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Prediction of Cracking Induced by Indirect Actions in RC Structures

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Abstract. Cracking of concrete plays a key role in reinforced concrete (RC) structures design, especially in serviceability conditions. A variety of reasons contribute to develop cracking and its presence in concrete structures is to be considered as almost unavoidable. Therefore, a good control of the phenomenon in order to provide durability is required. Cracking development is due to tensile stresses that arise in concrete structures as a result of the action of direct external loads or restrained endogenous deformations. This paper focuses on cracking induced by indirect actions. In fact, there is very limited literature regarding this particular phenomenon if compared to its high incidence in the construction practice. As a consequence, the correct prediction of the crack opening, width and position when structures are subjected to imposed deformations, such as massive castings or other highly restrained structures, becomes a compelling task, not so much for the structural capacity, as for their durability. However, this is only partially addressed by commonly used design methods, which are usually intended for direct actions. A set of non-linear analysis on simple tie models is performed using the Finite Element Method in order to study the cracking process under imposed deformations. Different concrete grades have been considered and analysed. The results of this study have been compared with the provisions of the most common codes.

1. Introduction

Many studies on the prediction of the crack width grounded on experimental data are available in literature, like those by Gergely and Lutz [1], Oh and Kang [2], Frosch [3] and Gerstle [4]. Crack width is related to the geometry and position of the reinforcing steel, and to the bond between steel bars and concrete, as presented by Goto [5].

More recent research focuses on the factors that influence the crack width itself, like the works of Borosnyói et Balázs [6] and Beeby [7] that evidenced how the transverse reinforcement plays a significant role on the crack spacing.

Crack widths are generally calculated by designers using simplified methods adopted by design codes (i.e. Model Code 1990 [8] and 2010 [9] or Eurocode 2 [10]). The methods proposed by these three codes are very similar and calculate the crack width by multiplying the maximum crack spacing by the strain of the steel reinforcement in the crack. Nevertheless, the formulations proposed are generally calibrated for direct action induced cracking, while limited attention has been given to implicit actions [11].

In this paper, grounding on the previous work of the authors [12], cracking induced by implicit actions like shrinkage or temperature variations has been studied on the simplest structure: a tie under pure axial actions subject to different static schemes in order to simulate different levels of restraint at the extremities.

Two concrete classes and several reinforcement configurations have been investigated. Crack width has been calculated by means of non-linear discrete cracking FEM (Finite Elements Model) models and finite differences hand calculations both applying explicit actions (an external force pulling the bars at one end of the tie) and applying imposed deformations (thermal cooling of concrete). The results have been compared with the output of the calculations done following the approach proposed by design codes (MC2010 and EC2).

The effect of the variation in time of the concrete Young modulus and the creep effect have not been taken into consideration in this work, but it is clear from the results of the present work and many other studies available in literature [13] [14] [15] that they will lead to a significant decrease of cracking phenomena when combined with shrinkage imposed deformations.

2. Model used to evaluate direct actions induced cracking

Direct actions induced cracking has been studied on a reinforced concrete tie with length L, concrete cross section A_c, and steel cross section A_s already described in [12].

The tie is perfectly restrained at its starting point, A, free at end point, B, and can be loaded in B as shown in figure 1(a) in two different ways:

1. applying the force F only to the bars;
2. applying the force F to both concrete and steel.

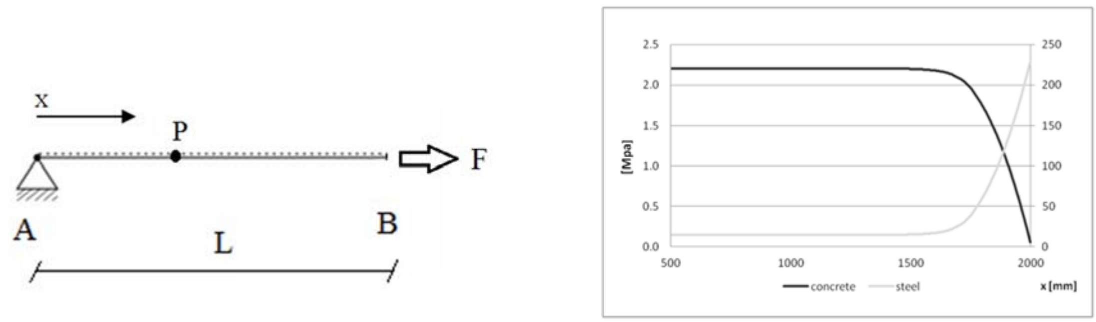

Figure 1. (a) Static scheme used to evaluate the effect of explicit actions, (b) stresses induced by explicit action before crack formation

