

Elastic Lateral Torsional Buckling of Simply Supported Beams under Concentrated Load and Linear Moment Gradient

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Abstract. Lateral torsional buckling is a limit state for I-shaped steel beams that may often be a controlling issue in structural steel design. When these members are not appropriately braced so as to prevent lateral deformations and torsion, they are subjected to risk of failure by lateral torsional buckling before reaching their ultimate capacity. This paper investigates elastic lateral torsional buckling of simply supported I-shaped steel beams under concentrated load and linear moment gradient using design standards and codes, approaches from the literature and finite element analysis. Several unbraced member lengths and end moment values are taken into account to compare and evaluate these approaches in terms of elastic critical moment and end moment ratios. Analysis results show that lateral torsional buckling is a crucial stability problem for I-shaped steel members that are under flexure and it is reflected with adequate safety in the design codes and standards considering finite element analysis outcomes.

1. Introduction

Structural stability problems have substantial effects on the design steps of steel structures. Lateral torsional buckling, where the behaviour changes from mainly in-plane bending to combined lateral deflection and twisting, is one of the most important stability problems and may often be a controlling factor in steel beam design. Therefore, various design standards and codes recommend methods in order to calculate lateral torsional buckling of steel members.

Lateral torsional buckling gained interest in recent years and many analysis approaches are suggested in order to calculate lateral torsional buckling behaviour of steel members [1, 2]. Different material and section properties are also reflected in the studies for identifying the structural behaviour in details [3, 4]. Besides, effects of lateral restraints on lateral torsional buckling are investigated for elastic and inelastic regions [5, 6]. Furthermore, curved members [7] and geometric imperfect members [8] are focused and influences on steel members are discussed. Also, lateral torsional buckling is investigated considering finite differences [9], finite elements [10] and experimental studies [11, 12]. On the other hand, design standards and codes have approaches based on uniform moment loading condition. In order to monitor alternative load cases, moment gradient correction factor, C_b is introduced in the literature and many studies are performed to determine the moment capacity with adequate accuracy [13, 14].

In this study, elastic lateral torsional buckling of I-shaped steel beams under concentrated load and linear moment gradient are examined using finite element analysis and some design standard and code equations. Since design standards and codes vary extensively from one another in the ways that they



characterize the lateral torsional buckling behaviour, this study aims to investigate these approaches using different unbraced member lengths and various end moment values. Lateral torsional buckling analysis results of steel members are shown in terms of elastic critical moment and end moment ratios.

2. Elastic moment capacity under lateral torsional buckling of steel members

Lateral torsional buckling is a stability problem for I-shaped flexural steel members and it depends on the minor axis moment of inertia of its compression flange and the torsional rigidity. For lateral torsional buckling type of failure, critical moment value, which is a function of lateral and torsional stiffness, should be reached under the applied loads or moments. Boundary conditions, unbraced length, material nonlinearities, load pattern and dimensions of the member cross section are some of the parameters that directly affect critical moment capacity. In this study, elastic lateral torsional buckling of I-shaped members under concentrated load and linear moment gradient is investigated for determining critical moment capacity and end moment ratios.

Critical elastic lateral torsional buckling moment capacities for I-shaped steel members are considered in various standards and codes. In this study, AISC 360-10 [15], AS 4100 [16], BS 5950 [17] and TSDC-2016 [18] are considered for comparison. Additionally, in order to improve the efficiency of the analysis approaches that are given in the design specifications, studies about moment gradient factor [19-21] are also presented for evaluation.

2.1 AISC 360-10

Lateral torsional buckling of steel members is studied and a detailed calculation methodology is presented in AISC 360-10 [15]. In this standard, unbraced length limits are calculated and steel member behaviours are classified as elastic or inelastic. If the member length is lower than the length limit given with Eq. (1), this type of buckling is inelastic. Otherwise, lateral torsional buckling is on the elastic range and the calculation process follows the elastic type of buckling routine as outlined in the present study.

$$L_r = 1.95 \sqrt{\frac{I_y C_w}{W_{ex}}} \frac{E}{0.7F_y} \sqrt{\frac{Jc}{W_{ex} h_0} + \sqrt{\left(\frac{Jc}{W_{ex} h_0}\right)^2 + 6.76 \left(\frac{0.7F_y}{E}\right)^2}} \quad (1)$$

where: L_r is length limit for elastic lateral torsional buckling, E is the modulus of elasticity of steel, F_y is the specified yield stress, W_{ex} is the elastic section modulus about strong axis, J is the torsional constant, i_y is the radius of gyration about weak axis, h_0 is the distance between the flange centroids, I_y is the moment of inertia about weak axis, C_w is the warping constant.

