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Method Suitable for Updating the Boundary Condition of Continuous Beam Bridges

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Abstract: The boundary support conditions of continuous beam bridges play the great influence on the results of the structural analysis, but it is difficult to accurately model the boundaries owing to the complexity structure of constraint conditions. To address this issue, a parameterized method is proposed to update the boundary support conditions in this study. First, the connection stiffness at boundary is considered as the optimization variable, and then the optimization problem of updating the boundary conditions are described in detail based on the theory of finite element model updating. Second, for verifying the proposed method, a loading test was conducted on an actual three-span continuous beam bridge. With the proposed method, the discrepancy between the measured modal parameters and the analytical results are greatly reduced; therefore, it is shown that the proposed method is effective for updating the boundary support conditions of actual continuous beam bridges.

1. Introduction

Finite element model (FEM) updating of bridges using vibration test data has received considerable attentions in recent years due to its crucial role in fields ranging from establishing a reality-consistent structural model for dynamic analysis and control, to providing baseline model for damage identification in structural health monitoring. Structural model updating is to correct the analytical finite element model using test data to produce a refined one that better predict the dynamic behaviour of structure. FEM model updating usually ends up with a nonlinear optimization problem. Many techniques have been developed to address the model updating problem, as discussed by Mottershead and Friswell [1,2]. Generally different techniques vary in the choice of the three [3,4]: (1) Objective function defined to be minimized; (2) Constraints placed to narrow down the domain for search; (3) Optimization technique used to achieve global minimum. Of course, choice of appropriate updating parameters is also very important [5].

This study addresses the correction of the boundary conditions of an actual bridge using the technique of FEM updating. A method suitable for updating the constraints of bridges is proposed in next section. The optimization function of FEM updating is described in detail in Section 3. Finally, the proposed method is applied to update the boundary of an actual continuous beam bridge using the measured data of load test.

2. Updating the boundary conditions

For structures where the boundary condition is not clear or well modeled in the finite element modeling, the structure support condition should be parameterized and included in the vector for updating. In this section, a parameterizing scheme for boundary condition is proposed.

Neglecting the inertia on boundaries, the equilibrium equation of structure is defined as,

$$\begin{bmatrix} \mathbf{K}_{\mathrm{b}} & \mathbf{K}_{\mathrm{bv}} \\ \mathbf{K}_{\mathrm{vb}} & \mathbf{K}_{\mathrm{v}} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_{\mathrm{b}} \\ \mathbf{u}_{\mathrm{v}} \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}_{\mathrm{b}} \\ \mathbf{f}_{\mathrm{v}} \end{Bmatrix} \tag{1}$$

where v is the degree of freedoms (DOFs) of the unconstrained structure and b is the DOFs at the boundary; $\mathbf{u}$ and $\mathbf{f}$ are the displacement and force vectors respectively. Then the following equation is obtained as,

$$\mathbf{K}^{*}\mathbf{u}_{\mathrm{b}} = \mathbf{f}^{*}. \tag{2}$$

where $\mathbf{K}^{*}$ and $\mathbf{f}^{*}$ are condensed stiffness matrix and force vector respectively in the form of

$$\begin{aligned} \mathbf{K}^{*} &= \mathbf{K}_{\mathrm{v}} \text{-} \mathbf{K}_{\mathrm{vb}}\mathbf{K}_{\mathrm{b}}^{-1}\mathbf{K}_{\mathrm{bv}} \\ \mathbf{f}^{*} &= \mathbf{f}_{\mathrm{v}} - \mathbf{K}_{\mathrm{vb}}\mathbf{K}_{\mathrm{b}}^{-1}\mathbf{f}_{\mathrm{b}} \end{aligned} \tag{3}$$

$\mathbf{K}^{*}$ can be rewritten as follows with the introduction of a diagonal matrix $\mathbf{Q}$,

$$\mathbf{K}^{*} = \mathbf{K}_{\mathrm{v}} \text{-} \mathbf{K}_{\mathrm{vb}}\left(\mathbf{Q}^{\mathrm{T}}\mathbf{K}_{\mathrm{b}}^{-1}\mathbf{Q}\right)\mathbf{K}_{\mathrm{bv}} \tag{4}$$

where $\mathbf{Q} = \begin{bmatrix} q_1 & & & & \\ & \ddots & & & \\ & & q_i & & \\ & & & \ddots & \\ & & & & q_B \end{bmatrix}$, $\left(0 \le q_i \le 1\right)$, and B is the total number of DOFs on the boundary.