In case 1, if the length of the tie L is longer than the transfer length l_s, when the applied force is $F_{cr,1} = f_{ct} \cdot A_0$, (where A_0 is the homogenized area $A_c + nA_s$) the tensile strength in concrete is reached, $\sigma_c = f_{ct}$, as shown in figure 1(b) and the first crack forms.

In case 2, the force F generates a uniform state of stress along the tie with $\sigma_s = n \cdot \sigma_c$. Therefore, the force $F_{cr,1}$ can also be interpreted as the force applied to both concrete and steel in point B, which causes a constant cracking stress in the tie. The position of the first crack, in this case, cannot be predicted with a deterministic approach, therefore it can be arbitrarily chosen exactly in point B as it does not affect the result of the investigation. The second crack will then appear for the same force in any point outside the transfer length exactly as in case 1.

3. Model used to evaluate implicit actions induced cracking

Implicit actions induced cracking has been studied on a tie with the same cross sections and materials, but a different static scheme, called 2, where the tie is perfectly restrained at its starting point A, and end point C, and it is long 2L as shown in figure 2(a).

If an implicit action like an imposed shortening strain $\bar{\varepsilon}_{cs}$ (i.e. thermal cooling or shrinkage) arises in concrete, it will tend to shorten. Therefore, an elastic deformation $\varepsilon_{ce} = - \bar{\varepsilon}_{cs}$ will occur in concrete and a consequent tensile stress $\sigma_c = E_c \cdot \varepsilon_{ce}$ will generate. However, due to the restrain level at beginning and end of the tie, no total deformation can arise in every point P of the tie. No deformation and no stress will be present in steel in such situation.

When the tensile stress in concrete reaches the tensile strength, $\sigma_c = f_{ct}$, the first crack will appear in the tie and a completely new configuration of strain and stresses will take place in both concrete and steel because of bond-slip interaction between the materials. Again, the position of the first crack cannot be predicted with a deterministic approach, therefore it can be arbitrarily chosen in any point of the tie. If chosen in the midspan, it leads to a third static scheme, where the tie has a length L, both concrete and steel are restrained in point A and only steel is restrained in point B, as shown in figure 2(b) and as described in [12].

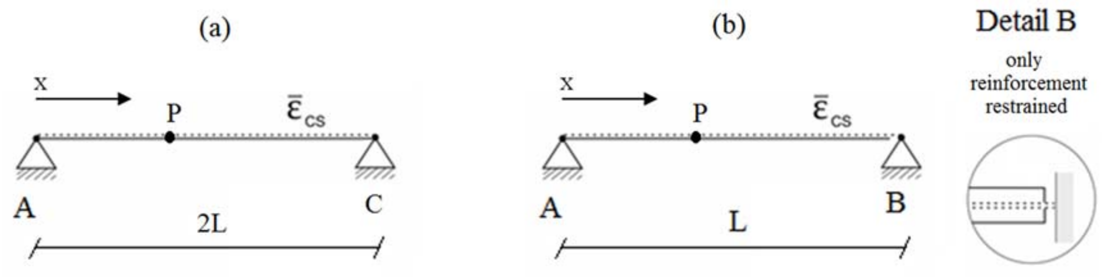

Figure 2. Static schemes 2 and 3 used to evaluate the effect of imposed strains

4. Solution procedures

Direct actions induced cracking has been studied in the model described in paragraph 2 by means of both non-linear discrete cracking FEM models [16] and finite differences hand calculations.

Implicit actions induced cracking has been investigated only by means of non-linear (N.L.) FEM analysis, as closed form solution is nontrivial. The obtained results for both direct and implicit actions, in terms of crack widths and crack spacing, have been compared with MC2010 [9] and EC2 [10] provisions.