Critical elastic lateral torsional buckling moment is given in AISC 360-10 [15] with Eq. (2).

$$M_{cr} = C_b \frac{\pi}{L_b} \sqrt{EI_z GI_t + \left(\frac{\pi E}{L_b}\right)^2 I_z I_w} \quad (2)$$

where: L_b is the lateral buckling length, C_b is the moment modification factor, I_t is the torsion constant, I_w is the warping constant, I_z is the second moment of area about the minor axis, E is the elastic modulus and G is the shear modulus.

In elastic lateral torsional buckling, moment modification factor plays a significant role in determining the critical moment capacity as exposed in Eq. (2). Therefore, moment modification factor is studied in the literature [19-21] for improving the capability of representing lateral torsional buckling behaviour. In the present study, moment modification factor approaches are considered from the literature and structural behaviour is evaluated from this perspective.

Moment modification factor is studied by Kirby and Nethercot (1979) and Eq. (3) for the calculation of C_b , which is applicable for any shape of moment diagrams, is presented [19]. Also, Eq. (3) is given in AISC360-10 for any moment distribution [15].

$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C} \quad (3)$$

where: M_{max} is the absolute maximum moment along L_b , M_A , M_B and M_C , are the absolute moment values at the quarter, center, and three-quarter point, respectively.

Likewise, Serna et al. (2006) provide an equation for moment modification factor by curve fitting of the finite difference analysis results considering wide range of loading and end support conditions [20]. This alternative moment modification factor targets to account lateral rotation and warping restraints at the brace points efficiently and it is presented with Eq. (4) [20].

$$C_b = \sqrt{\frac{35M_{max}^2}{M_{max}^2 + 9M_A^2 + 16M_B^2 + 9M_C^2}} \quad (4)$$

On the other hand, Wong and Driver (2008) offered moment modification factor with Eq. (5) [21].

$$C_b = \frac{4M_{max}}{\sqrt{M_{max}^2 + 4M_A^2 + 7M_B^2 + 4M_C^2}} \leq 2.5 \quad (5)$$

2.2 AS 4100

Nominal member moment capacity under elastic lateral torsional buckling is provided with Eq. (6) in Australian steel design standard (AS 4100) [16].

$$M_b = \alpha_m \alpha_s M_s \leq M_s \quad (6)$$

where: M_b is the nominal member moment capacity, M_s the nominal section moment capacity, M_{oa} is the reference buckling moment, α_m and α_s are determined in Eq. (7) and Eq. (8). M_A , M_B and M_C , are the absolute moments at the quarter, the centre, and the three-quarter point of the member respectively.

$$\alpha_m = \frac{1.7M_m}{\sqrt{M_A^2 + M_B^2 + M_C^2}} \leq 2.5 \quad (7)$$

$$\alpha_s = 0.6 \left[\sqrt{\left(\frac{M_s}{M_{oa}} \right)^2 + 3} - \left(\frac{M_s}{M_{oa}} \right) \right] \quad (8)$$

2.3 BS 5950

In British code for steelworks in buildings (BS 5950), critical elastic lateral torsional buckling moment capacity for the case of beams with doubly symmetric sections and simply supported ends that are subjected to a constant moment over the laterally unbraced length is presented with Eq. (9) [17].

$$M_{cr} = C_1 \frac{\pi^2 EI_z}{(kL)^2} \left[\sqrt{\left(\frac{k}{k_w} \right)^2 \frac{I_w}{I_z} + \frac{(kL)^2 GI_t}{\pi^2 EI_z}} \right] \quad (9)$$

where: I_t is the torsion constant, I_w is the warping constant, I_z is the second moment of area about the minor axis and L is the length of the beam between points which have lateral restraint, E is the modulus of the elastic. The effective length factors k and k_w vary from 0.5 for full fixity to 1.0 for no fixity, with 0.7 for one end fixed and one end free.