Matrix $\mathbf{Q}$ is to describe the support condition at the boundary, with its element $q_i (i = 1, 2, ..., B)$ varies between 0 for fully constrained support and 1 for free of constraint.

3. The optimization problem for model updating

The main objective of model updating is to identify the uncertain parameters of structures, such as stiffness and mass elements, by minimizing the discrepancies between experimental and analytical modal data. The uncertain parameters are the variables to be updated. Assuming the difference between m pairs of modes is to be minimized, the objective function of model updating is defined as [6]

$$\boldsymbol{\Phi}(\boldsymbol{a}) = \mathbf{D}_f(\boldsymbol{a})^{\mathrm{T}}\mathbf{W}\mathbf{D}_f(\boldsymbol{a}) \tag{5}$$

where $\mathbf{W}$ is the weight matrix, and $\boldsymbol{a}$ are the vector of variables to be updated; $\mathbf{D}_f = \left\{D_{f,1}, D_{f,2}, \cdots D_{f,m}\right\}$ is the residual vectors of the discrepancy of frequencies, of which the components are written in Eqs. (6) .

$$D_{f,j} = \frac{\lambda_j(\boldsymbol{a}) - \lambda_j^e}{\lambda_j^e} \tag{6}$$

where $\lambda_j(\boldsymbol{a})$ and λ_j^e are the jth eigenvalues of structures obtained by FE analysis and modal test respectively. The gradient matrix $\mathbf{g}$ and Hessian matrix $\mathbf{G}$ are given by Eqs. (7) and (8) respectively.

$$\mathbf{g} = \nabla \mathbf{\Phi}(\boldsymbol{a}) = \sum_{j=1}^{m} D_{f,j}(\boldsymbol{a}) \nabla D_{f,j}(\boldsymbol{a})^{\mathrm{T}} = \mathbf{J}_a(\boldsymbol{a})^{\mathrm{T}} D_{f,j}(\boldsymbol{a}) \tag{7}$$

where $\mathbf{J}_a(\boldsymbol{a})$ is the Jacobian matrix, and m is the number of parameters to be updated.

$$\mathbf{G} = \nabla^2 \mathbf{\Phi}(\boldsymbol{a}) = \mathbf{J}_a(\boldsymbol{a})^{\mathrm{T}} \mathbf{J}_a(\boldsymbol{a}) + \sum_{j=1}^{m} D_{f,j}(\boldsymbol{a}) \nabla^2 D_{f,j}(\boldsymbol{a})^{\mathrm{T}} \approx \mathbf{J}_a(\boldsymbol{a})^{\mathrm{T}} \mathbf{J}_a(\boldsymbol{a}) \tag{8}$$

where $\nabla D_{f,j}(\boldsymbol{a})$ is to be calculated by Eq. (9) .

$$\frac{\partial D_{f,j}}{\partial \boldsymbol{a}} = \frac{1}{\lambda_j^e} \frac{\partial \lambda_j(\boldsymbol{a})}{\partial \boldsymbol{a}} \tag{9}$$

where $\frac{\partial \lambda_j(\boldsymbol{a})}{\partial \boldsymbol{a}}$ is the eigenvalue sensitivity.

4. FEM updating of an actual continuous beam bridge

In this section, an actual continuous beam bridge is taken as an example to validate the proposed method.

4.1. Brief introduction of the actual bridge

This practical example is a three-span continuous beam bridge with a span of 10m+10m+10m. It is tilted 40 degrees, as shown in Figure 1. The full width of the standard section is 10.75 m, including a 2.5 m sidewalk.