In the non-linear finite element model, concrete and steel are modelled with truss elements connected by interface elements according to MC2010 bond-slip law [9], [17], [18]. Incremental load stepping is applied. Steel is modelled as elastoplastic material. With regards to concrete, discrete cracking approach is followed substituting concrete elastic elements with brittle elements where tensile strength of concrete is reached.

If concrete tensile strength is reached contemporaneously in many different elements, the position of the crack is arbitrarily chosen at the minimum distance from the closest one. This choice does not influence the results and their interpretation, as shown in the following. Several tests using different elements lengths have been done to exclude results mesh-dependence.

5. Test Cases

Concrete classes C20 and C50 have been considered in the simulations in order to represent a low strength concrete and a high strength one. The mechanical parameters of steel and concrete used in the analyses are resumed in table 1.

Table 1. Material properties

Concrete class	f_{cm} [MPa]	f_{ctm} [MPa]	E [GPa]	*Steel*	f_{ym} [MPa]	E [GPa]
C20	28	2.20	30.3	B450C	490	200
C50	58	4.10	38.6			

Concrete cross section is circular with diameter of 200mm. Seven reinforcement layouts have been compared in order to study the effect of different geometrical reinforcement ratios $\rho = A_s / A_c$, mechanical reinforcement ratios $\omega = (A_s \cdot f_{ym}) / (A_c \cdot f_{ctm})$ and bond ratios $\rho = u_s / A_c$, where u_s is the perimeter of the steel bars. The layouts are presented in table 2.

Table 2. Reinforcement layouts and geometry

Layout	rebar's [-]	ρ [-]	ρ_b [mm^{-1}]	ω C20 [-]	ω C50 [-]
1	2ϕ10	0.50%	0.20%	1.1	0.60
2	4ϕ10	1.00%	0.40%	2.2	1.20
3	1ϕ14	0.49%	0.14%	1.1	0.59
4	2ϕ14	0.98%	0.28%	2.2	1.17
5	4ϕ14	1.96%	0.56%	4.4	2.34
6	1ϕ20	1.00%	0.20%	2.2	1.20
7	2ϕ20	2.00%	0.40%	4.5	2.39

Each specimen has been subjected to external axial force according to load scheme n°1, as shown in Figure 1, obtaining crack spacing and crack opening in good accordance with crack previsions given in MC2010 for explicit actions. Once validated the model with direct loading, internal imposed deformations ε_{cs} have been applied in load scheme n°3.

6. Crack width calculations following codes provisions

The relationships proposed by *fib* MC2010 [9] and EC2 [10] for the calculation of cracks opening w, are presented in this paragraph for comparison with FEM results. Both codes require a minimum reinforcement in order to avoid yielding at first cracking as follows:

$$A_{s,\min} = \frac{A_c \cdot f_{ct}}{f_y} \tag{1}$$

that is to say a minimum mechanical reinforcement ratio equal to one should be expected.

$$\omega_s = \frac{A_s \cdot f_y}{A_c \cdot f_{ct}} > \omega_{s,\min} = 1 \tag{2}$$

Once minimum reinforcement is provided, crack width calculation can be done according to the following paragraphs. It is important to underline that reinforcement layouts 1 and 3 (2ϕ10 and 1ϕ14) do not respect the minimum reinforcement requirement when associated to C50, as can be seen in table 2, therefore will undergo yielding after cracking.

The crack width can be calculated following MC2010 and EC2 respectively using equations (3) and (4). MC2010 provides a design width w_d whereas EC2 provides a characteristic value w_k.

$$MC2010 \quad w_d = 2 \cdot l_{s,\max} \left(\varepsilon_{sm} - \varepsilon_{cm} - \varepsilon_{cs} \right) \tag{3}$$

$$EC2 \quad w_k = 2 \cdot l_{s,\max} \left(\varepsilon_{sm} - \varepsilon_{cm} \right) \tag{4}$$

where:

- $l_{s,max}$ is the transfer length, that is the distance between the crack and the point where the maximum stress in concrete is reached or, in other words, the length over the slip between concrete and steel occur;
- ε_{sm} is the average steel strain along $l_{s,max}$;
- ε_{cm} is the average concrete strain along $l_{s,max}$;
- ε_{cm} is the strain of the concrete due to (free) shrinkage.

