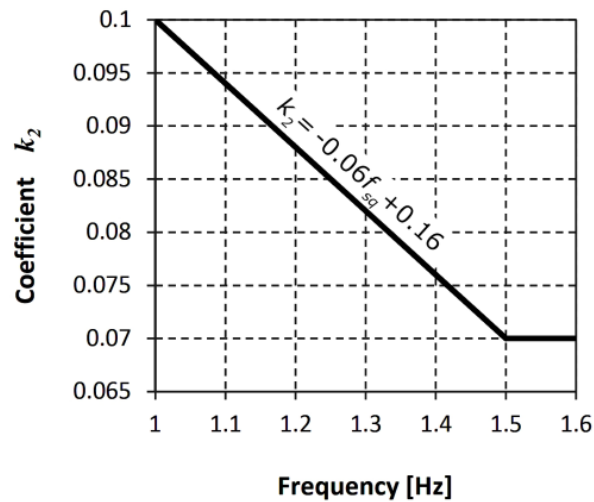
For frequencies  $f_{sq} > 1.60$  Hz the  $VGRF$  curves are similar to the sine curve  $y = B + A\sin(x)$  (figure 2 e-i).

In further stages of researches the parameters of basic mathematical function were analysed in order to match the function to the results of laboratory tests. Finally, equations (1), (2) and (3) were proposed.

The equation (1) was proposed for frequencies  $f_{sq} = 1.00 \div 1.60$  Hz:

$$F_{VGRF}(t) = k_1 \cdot G \cdot \left( 0.5 + k_2 \cdot \frac{\sin(4 \cdot \pi \cdot f_{sq} \cdot \Delta t)}{\Delta t} \right) \quad (1)$$

where:  $G$  – the body weight of a person,  $f_{sq}$  – frequency of the squats,  $k_1$  – coefficient:  $k_1 = 1.35$  for  $G < 0.7$  kN,  $k_1 = 1.0$  for  $G \geq 0.7$  kN,  $k_2$  – coefficient (figure 4):  $k_2 = -0.06 \cdot f_{sq} + 0.16$  for  $f_{sq} = 1.00 \div 1.50$  Hz,  $k_2 = 0.07$  for  $f_{sq} = 1.50 \div 1.60$  Hz,  $\Delta t$  – time step:  $\Delta t \in (-0.5T_{sq}, 0.5T_{sq})$ ,  $T_{sq}$  – period of the squat  $T_{sq} = 1/f_{sq}$ .



**Figure 4.** Coefficient  $k_2$  for equation (1)

Alternatively for  $f_{sq} = 1.00 \div 1.60$  Hz the model in a form of triangle function was elaborated:

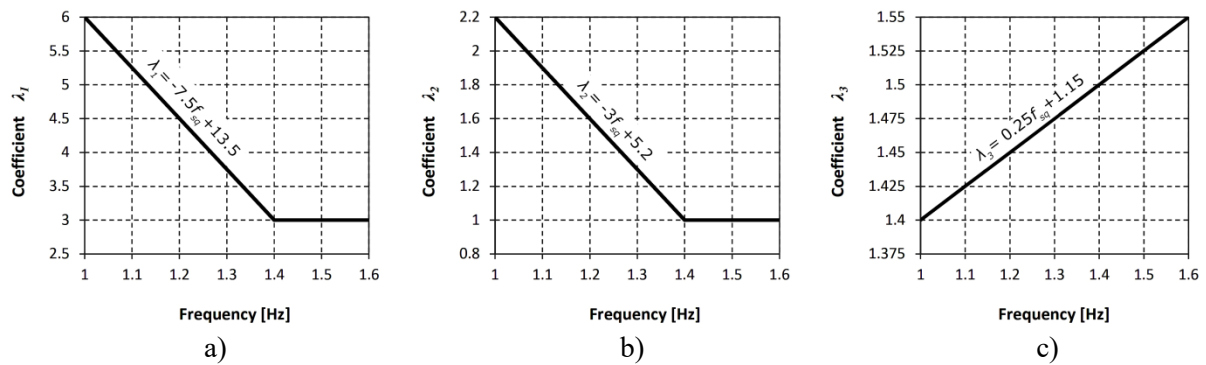
$$F_{VGRF}(t) = \begin{cases} G \cdot \left( \lambda_1 \cdot f_{sq} \cdot |\Delta t| - \lambda_2 \right) & \text{for } \Delta t \in \langle -0.5T, -0.4T \rangle \wedge \Delta t \in \langle 0.4T, 0.5T \rangle \\ G \cdot \left( 0.4 - 3 \cdot \lambda_3 \cdot f_{sq} \cdot |\Delta t| + \lambda_3 \right) & \text{for } \Delta t \in (-0.4T, 0.4T) \end{cases} \quad (2)$$