BS 5950 provides an equivalent uniform moment factor with C_1 as in Eq. (10) [17].

$$C_1 = \frac{M_{max}}{0.2M_{max} + 0.15M_A + 0.5M_B + 0.15M_C} \leq 2.273 \quad (10)$$

where: values of M_{max} is the absolute maximum moment along L_b , M_A , M_B and M_C , are the absolute moments at the quarter, center, and three-quarter point, respectively.

2.4 TSDC-2016

In TSDC-2016 [18], in the case of elastic lateral torsional buckling, Eq. (11) and Eq. (12) are used for calculating elastic lateral torsional buckling moment M_n .

$$M_n = F_{cr} W_{ex} \leq M_p \quad (11)$$

$$F_{cr} = \frac{C_b \pi^2 E}{\left(\frac{L_b}{i_{ts}} \right)^2} \sqrt{1 + 0.078 \frac{Jc}{W_{ex} h_0} \left(\frac{L_b}{i_{ts}} \right)^2} \quad (12)$$

where: F_{cr} is the critical yielding point, E is the modulus of elasticity of steel, L_b is the length of the unbraced segment of the member, W_{ex} is the elastic section modulus about strong axis, J is the torsional constant, h_0 is the distance between the flange centroids.

On the other hand, Eurocode 3 [22] does not provide information on how to compute moment modification factor and critical elastic moment under concentrated load and moment gradient.

3. Numerical example

Elastic lateral torsional buckling behaviours of simply supported I-shaped steel beams under concentrated load and linear moment gradient are investigated considering steel IPE300 section. Load details of simply supported steel beam are given in Figure 1. Geometric and section properties of IPE300 is presented in Figure 2.

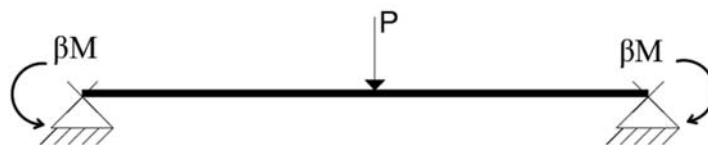


Figure 1. Simply supported I-shaped member under concentrated load and linear moment gradient

Section properties for IPE300 profile is defined considering Figure 2. Height of the section, h is 300 mm, width of the flange, b_f is 150 mm, radius of the section, r is 15 mm, thickness of the flange, t_f is 10.7 mm, thickness of the web, t_w is 7.1 mm, torsion constant, I_t is 198681 mm⁴, warping constant, I_w is 1.26 x 10¹¹ mm⁶, second moment of area about the minor axis, I_z is 6037800 mm⁴.

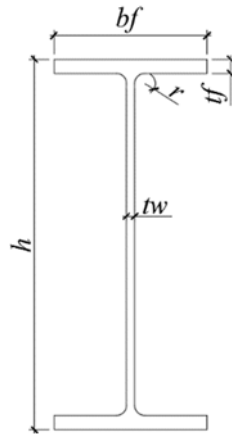


Figure 2. I-shaped steel member section

For the numerical example, methods presented in the design standards and codes, moment gradient factor equations are considered and lateral torsional buckling analysis are performed. Moreover, LTBeam [23] and finite element analysis are applied in order to evaluate the outcomes of the study.

In order to plot the end moment ratio to elastic lateral torsional buckling moment capacity, different unbraced length conditions are considered in the present study. These unbraced lengths are selected as 6 m, 8 m, 10 m, and 12 m in which lateral torsional buckling occurs in the elastic range.

4. Analysis results

Lateral torsional buckling behaviours of I-shaped steel beams are calculated for simply supported steel IPE300 beam considering unbraced lengths. In the study, AISC360-10 [15], AS4100 [16], BS5950 [17] and TSDC-2016 [18] methods are compared with finite element analysis and LTBeam [23] outcomes. Furthermore, moment gradient factor proposed by researchers [19-21] are considered as well and the results are presented for evaluation.

Elastic lateral torsional buckling moment capacity considering the changes in the end moment ratio values for 6 m unbraced length is given in Figure 3. Likewise, end moment ratios and elastic buckling moment capacity relations for different moment gradient factors are presented in Figure 4.