The value of $l_{s,max}$ can then be calculated respectively using equations (5) and (6):

$$MC2010 \quad l_{s,\max} = k \cdot c + \frac{1}{4} \cdot \frac{f_{ctm}}{\tau_{bms}} \cdot \frac{\phi}{\rho} \tag{5}$$

$$EC2 \quad 2l_{s,\max} = k_3 \cdot c + k_1 k_2 k_4 \cdot \frac{\phi}{\rho} \tag{6}$$

where:

- k, is an empirical parameter to account for cover and is taken here equal to one;
- c, is the concrete cover;
- f_{ctm}, is the mean cylinder compressive strength;
- ϕ, is the longitudinal bar diameter;
- ρ, is the reinforcement ratio calculated as in section 5;
- τ_{bms} , is the mean bond strength between steel and concrete (assumed for the case of short term crack formation stage as equal to $1.8 \cdot f_{ctm}$).
- k_1 is a coefficient assumed as 0.8 for good bond conditions;
- k_2 is a coefficient assumed as 1 in the case of a simple tie;
- k_3 and k_4 are coefficient set equal to 3.4 and 0.425 as recommended;

The relative mean strain defined in eq.(3) and (4) may be evaluated as follows. MC2010 allows to account for the effect of implicit actions as the value of imposed deformation can be introduced in eq. (7) whereas EC2 does not.

$$MC2010 \quad \varepsilon_{sm} - \varepsilon_{cm} - \varepsilon_{cs} = \frac{\sigma_s - \beta \cdot \sigma_{sr}}{E_s} + \eta_r \cdot \varepsilon_{sh} \tag{7}$$

$$EC2 \quad \varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - \beta \cdot \sigma_{sr}}{E_s} \geq 0.6 \cdot \frac{\sigma_s}{E_s} \tag{8}$$

where:

- $\sigma_s = (A_c + \alpha_e A_s) \cdot f_{ctm}/A_s$, is the steel stress in a crack ;
- $\sigma_{sr} = (f_{ctm}/\rho) \cdot (1+\alpha_e \cdot \rho)$, is the maximum steel stress in a crack into crack formation stage;
- $\alpha_e = E_c/E_s$ is the modular ratio,
- β is an empirical coefficient set equal to 0.6 for short term/instantaneous loading;
- η_r, is a coefficient for considering the shrinkage effect, in this case assumed as unit;
- ε_{sh} is the imposed strain ($\varepsilon_{sh} = \acute{\varepsilon}_{cs}$ in this work).

No relation between design crack widths, w_d , characteristic ones, w_k, and mean ones, w_m , is given in the codes. The coincidence between w_d and w_k is generally accepted, being the safety coefficient in serviceability equal to one. The mean value of crack width w_m should be smaller than w_k; ENV1992-1-

1 [19] suggests the relation w_k = 1.7 w_m. The authors found no statistical evidence on this subject in literature.

7. Results comparison

7.1. Direct actions induced cracking

The comparison between the crack spacing and crack opening calculated for concrete C20 on static scheme 1, described in paragraph 2, are presented in table 3.

Table 3. Results of direct actions induced cracking

			Finite differences		N.L. FEM		MC2010		EC2	
Layout	rebars [-]	F [kN]	w_m [mm]	$l_{s,min}$ [mm]	w_m [mm]	$l_{s,min}$ [mm]	w_d [mm]	l_s [mm]	w_k [mm]	l_s [mm]
1	2f10	71.3	0.47	345	0.46	495	0.67	368	0.90	493
2	4f10	73.6	0.17	250	0.18	440	0.21	229	0.30	323
3	1f14	71.3	0.61	440	0.60	550	0.91	490	1.19	644
4	2f14	73.5	0.23	325	0.23	725	0.27	284	0.37	389
5	4f14	78.0	0.09	235	0.09	540	0.09	185	0.14	268
6	1f20	73.6	0.29	415	0.29	855	0.35	368	0.46	493
7	2f20	78.2	0.11	300	0.11	645	0.11	229	0.15	306