where:  $G$ ,  $f_{sq}$ ,  $T_{sq}$  and  $\Delta t \in (-0.5T_{sq}, 0.5T_{sq})$  as previously,  $\lambda_1$ ,  $\lambda_2$ ,  $\lambda_3$  – coefficients presented in figure 5.

For frequencies  $f_{sq} > 1.60$  Hz the equation (3) was proposed:

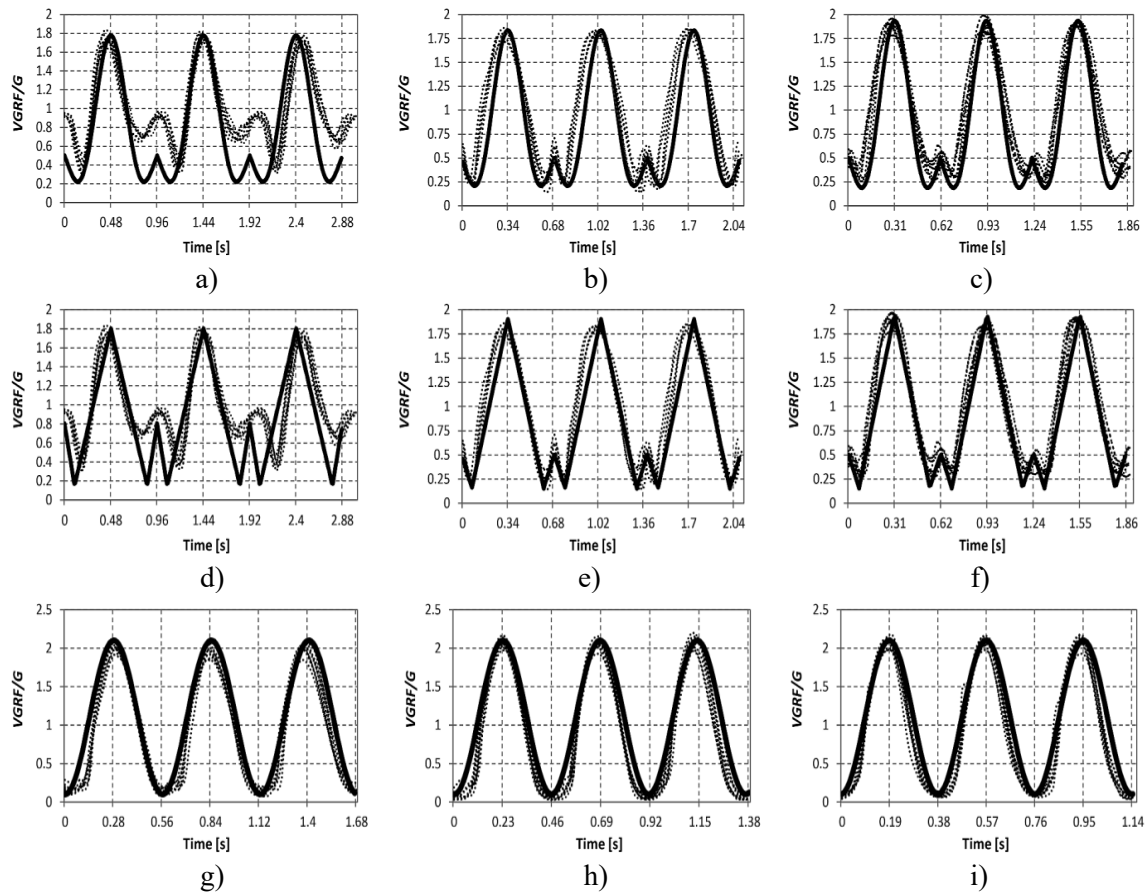
$$F_{VGRF}(t) = G \cdot \left[ 1.1 + \sin(2 \cdot \pi \cdot f_{sq} \cdot \Delta t) \right] \quad (3)$$