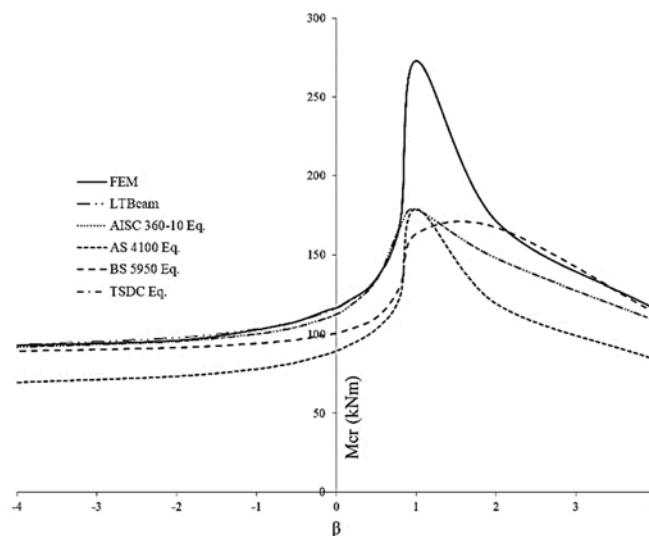


Figure 3. End moment ratio (β) and elastic critical moment (M_{cr}) for I-beam with $L_b = 6$ m

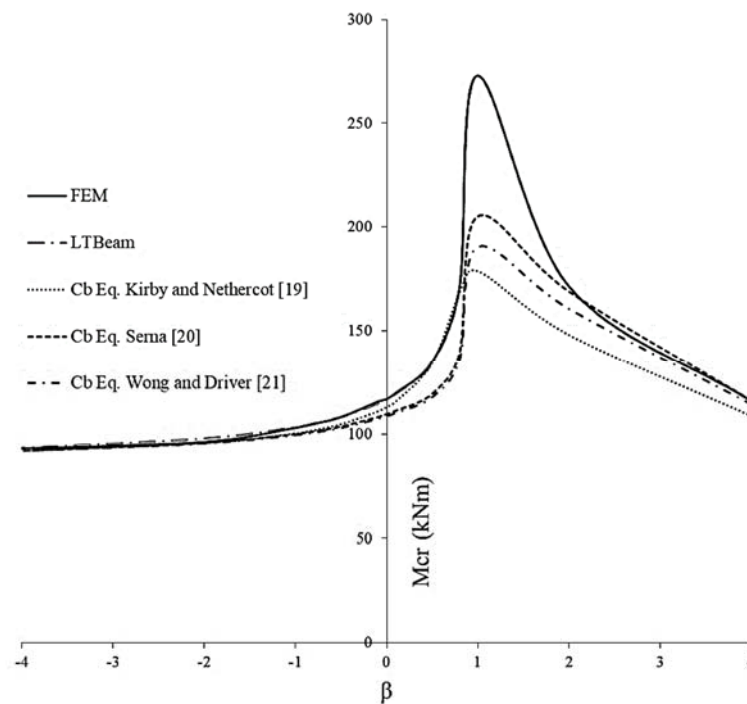


Figure 4. End moment ratio (β) and elastic critical moment (M_{cr}) for I-beam with $L_b = 6$ m considering C_b factors