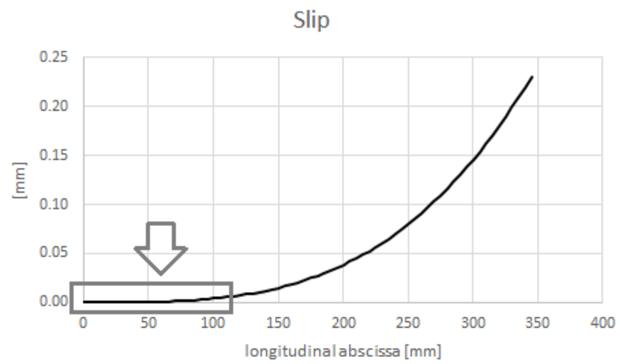

Figure 3. Part of the slip curve that can lead to poor accuracy in calculation of crack spacing

The following considerations can be drawn:

1. N. L. FEM and hand calculations are done using mean material properties, being non- linear analyses, and provide excellent accordance in terms of crack opening, but not in term of crack spacing. N. L. FEM seems not to be able to estimate precisely crack spacing as provides the same spacing for different bar layouts.
2. The lack of accuracy of N.L. FEM in predicting crack spacing does not affect the result in term of crack opening. The part of the slip curve relative to the zones where concrete stress undergoes a very small variation (shown in the gray box in Figure 3) can be easily miscalculated by FEM; however, even a significant error in the abscissa leads to a very small inaccuracy in the slip calculation, allowing a correct evaluation of crack opening.
3. MC2010 provides both crack width and crack spacing in good accordance with finite differences solution. The design values "d" seems then to be equal to mean ones "m".

4. EC2 generally provides wider crack spacing than MC2010 and therefore bigger crack widths. It must be underlined that for uniformity of comparison the limit $(\varepsilon_{sm}-\varepsilon_{cm})\geq 0.6\sigma_s/E_s$ has been neglected otherwise $\varepsilon_{sm}-\varepsilon_{cm}$ would have been equal to $0.6\sigma_s/E_s$ for all layouts.

7.2. Implicit actions induced cracking

The results of the N.L. FEM simulations in terms of number of cracks n_c, average crack opening w_m, average stress in the steel bars in correspondence of the cracks $\sigma_{s,m}$ and maximum stress in concrete between cracks $\sigma_{c,max}$ are presented in table 4 and 5 respectively for concrete class C20 and C50.

Table 4. Results of numerical simulations for C20.

Rebars	$\acute{\varepsilon}_{cs}$	n_c	w_m	$\sigma_{s,m}$	$\sigma_{c,max}$	Rebars	$\acute{\varepsilon}_{cs}$	n_c	w_m	$\sigma_{s,m}$	$\sigma_{c,max}$
	[-]	[-]	[mm]	[MPa]	[MPa]		[-]	[-]	[mm]	[MPa]	[MPa]
2ϕ10 ρ 0.50%	7.26E-05	1	0.16	193	1.05	4ϕ14 ρ 1.96%	7.26E-05	1	0.07	82	1.76
	1.85E-04	1	0.46	404	2.2		9.25E-05	1	0.09	103	2.2
	1.85E-04	2	0.2	212	1.23		9.25E-05	2	0.06	70	1.6
	4.13E-04	2	0.46	366	2.2		1.34E-04	2	0.09	100	2.2
	4.13E-04	3	0.3	252	1.64		1.34E-04	3	0.07	75	1.77
	6.40E-04	3	0.46	322	2.2		1.76E-04	3	0.09	92	2.2
4ϕ10 ρ 1.00%	7.26E-05	1	0.1	136	1.49		1.76E-04	4	0.07	72	1.86
	1.14E-04	1	0.18	202	2.2		2.17E-04	4	0.09	84	2.2
	1.14E-04	2	0.09	124	1.41		2.17E-04	5	0.07	67	1.92
	1.99E-04	2	0.18	190	2.2		2.58E-04	5	0.09	76	2.2
	1.99E-04	3	0.12	137	1.69		2.58E-04	6	0.08	64	1.86
	2.84E-04	3	0.18	175	2.2		3.30E-04	6	0.1	74	2.2
	2.84E-04	4	0.13	135	1.82	1ϕ20 ρ 1.00%	7.26E-05	1	0.13	117	1.27
	3.69E-04	4	0.18	159	2.2		1.41E-04	1	0.29	201	2.2
1ϕ14 ρ 0.49%	7.26E-05	1	0.17	174	0.93		1.41E-04	2	0.14	110	1.31
	2.20E-04	1	0.61	409	2.2		2.80E-04	2	0.29	177	2.2
	2.20E-04	2	0.25	204	1.2		2.80E-04	3	0.19	121	1.66
	5.10E-04	2	0.6	357	2.2		4.15E-04	3	0.29	152	2.2
	5.10E-04	3	0.37	241	1.62		4.15E-04	4	0.21	110	1.8
	8.00E-04	3	0.59	303	2.2		5.63E-04	4	0.29	125	2.2
2ϕ14 ρ 0.98%	7.26E-05	1	0.12	129	1.37	2ϕ20 ρ 2.00%	7.26E-05	1	0.08	77	1.68
	1.27E-04	1	0.23	206	2.2		9.73E-05	1	0.11	101	2.2
	1.27E-04	2	0.12	120	1.36		9.73E-05	2	0.08	66	1.54
	2.37E-04	2	0.23	185	2.2		1.50E-04	2	0.11	93	2.2
	2.37E-04	3	0.15	133	1.67		1.50E-04	3	0.08	69	1.74
	3.48E-04	3	0.23	170	2.2		2.02E-04	3	0.11	85	2.2
	3.48E-04	4	0.17	128	1.82		2.02E-04	4	0.09	65	1.86
	4.58E-04	4	0.23	148	2.2		2.50E-04	4	0.11	75	2.2
	4.58E-04	5	0.18	116	1.76		2.50E-04	5	0.09	59	1.76
	6.80E-04	5	0.28	137	2.2		3.50E-04	5	0.13	71	2.2