Figure 1. Overview of an actual continuous beam bridge

4.2. Case design for the dynamic load test

The dynamic test is that one of the above standard load trucks passed bridge by 30km/h speed. The acquisition part of dynamic data utilizes the acceleration sensors installed in the bridge structure and uses the Spectral Acquisition in the dynamic data acquisition system (LMS SCADAS □ data acquisition system) to obtain the acceleration of the bridge structure.

The connection stiffness at boundary is considered to be a rigid connection. Eigen system Realization Algorithm (ERA) [7] were applied to extract the modal parameters from the measured accelerations in MATLAB environment. The specific steps are as follows: 1) The frequency of the spectrum corresponding to each peak is calculated by the mutual power spectrum of the different acceleration sensor information and the auto-power-spectrum density of each acceleration sensor information. 2) The frequencies, damping ratio and modal shapes of the structure are identified by ERA. Great discrepancy between the natural frequencies by the FE analysis and those by identification is observed by comparing with the theoretical finite element calculation results.

Ten modes were identified as listed in Table 1. The acceleration signal is shown in Figure 2, and the auto-power-spectrum density is shown in Figure 3.

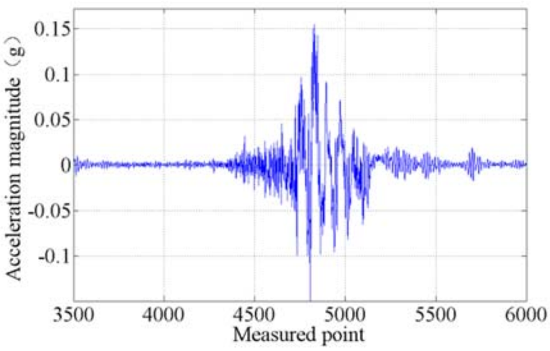

Figure 2. The acceleration signal

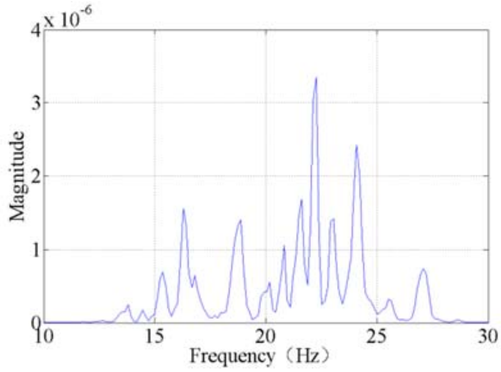

Figure 3. The auto-power-spectrum density

Table 1. Analytical and identified frequencies of the structure (Hz)

Mode	1	2	3	4	5	6	7	8	9	10
FE	13.73	14.66	16.51	17.01	18.47	20.02	22.68	23.03	23.38	24.06
ERA	15.36	16.28	18.88	20.18	20.83	21.61	22.27	23.05	24.09	25.52

4.3. Case design for the static load test

Four cases are designed for the load test on this bridge, Case 1 for an asymmetric load acting on the side span, Case 2 for a symmetrical load acting on the side span, Case 3 for a asymmetric load acting on the middle span and Case 4 for an symmetrical load acting on the middle span. The load placements for the case 1 are shown in Figure 4.

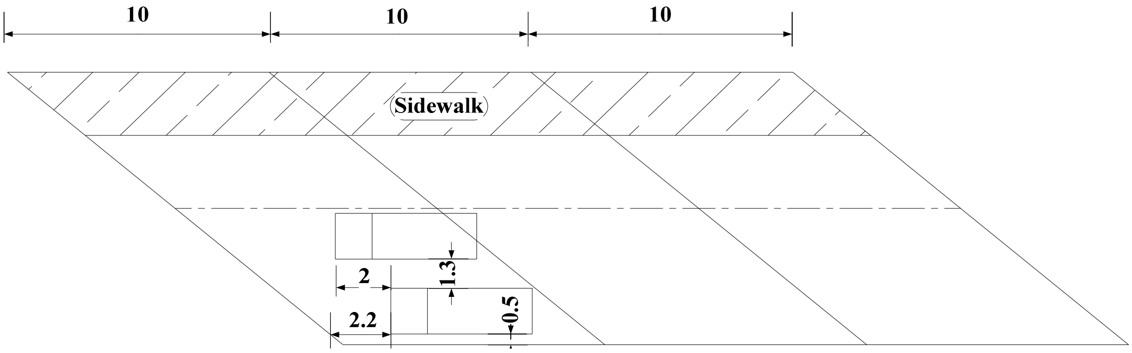

Figure 4. Loading positions of Case 1 (units: m)