where:  $G$ ,  $f_{sq}$ , as previously ( $f_{sq} > 1.60$  Hz),  $\Delta t$  – time steep:  $\Delta t \geq 0$ .



**Figure 5.** Coefficients  $\lambda_1, \lambda_2, \lambda_3$  for equation (2)

The  $VGRF/G$  curves calculated using equations (1), (2) and (3) in relation to  $VGRF/G$  curves obtained from results of the laboratory tests are presented in figure 6.



**Figure 6.** The  $VGRF/G$  calculated using equations (1), (2) and (3) in relation to  $VGRF/G$  measured during laboratory tests for different frequency: a), b), c) equation (1) for frequency 1.04 Hz, 1.43Hz, 1.63 Hz respectively, d), e), f) equation (2) for frequency 1.04 Hz, 1.43Hz, 1.63 Hz respectively, g), h), i) equation (3) for frequency 1.78 Hz, 2.17 Hz, 2.63 Hz respectively

The other proposal of mathematical model of  $VGRF$  generated during squats was presented in [4] where the  $VGRF$  curve was approximated by means of harmonic function with damping (4).

$$F_{VGRF}(t) = G \cdot \left[ 1 + A \cos \left( 4.8 \pi |\Delta t| - \varphi \right) e^{-|\Delta t| \delta} \right] \quad (4)$$

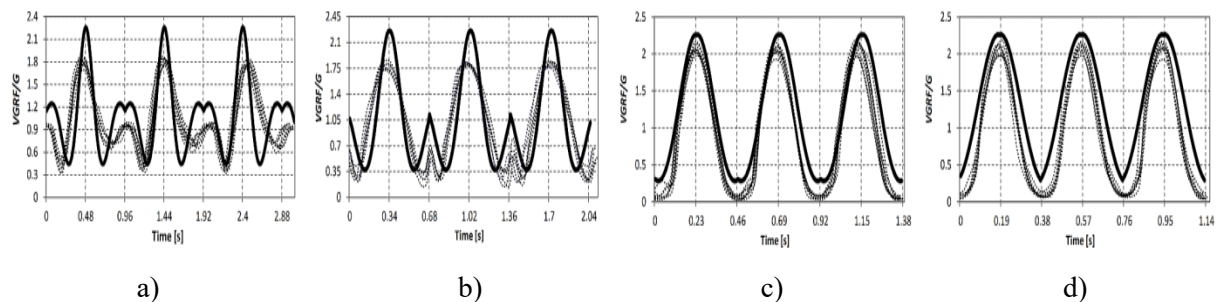
where:  $G$ ,  $\Delta t$ ,  $T_{sq}$  as previously ( $\Delta t \in (-0.5T_{sq}, 0.5T_{sq})$ ),  $A$  – the amplitude of the dynamic component of the load,  $\delta$  – damping  $\delta = 4\sqrt{T_{sq}}$ ,  $\varphi$  – phase shift  $\varphi = 0.25$  (constant value) [4].