Table 5. Results of numerical simulations for C50.

Rebars	$\acute{\varepsilon}_{cs}$ [-]	n_c [-]	w_m [mm]	$\sigma_{s,m}$ [MPa]	$\sigma_{c,max}$ [MPa]	Rebars	$\acute{\varepsilon}_{cs}$ [-]	n_c [-]	w_m [mm]	$\sigma_{s,m}$ [MPa]	$\sigma_{c,max}$ [MPa]
2ϕ10 ρ 0.50%	1.06E-04	1	0.25	327	1.74	4ϕ14 ρ 1.96%	1.06E-04	1	0.11	149	3.11
	1.94E-04	1	0.50	490	2.65		1.44E-04	1	0.17	196	4.10
	5.80E-04	1	1.98	560	3.39		1.44E-04	2	0.10	131	2.87
1ϕ14 ρ 0.49%	1.06E-04	1	0.27	293	1.53		2.22E-04	2	0.17	184	4.10
	2.19E-04	1	0.63	490	2.60		2.22E-04	3	0.12	137	3.25
	5.72E-04	1	1.96	560	3.25		2.99E-04	3	0.17	169	4.10
4ϕ10 ρ 1.00%	1.06E-04	1	0.17	241	2.56	1ϕ20 ρ 1.00%	1.06E-04	1	0.21	201	2.15
	1.86E-04	1	0.34	385	4.10		2.37E-04	1	0.55	379	4.10
	1.86E-04	2	0.17	229	2.56		2.37E-04	2	0.24	202	2.39
	3.46E-04	2	0.33	358	4.10		4.94E-04	2	0.54	330	4.10
	3.46E-04	3	0.22	259	3.15		4.94E-04	3	0.35	225	3.10
	5.07E-04	3	0.33	327	4.10		7.47E-04	3	0.53	279	4.10
2ϕ14 ρ 0.98%	1.06E-04	1	0.17	224	2.34	2ϕ20 ρ 2.00%	1.06E-04	1	0.13	139	2.95
	2.11E-04	1	0.42	390	4.10		1.53e-04	1	0.21	192	4.10
	2.11E-04	2	0.19	219	2.46		1.53E-04	2	0.12	121	2.76
	4.20E-04	2	0.43	352	4.10		2.50E-04	2	0.21	175	4.10
	4.20E-04	3	0.28	312	3.12		2.50E-04	3	0.14	126	3.21
	6.30E-04	3	0.43	240	4.10		3.74E-04	3	0.21	156	4.10

Furthermore, table 6 shows the minimum crack spacing and the average bond stress.

Table 6. Minimum crack spacing and average bond stresses.