For each load case, loading trucks with an average weight of 400kN are used to apply the static loads, and the details of the loading trucks are shown in Figure 5. During the test, the vertical displacement of the main girder is measured precisely. Two mid-span sections of the bridge are measured, and each section has 10 measurement points. The measured data are listed in Table 2. For Case 1 and Case 3, the vertical displacements from 6# girder to 10# girder are negligible.

4.4. The results of FEM updating

The FEM updating is based on the experimental modal data to update the parameters of the structural finite element model. The process of updating is an iterative process. The main objective of model updating is to identify the uncertain parameters of structures, by minimizing the discrepancies between experimental and analytical modal data. Taking into account the difference between the finite element simulation and the actual boundary conditions, the six degrees of freedom of the bearing which translating and rotating along three axes are considered as the updating parameters, the frequencies difference of testing and analysing are used as the objective function. Then, the boundary condition is updated by solving the optimization problem of model updating.

The frequencies of models and their difference before and after updating are listed in Table 3, and it is shown that the updated finite element model and test model match well. The discrepancy of modal frequencies is greatly reduced, and generally a discrepancy after modal updating is less than 3%.

In addition, the method of the model updating is further verified by the displacement data of the static loading test. The discrepancy of displacements is greatly reduced, and generally a discrepancy after modal updating is less than 20%. Otherwise, Figure 6 and Table 4 compare the displacement values and displacement errors before and after FEM updating.

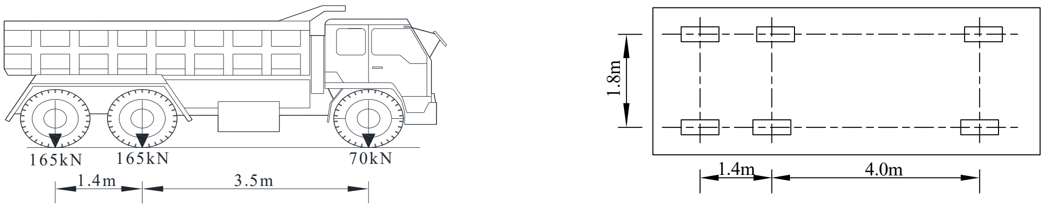

Figure 5. Detailed information of the loading trucks

Table 2. The measured displacement of four cases of load test

Measured points	Case 1 (mm)	Case 2 (mm)	Case 3 (mm)	Case 4 (mm)
1	1.95	0.15	1.65	0.18
2	1.87	0.31	1.60	0.27
3	1.13	0.67	0.97	0.64
4	1.22	0.65	1.19	0.68
5	0.81	1.40	0.61	1.19
6	-	1.46	-	1.34
7	-	1.01	-	0.83
8	-	0.90	-	0.95
9	-	0.55	-	0.41
10	-	0.32	-	0.22

Table 3. Frequency of test model and models before and after updating

Mode	Identified frequencies (Hz)	Before updating		After updating	
		Frequency (Hz)	Difference (%)	Frequency (Hz)	Difference (%)
1	14.59	13.73	11.87	14.45	0.96
2	16.01	14.66	11.05	15.84	1.06
3	17.22	16.51	14.35	17.03	1.10
4	18.61	17.01	18.64	18.50	0.59
5	20.31	18.47	12.78	20.07	1.18
6	22.41	20.02	7.94	22.37	0.18
7	23.98	22.68	-1.81	23.44	2.25
8	25.21	23.03	0.09	25.02	0.75
9	26.23	23.38	3.04	25.79	1.68
10	27.93	24.06	6.07	27.58	1.25