The amplitude of the dynamic component of the load  $A$  was determined based on the researches of 33 persons (12 women, 21 men), performing squats with frequency  $f_{sq} = 2.0$  Hz. The mean value of  $A$  was found  $A_{mean} = 1.3$  with  $A_{min} = 1.07$ ,  $A_{max} = 1.52$  and variance  $Var(A) = 0.128$  [4]. Finally, proposed equation was written for the assumed typical weight of the person  $G = 0.75$  kN and  $A_{mean} = 1.3$  as (5):

$$F_{VGRF}(t) = 0.75 \cdot \left[ 1 + 1.3 \cos \left( 4.8 \pi |\Delta t| - 0.25 \right) e^{-|\Delta t| \delta} \right] \quad (5)$$

It should be noted that  $VGRF$  model (5) is prepared with assumption of the constant value of  $G = 0.75$  kN. For this reason, its comparison with laboratory tests results and with other models require to divide the values calculated using model (5) by 0.75 (note: all comparisons of the  $VGRFs$  presented in the paper are made using dimensionless values of compared forces  $VGRF/G$ ).

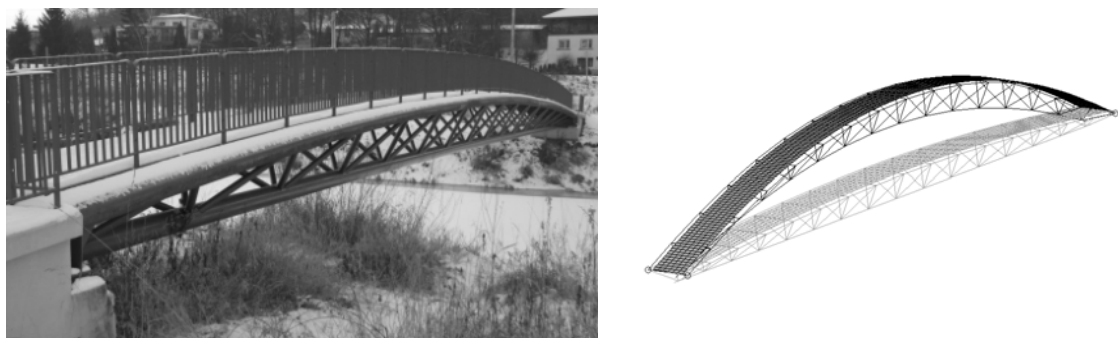
The comparisons of the  $VGRF/G$  curves calculated by means of equation (5) for  $G = 0.75$  kN and different frequencies of the squats with the  $VGRF/G$  curves obtained from results of the laboratory tests are presented in figure 7.



**Figure 7.** The  $VGRF/G$  calculated using the equation (5) for  $G = 0.75$  kN and different frequencies of the squats a), b), c), d) for 1.04 Hz, 1.43 Hz, 2.20 Hz, 2.63 Hz respectively

#### 4. Dynamic analyses of the footbridge

In order to check the effectiveness and correctness of the proposed load models the dynamic analyses of an exemplary footbridge (figure 8) were carried out using  $VGRFs$  generated by equations (1), (2), (3) and (5).



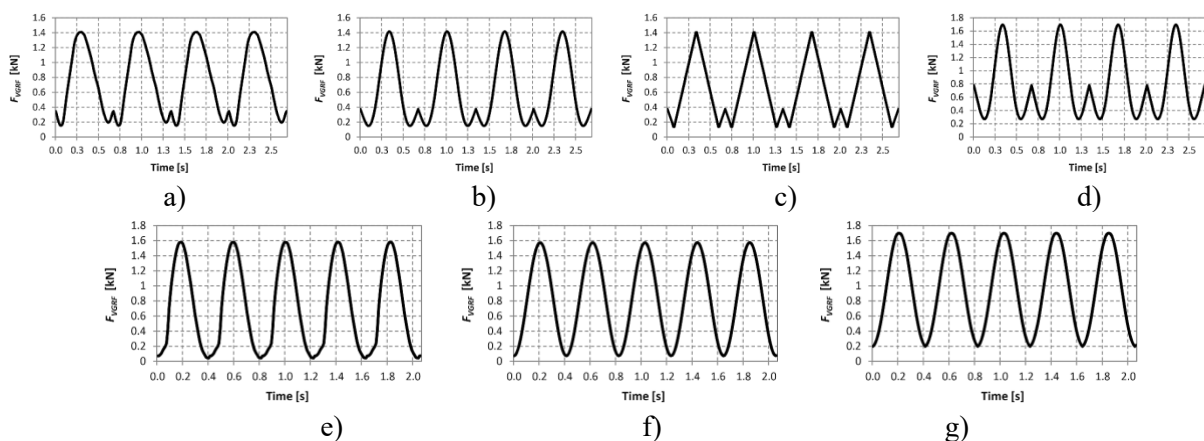
**Figure 8.** General view of the footbridge and fundamental mode shape of the footbridge