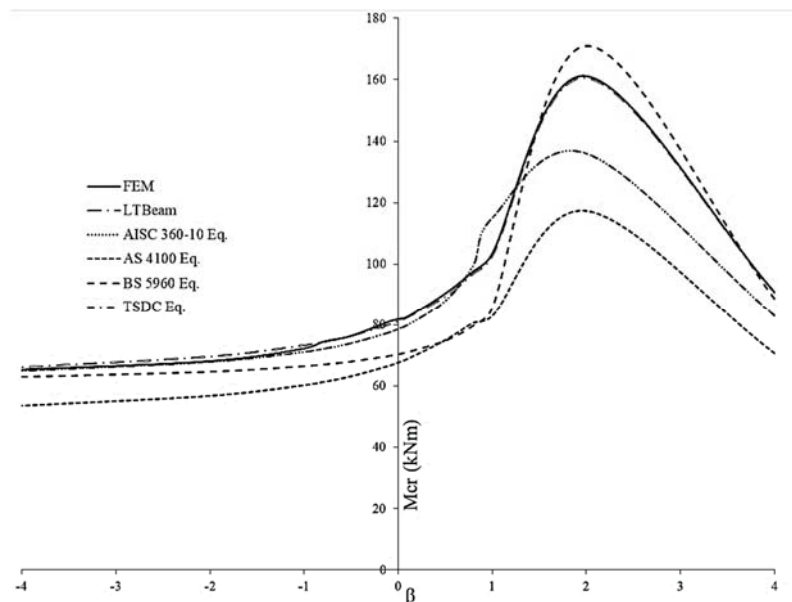


Figure 5. End moment ratio (β) and elastic critical moment (M_{cr}) for I-beam with $L_b = 8$ m

Elastic lateral torsional buckling moment capacity considering the changes in the moment ratio values for 8 m unbraced length is given in Figure 5. Likewise, end moment ratios and elastic buckling moment capacity relations for different moment gradient factors are presented in Figure 6.

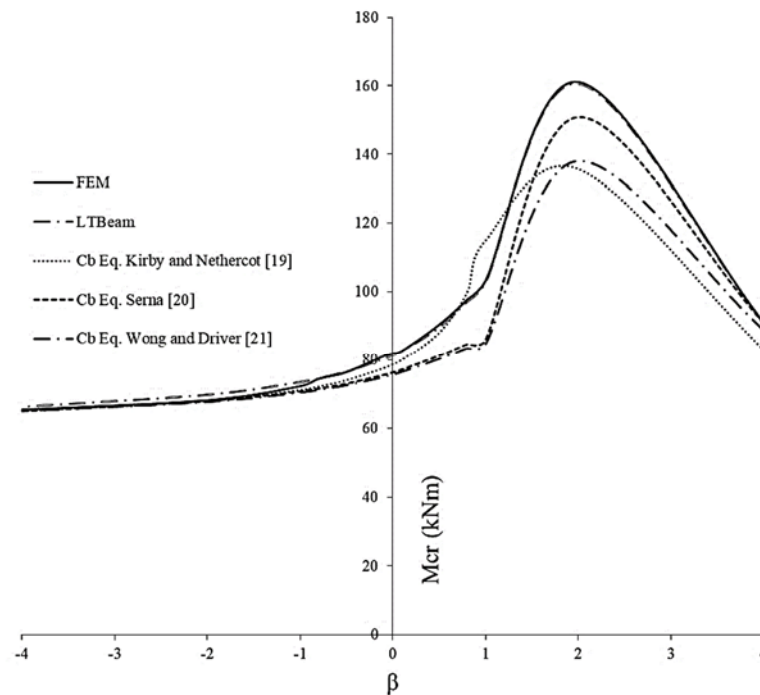


Figure 6. End moment ratio (β) and elastic critical moment (M_{cr}) for I-beam with $L_b = 8$ m considering C_b factors

Elastic lateral torsional buckling moment capacity considering the changes in the moment ratio values for 10 m unbraced length is given in Figure 7. Likewise, end moment ratios and elastic buckling moment capacity relations for different moment gradient factors are presented in Figure 8.