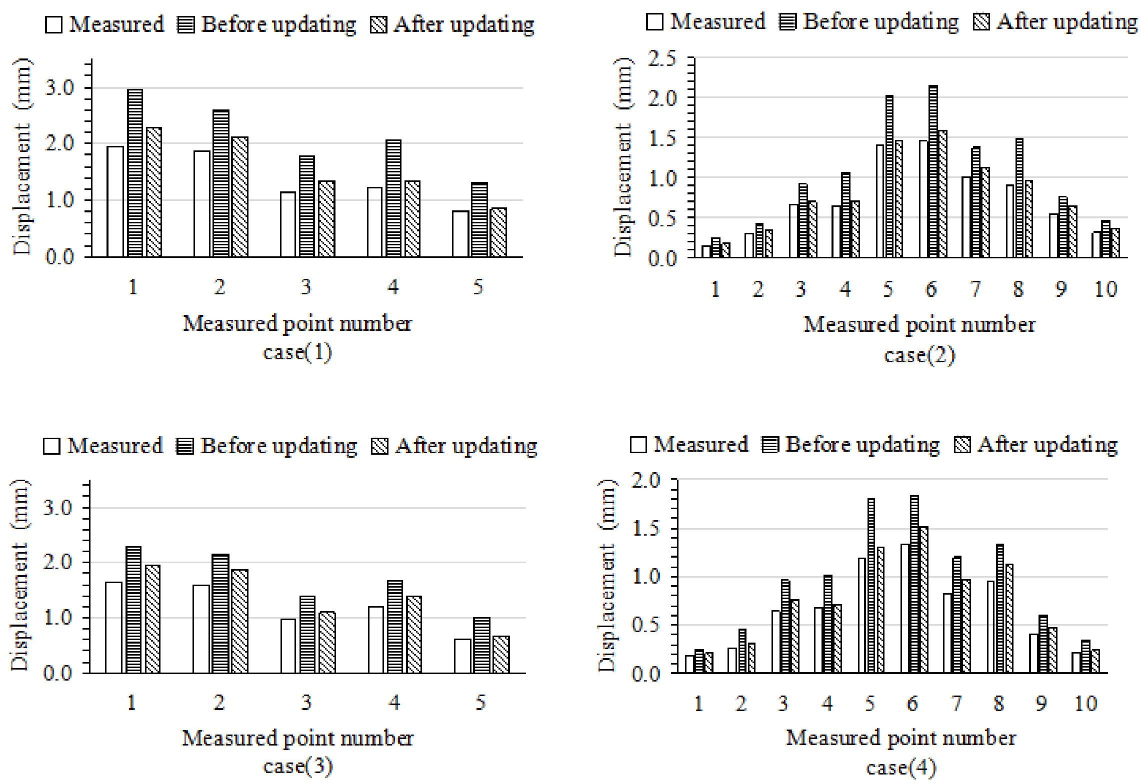

Figure 6. Comparison of displacement values before and after FEM updating

Table 4. Comparison of displacement errors before and after FEM updating

Measured points	Errors (%)							
	Case 1		Case 2		Case 3		Case 4	
	Before updating	After updating	Before updating	After updating	Before updating	After updating	Before updating	After updating
1	51.28	17.32	66.67	19.23	38.79	18.23	38.89	15.32
2	39.04	13.66	38.71	13.67	35.00	16.57	66.67	13.65
3	58.41	19.37	37.31	4.33	43.30	13.29	50.00	18.33
4	69.67	10.33	64.62	10.44	40.34	16.32	50.00	5.36
5	60.49	5.37	45.00	5.37	65.57	9.56	51.26	9.35
6	-	-	47.26	9.34	-	-	36.57	12.58
7	-	-	36.63	12.32	-	-	44.58	16.35
8	-	-	64.44	7.37	-	-	41.05	19.33
9	-	-	38.18	18.26	-	-	46.34	14.32
10	-	-	46.88	15.36	-	-	54.55	10.11

5. Conclusions

A parameterized method is proposed to update the boundary support conditions. Though solving the optimization problems of structural model updating, the boundary conditions are effectively updated. The updated finite element model predicts the frequencies of the structure within 3% difference comparing with that identified by modal test. The method of the model updating is further verified by the displacement data of the static loading test. The discrepancy of displacements after modal updating is less than 20%. Therefore, this method can provide reference for practical engineering application.

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