The footbridge was designed as a truss structure with main girder in a form of spatial truss and span length 47.0 m. Two constructional variants of the structure were considered: 1) composite footbridge with steel girder and concrete deck and 2) footbridge made entirely of steel (footbridge with steel girder and steel deck). Finally, the footbridge was built in 2011 as a steel structure.

Different dynamic characteristics of the constructional variants of the footbridge enabled to check the dynamic response of the footbridge using described models of the  $VGRFs$  for different frequencies of the squats. Additionally, after finishing the construction of the footbridge, it was possible to perform the dynamic field tests of the structure under dynamic loads in form of squats. The results of the field tests were used to evaluate the  $VGRF$  models.

The fundamental mode shape of the footbridge is presented in figure 8. The vibration frequencies for two constructional variants of the structure were:  $f_{co} = 1.49$  Hz – for composite structure and  $f_{st} = 2.43$  Hz – for steel structure. The vibration frequency of the erected steel footbridge identified during the field tests was  $f_{st,t} = 2.44$  Hz. The mean value of the logarithmic decrement of the steel structure determined using vibration signal acquired during dynamic field tests was  $\Delta = 0.044$  (damping ratio  $\zeta \approx 0.007$ ). In dynamic analyses of the composite structure the same value of  $\Delta$  was assumed taking into account recommendations presented in [5].

The  $VGRFs$  generated during squats was modelled using the data acquired during the  $VGRF$  laboratory tests (selected single impulse of the  $VGRF$  repeated periodically, compare figure 3 and chapter 5) and data generated appropriately to the frequency of the squats using equations (1), (2), (3) and (5) for two frequencies of the squats  $f_{sq,1} = 1.49$  Hz,  $f_{sq,2} = 2.43$  Hz and  $G = 0.75$  kN (figure 9).