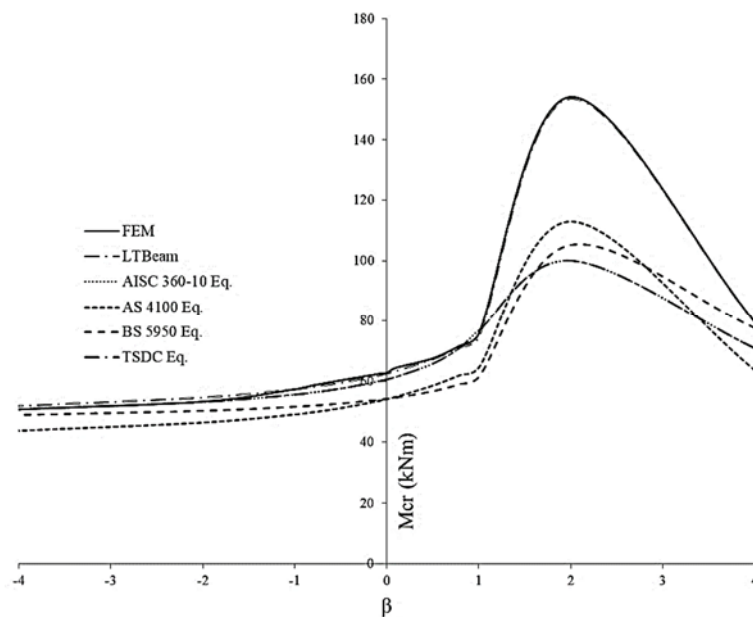


Figure 7. End moment ratio (β) and elastic critical moment (M_{cr}) for I-beam with $L_b = 10$ m

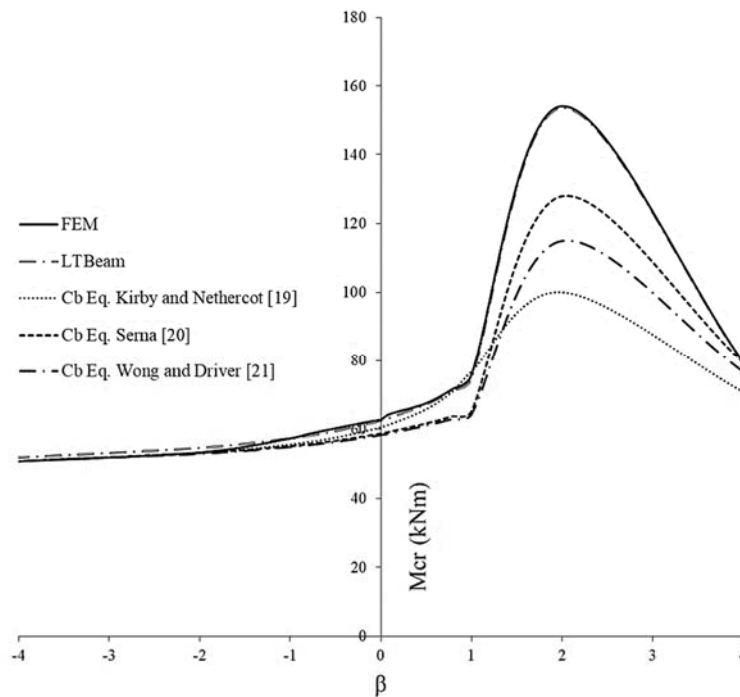


Figure 8. End moment ratio (β) and elastic critical moment (M_{cr}) for I-beam with $L_b = 10$ m considering C_b factors

Finally, elastic lateral torsional buckling moment capacity considering the changes in the moment ratio values for 12 m unbraced length is given in Figure 9. Likewise, end moment ratios and elastic buckling moment capacity relations for different moment gradient factors are presented in Figure 10.

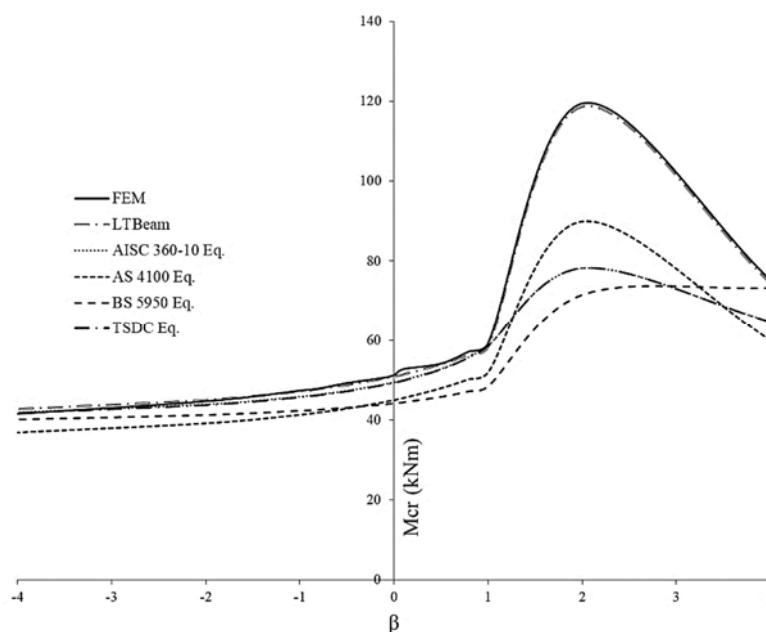


Figure 9. End moment ratio (β) and elastic critical moment (M_{cr}) for I-beam with $L_b = 12$ m