		C20		*C50*	
Bar layout	rebars [-]	l_s [mm]	τ_{bm}/f_{ctm} [-]	l_s [mm]	τ_{bm}/f_{ctm} [-]
1	2ϕ10	436	2.29	*	*
2	4ϕ10	410	1.22	482	1.04
3	1ϕ14	495	2.89	*	*
4	2ϕ14	463	1.54	489	1.46
5	4ϕ14	401	0.89	492	0.73
6	1ϕ20	521	1.92	499	2.00
7	2ϕ20	471	1.06	484	1.03

The following considerations can be drawn from the analysis of the results:

1. The imposed deformation that generates the first crack is the same for all the specimens and related only to concrete class ($\varepsilon_{cr,1}$=7.26E-05 for C20 and $\varepsilon_{cr,1}$=1.06E-04 for C50).
2. As soon as the first crack in point B arises, keeping $\acute{\varepsilon}_{cs} = \varepsilon_{cr,1}$, the stress in concrete drops in the whole specimen to a non-cracking level.
3. Increasing the imposed deformation beyond $\varepsilon_{cr,1}$ further cracks arise. After the formation of each crack, keeping $\acute{\varepsilon}_{cs}$ constant, the stress in concrete drops to a non-cracking level.

4. Specimens with low reinforcement ratios, ρ, needs higher imposed deformations to generate new cracks because of the lower stiffness of steel.
5. The transfer length, l_s, does not vary sensibly changing the reinforcement layout or the concrete class as can be seen in table 4 and has already been discussed in paragraph 7.1.
6. Maximum tensile stresses in steel bars decrease after the formation of each crack.
7. If yielding occurs no further cracks arise as the stiffness of the yielded bar is too low to let concrete stress increase because of restrained deformation.

8. Conclusions
The following conclusions can be achieved from the study:
1. The physical rules that govern cracking induced by implicit actions are different from the ones related to explicit ones. Stresses induced by implicit actions are function of the stiffness of the structure itself whereas stresses induced by external actions on a statically determined elements are only a function of the applied loads. Therefore, imposed deformations cause narrower cracks with respect to explicit actions starting from an equal level of elastic stresses.
2. No specimen, regardless of concrete class, reached stabilized cracking when subjected to imposed strains. Some ties developed few cracks in part of their length whereas other were cracked on the full length but not in stabilized mode. Cracks with a spacing of a few meters, as experimentally seen in earth retaining walls, are then compatible with the achieved results.
3. High levels of imposed strains (up to 8.0E-04) have been applied to the ties, if compared to the standard deformations values expected in structural elements. A common value of shrinkage is typically around 3.0E-04 and a deformation of 1.0E-4 corresponds to about 10°C of temperature variation.
4. When non-linear analysis is performed, crack spacing is difficult to be correctly evaluated, as described in par. 7, but crack opening can be accurately predicted, regardless of the mistake done on crack spacing.
5. The results of FEM simulations are substantially different from the ones found with codes provisions. In fact, both Model Code 2010 and Eurocode 2 overestimate the values of crack width in case of imposed deformation as can be seen in table 7.

Table 7. Comparison of crack width w between FEM simulations and codes provisions for implicit actions

		C20			C50		
1ϕ20	Imposed strain ε_{cs}	7.26E-05	1.41E-04	2.80E-04	1.06E-4	2.37E-04	4.94E-04
	MC 2010 w_d	0.40	0.45	0.55	0.71	0.81	1.00
	Eurocode 2 w_k	0.46	0.46	0.46	0.85	0.85	0.85
	FEM w_m	0.13	0.14	0.19	0.21	0.24	0.35
4ϕ10	Imposed strain ε_{cs}	7.26E-5	1.14E-4	1.99E-4	1.06E-5	1.86E-4	3.46E-4
	MC 2010 w_d	0.25	0.27	0.31	0.44	0.48	0.55
	Eurocode 2 w_k	0.31	0.31	0.31	0.56	0.56	0.56
	FEM w_m	0.10	0.09	0.12	0.17	0.17	0.22

Moreover, the effect of cover and transverse confinement has not been taken into consideration in this work. Further research is now ongoing to extend these results to all concrete classes and to derive from this simulation a simple model to predict crack width and spacing due to implicit actions. The favourable effect of creep on shrinkage imposed deformations is also object of study at the present time.

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