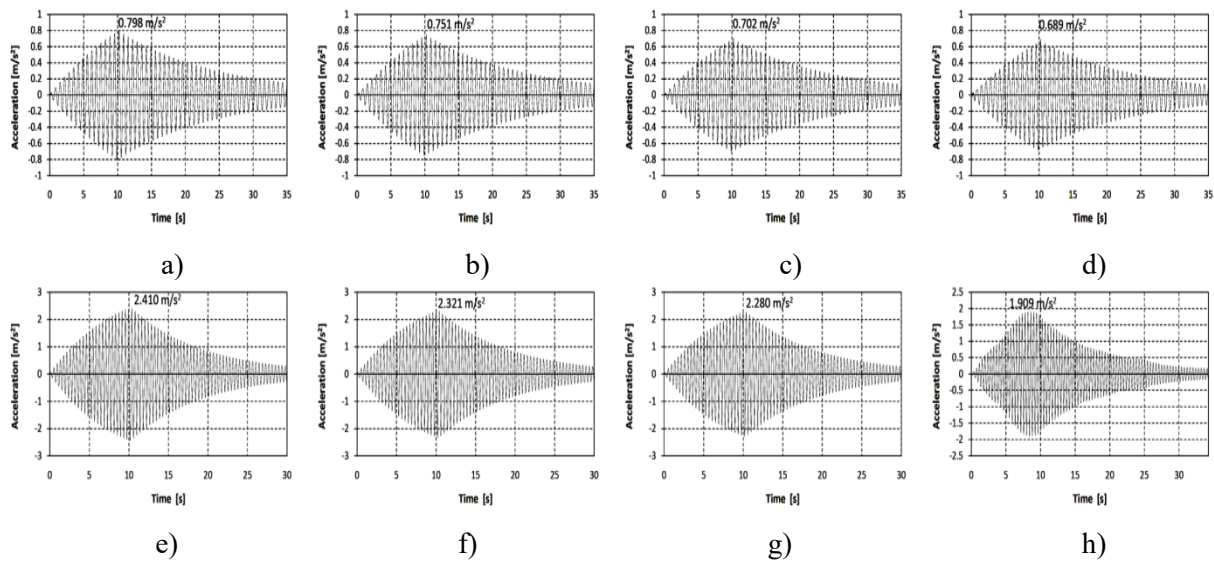


**Figure 9.** The  $F_{VGRF}$  obtained for  $G = 0.75$  kN from: a) laboratory tests for  $f_{sq,1} = 1.49$  Hz, b) equation (1) for  $f_{sq,1} = 1.49$  Hz, c) equation (2) for  $f_{sq,1} = 1.49$  Hz, d) equation (5) for  $f_{sq,1} = 1.49$  Hz, e) laboratory tests for  $f_{sq,2} = 2.43$  Hz, f) equation (3) for  $f_{sq,2} = 2.43$  Hz, g) equation (5) for  $f_{sq,2} = 2.43$  Hz.

The dynamic loads were applied in the middle of the footbridge span in a form of two concentrated forces acting for 10 sec and changing in time in accordance with the course of  $VGRF$  curve. For each force the load multiplier of 0.5 was used. Calculated values of vibration acceleration are presented in figure 10 and in table 1. In table 1 the values of acceleration reached after about 8.6 sec was additionally presented to compare the results of the numerical dynamic analyses of steel structure with the results of the dynamic field tests of the footbridge (8.6 sec – duration of the squats during field test of the structure).

## 5. Evaluation of the $VGRF$ models

The presented dynamic load models were elaborated on the basis of the results obtained during laboratory tests of the  $VGRFs$  generated during partial squats carried out in frequency range 0.6 – 2.8 Hz. The load models were proposed for frequency range 1.00 – 2.60 Hz.



**Figure 10.** Vibration acceleration obtained for *VGRF* from a) laboratory tests for  $f_{sq,1} = 1.49$  Hz, b) equation (1) for  $f_{sq,1} = 1.49$  Hz, c) equation (2) for  $f_{sq,1} = 1.49$  Hz, d) equation (5) for  $f_{sq,1} = 1.49$  Hz, e) laboratory tests for  $f_{sq,2} = 2.43$  Hz, f) equation (3) for  $f_{sq,2} = 2.43$  Hz, g) equation (5) for  $f_{sq,2} = 2.43$  Hz, h) field test result for  $f_{sq,t} = 2.44$  Hz

**Table 1.** Vibration acceleration for different *VGRFs* and two constructional variant of the footbridge

Load case		Frequency [Hz]	Acceleration after 8.6 sec [m/s <sup>2</sup> ]	Maximal acceleration after 10 sec [m/s <sup>2</sup> ]
Composite	Laboratory tests	1.49	0.728	0.798
	Equation (1)	1.49	0.677	0.751
	Equation (2)	1.49	0.635	0.702
	Equation (5)	1.49	0.602	0.689
Steel	Laboratory tests	2.43	2.261	2.410
	Equation (3)	2.43	2.155	2.321
	Equation (5)	2.43	2.124	2.280
	Field tests (8.6 sec)	2.44	1.909	1.909

As can be seen in figure 6 the *VGRFs* curves generated using equations (1) – (3) fits well to the laboratory tests results in frequency range  $f_{sq} = 1.40 - 2.60$  Hz. The results of performed analyses indicate that this range can be extended to  $f_{sq} = 1.20 - 2.80$  Hz. In the case of slowly performed squats (squats in frequency range of about 0.8 Hz to 1.1 Hz) the differences between load models and tests results are significant. The *VGRF* generated during slowly performed squats has an asymmetrical shape which leads to larger discrepancies in the results. Modifications of the proposed load models or elaboration of new model for slowly performed squats are required.