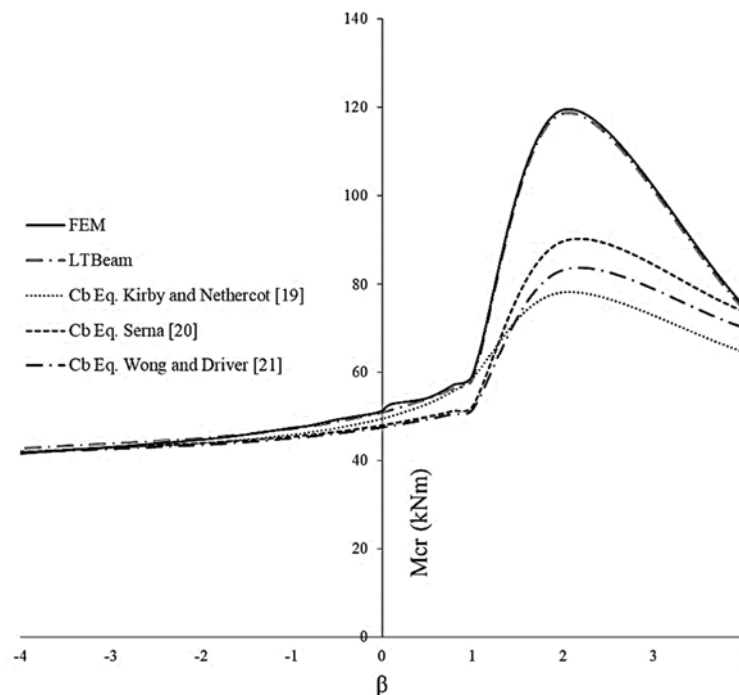


Figure 10. End moment ratio (β) and elastic critical moment (M_{cr}) for I-beam with $L_b = 12$ m considering C_b factors

5. Conclusion

Elastic lateral torsional buckling behaviours of I-shaped steel beams are presented considering finite element analysis outcomes, design standard and code approaches for concentrated load and linear moment gradient with different unbraced member lengths. End moment ratios and elastic critical moment relations are shown in graphics and calculation results are plotted.

1. Lateral torsional buckling is significant for steel I-shaped beams that are under flexure and it is reflected acceptably in the considered design codes and standards due to finite element analysis results. However, in order to reflect more accurate results, improvements about moment gradient factor and lateral torsional buckling is required.
2. Numerical examples of the study show that AS 4100 generally give more conservative results than the other standards and codes. Since AISC 360-10 and TSDC-2016 have similar elastic lateral torsional buckling equations, analysis results are identical. BS 5950 analysis results are close to FEM and LTBeam results for some cases. Indeed, for 8 m unbraced length case, BS 5950 give critical moment values higher than FEM analysis results. This is due to moment modification factor equation of BS 5950 in which midpoint moment have significant effect on the behaviour.
3. Unbraced member length increases cause major decrease in elastic moment capacity under lateral torsional buckling. On the other hand, increase in unbraced member length and end moment ratios cause closer elastic critical moment results for all approaches.
4. In order to enhance the precision of determining the elastic critical moment capacity, moment gradient factors from the literature are discussed. Results of the numerical examples of this study show that moment modification factor equation provided by Serna et al. (2006) give closer results than the other approaches considered in the study considering FEM analysis results.