As can be seen in figure 7 the *VGRFs* curves generated using equations (5) fits well to the laboratory test results obtained for frequency  $f_{sq} = 2.20$  Hz. Performed analyses have shown that proposed load model fits well to the test results in frequency range  $f_{sq} = 2.0 - 2.4$  Hz. It should be remembered that parameters  $A$  and  $\delta$  of the load model (5) were determined for the squats performed with frequency  $f_{sq} = 2.0$  Hz. In the case of using the model (5) for other frequencies an adjustment of its basic parameters  $A$  and  $\delta$  is required.

The results of numerical dynamic analyses of exemplary footbridge shows that proposed load models (1), (2), (3), (5) allow to determine the vibration acceleration that are comparable to the values

of acceleration obtained during simulations made with the use of forces acquired during laboratory tests. In the analyses the *VGRF* from laboratory test was modelled in a form of selected single impulse of the force (compare figure 3) repeated periodically. Single impulse of the *VGRF* repeated periodically was used in order to simulate the resonant excitation of vibration of the footbridge without any shifts in frequency occurring in the original signal recorded during the laboratory tests. In this way the greater compatibility between idealised mathematical model (1), (2), (3), (5) and idealised *VGRFs* from laboratory tests was achieved (the use of original *VGRF* signal recorded during laboratory tests in dynamic analyses (without their idealisation) leads to lower values of vibration acceleration of the structure because of aperiodic (imperfect) repetition of the force impulses. Idealisation of the load model created based on the laboratory tests results allowed to create of a comparable conditions of application of the analysed load models).

The results of analyses compared with the results of dynamic field tests carried out for steel structure confirm that proposed load models allow to correctly estimate the values of vibration acceleration of the structure. The estimated vibration acceleration values are slightly larger (about 20% larger) than the values of vibration acceleration measured during the field tests. This is mainly due to idealized way of modelling of the structure and loads. It should be remembered that due to scatter and randomness of the values of the parameters of the load models exact estimation of the vibration acceleration is very hard and possible only in individual analyses of particular case.

It is worth noting that the very easy to use load model (3) allows for a very accurate estimation of the values of the *VGRF* generated during squats with frequency  $f_{sq} > 1.6$  Hz and accurate estimation of the vibration acceleration of the structures.

The load model (5) despite the discrepancies between force estimated by means of the model and forces recorded during laboratory tests, occurring in the case of frequencies outside the range 2.0 – 2.4 Hz, allows to estimate the vibration acceleration of the structure with a certain margin of safety. The model (5) is easy to use and can be developed and adjusted to wide range of vibration frequency.

## 6. Conclusions

The proposed load models are the idealisation of the real dynamic forces generated during partial squats, which allow to correctly estimate the value of the *VGRF* and vibration acceleration of the structure. Estimated values of vibration acceleration are slightly larger than the measured values what allows to preserve the certain margin of safety in the case of assessment of the comfort of use of the structure.

It is advisable to develop a load model that allows to determine the *VGRF* generated during slow squats (squats with frequency  $f_{sq} \leq 1.1$  Hz).

Further numerical analyses and field or laboratory tests can be performed to check the effectiveness and correctness of the proposed dynamic load models over a wide range of vibration frequencies.

## References

- [1] S. Dali, M. Justine, H. Ahmad, Z. Othman, "Comparison of ground reaction force during different angle of squatting", *University of Alicante J. Human Sport & Exercise*, Vol. 8, Iss. 3, pp. 778-787, 2013.
- [2] B.J. Schoenfeld, Squatting kinematics and kinetics and their application to exercise performance, *NSCA J. Strength and Conditioning Research*, Vol 24, No. 12, pp. 3497–3506, 2010.
- [3] E. Caetano, Á. Cunha, C. Moutinho, "Vandal loads and induced vibrations on a footbridge" *ASCE J. of Bridge Engineering*, Vol. 6, Iss. 11, pp. 375-382, 2011.
- [4] K. Żółtowski, "Pedestrian on footbridges - loads and response", Publishing House of Gdansk University of Technology, Monograph 82, 2007 (in Polish).
- [5] Bachmann et al., "Vibration Problems in Structures: Practical Guidelines", Birkäuser Verlag, 1995.