References

- [1] A.B. Benyamina, S.A. Meftah, F. Mohri and E.M. Daya , “Analytical solutions attempt for lateral

- torsional buckling of doubly symmetric web-tapered I-beams,” *Engineering Structures*, vol. 56, pp. 1207-1219, 2013.
- [2] H. Ozbasaran, R. Aydin, and M. Dogan, “An alternative design procedure for lateral–torsional buckling of cantilever I-beams”, *Thin-Walled Structures*, vol. 90, pp. 235-242, 2015.
 - [3] Y.Q. Wang, H.X. Yuan, Y.J. Shi, and M. Chen, “Lateral-torsional buckling resistance of aluminium I-beam,” *Thin-Walled Structures*, vol. 50, pp. 24-36, 2012.
 - [4] L.S. da Silva, C. Rebelo, D. Nethercot, L. Marques, R. Simões, and P.M.M. Real, “Statistical evaluation of the lateral–torsional buckling resistance of steel I-beams, Part 2: Variability of steel properties,” *Journal of Constructional Steel Research*, vol. 65, pp. 832-849, 2009.
 - [5] N.S. Trahair, “Buckling analysis design of steel frames”, *Journal of Constructional Steel Research*, vol. 65, pp. 1459-1463, 2009.
 - [6] X.X. Li, “Flexural strength for general lateral-torsional buckling”, *Journal of Structural Engineering*, vol. 133, pp. 674-682, 2007.
 - [7] T. Usami, and S. Y. Koh, "Large displacement theory of thin-walled curved members and its application to lateral torsional buckling analysis of circular arches." *Journal of Solids and Structures*, vol. 16, pp. 71-95, 1980.
 - [8] A. Agüero, F.J. Pallarés, and L. Pallarés, “Equivalent geometric imperfection definition in steel structures sensitive the lateral torsional buckling due to bending moment,” *Engineering Structures*, vol. 96, pp. 41-55, 2015.
 - [9] B. Suroatmono, D. Ho, “The-moment gradient factor in lateral-torsional buckling on wide flange steel sections,” *Journal of Constructional Steel Research*, vol. 45, pp. 1247-1264, 2002.
 - [10] M.R. Aydin, “Elastic flexural and lateral torsional buckling analysis of frames using finite elements,” *KSCCE Journal of Civil Engineering*, vol. 14, pp. 25-31, 2010.
 - [11] J. Singer, J. Arbocz, and T. Weller, “Buckling experiments: experimental methods in buckling of thin-walled structures,” *John Wiley & Sons*, Vol. 1, 1998.
 - [12] X. Zhang, K.J.R. Rasmussen and H. Zhang, “Experimental investigation of locally and distortionally buckled portal frames,” *Journal of Constructional Steel Research*, vol. 122, pp. 571-583, 2016.
 - [13] D.W. White, Y.D. Kim, “Unified flexural resistance equations for stability design of steel I-section members: Moment gradient tests,” *Journal of Structural Engineering*, vol. 134, pp. 1471-1486, 2008.
 - [14] N.H. Lim, N.H. Park, Y.J. Kank, and I.H. Sung, “Elastic buckling of I-beams under linear moment gradient,” *Journal of Solids and Structures*, vol. 40, pp. 5635-5647, 2003.
 - [15] Specification for Structural Steel Buildings. ANSI/AISC 360-10. 2010. American Institute of Steel Construction, Chicago.
 - [16] Steel structures. Standards Australia. AS4100. 1998. NSW, Australia.
 - [17] Structural use of steelwork in buildings. Code of practice for design. Rolled and welded sections. BS 5950-1. 2000. British Standards Institution.
 - [18] Turkish Steel Design Code. TSDC. 2016. Ministry of Environment and Urbanization, Ankara, Turkey.
 - [19] P. A. Kirby, and D.A. Nethercot, “Design for structural stability,” *Halsted Press*, 1979.
 - [20] M.A. Serna, A. Lopez, I. Puente, and D.J. Yong, “Equivalent uniform moment factors for lateral-torsional buckling of steel members,” *Journal of Constructional Steel Research*, vol. 62, pp. 566-580, 2006.
 - [21] E. Wong, and R.G. Driver, “Critical Evaluation of Equivalent Moment Factor Procedures for Laterally Unsupported Beams.” *Engineering Journal*, American Institute of Steel Construction, vol. 47, First Quarter, pp. 1-20, 2010.
 - [22] European Committee for Standardization. EN 1993-1-1. Eurocode 3: Design of steel structures. Part 1–1: General rules and rules for buildings. 2005. Brussels.
 - [23] CTICM, Centre Technique Industriel de la Construction Méallique: LTBeam v. 1.0.10. 2010. Documentation, Saint-Aubin, France, CTICM.