

Dynamic Eigenvalue Problem of Concrete Slab Road Surface

Urszula Pawlak ¹, Michał Szczecina ²

¹ Department of Mechanics, Metal Structures and Computer Methods, Faculty of Civil and Environmental Engineering, Kielce University of Technology, Kielce, Poland

² Department of Mechanics, Metal Structures and Computer Methods, Faculty of Civil and Environmental Engineering, Kielce University of Technology, Kielce, Poland

u.pawlak@tu.kielce.pl

Abstract. The paper presents an analysis of the dynamic eigenvalue problem of concrete slab road surface. A sample concrete slab was modelled using Autodesk Robot Structural Analysis software and calculated with Finite Element Method. The slab was set on a one-parameter elastic subsoil, for which the modulus of elasticity was separately calculated. The eigen frequencies and eigenvectors (as maximal vertical nodal displacements) were presented. On the basis of the results of calculations, some basic recommendations for designers of concrete road surfaces were offered.

1. Introduction

Road surface is an engineering structure which transfers pre-set static and dynamic loads, from vehicles or environmental impacts (e.g., thermal, seismic) to the subsoil or another engineering structure. That must be done in such a way so that traffic safety and predicted life service are ensured [3].

In transport infrastructure, different types of road surfaces are used. Due to the material of the surface course, road surfaces can be categorised as cement concrete, natural stone, clay block, bituminous (asphalt or tarmac), crushed stone, gravel and concrete slab ones. With respect to deformation capacity, road surfaces can be classified as rigid and flexible.

Rigid pavements, e.g. from cement concrete (Figure 1), undergo elastic deformation, whereas in flexible ones, plastic deformations are observed and permanent deformations develop (e.g. bituminous surfaces on flexible base courses, crushed stone or paved surfaces) [4].

First concrete road surfaces were constructed in Poland, in Cracow, in 1912. It was only at the end of the 20th cent. that the level of interest in such structures increased. That is mainly related to the fact that existing surfacing has *rutted* badly in asphalt road pavements. Those permanent deformations contribute to the deterioration of traffic safety [6].

Properly designed and executed concrete road surface offers many advantages including: high bearing capacity, high ability to transfer loads, resistance to permanent deformation, light colour, good operational properties, and low maintenance costs.

Each road surface rests on subsoil, or another engineering structure (bridge, tunnel, flyover, culver). As a result, subsoil material must be taken into account in all types of buildings and engineering structures.

Road slab structure is affected by the laws of dynamics because of dynamic loads acting on it, and also due to the properties of the system itself, which is influenced by such impacts.





Figure 1. Concrete-surfaced road [7]

In the study, the analysis of the dynamic eigenvalue problem of road concrete slab resting on elastic subsoil is performed. Eigenvalues and eigenvectors, i.e. eigenfrequencies and vibration modes of the system were determined. When eigenfrequencies are known, it is possible to avoid resonance, which is dangerous to the structure. In resonance, the frequency of forced vibration is approximately the same as the structure eigenvibration, and an increase in displacement amplitudes is unlimited. Computations were based on Finite Element Method, and Autodesk Robot software was used.

2. Concrete surface roads

Concrete surface roads are characterised by high capability of transferring loads, even when those are concentrated at a single point. Concrete surface roads are highly resistant to deformation over the whole range of temperatures, as a result their surface does not become rutted and surface water can be effectively drained from the top layer. Light colour of the surface contributes to increased traffic safety, especially at dusk, or in rain, and also to reduction in costs of road lighting. For a properly constructed concrete surface, 20-year, and even 30-year service life can be assumed.

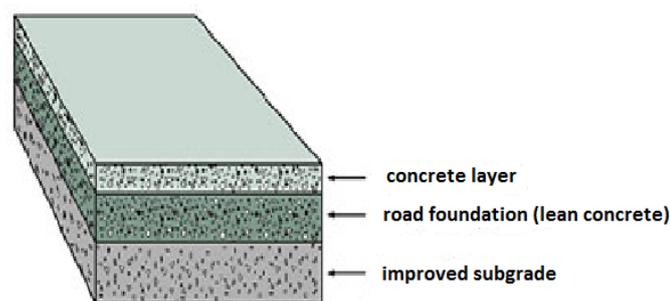


Figure 2. Diagram of the layers of concrete road surface [6]

The structure of a cement concrete road surface is made of a set of layers resting on natural or compacted subgrade. The road surface takes wheel loads and other external loads, and transmits those to the subsoil. That function must be performed throughout the designed service life of the road. The evenness and roughness of the surface course ensure comfortable travel and traffic safety. The structure of the concrete road surface, shown in Figure 2, consists of the following layers: concrete slab, base course, compacted subgrade (frost-protective layer, which, strengthens the subgrade) and natural subgrade (existing soil).

Numerous and unquestionable advantages of concrete road surfaces should enhance their popularity and make those a technological option in addition to asphalt road surfaces and cobblesett block paving [6].

3. Eigenvalue problem in structural dynamics

Structure dynamics is the science dealing with vibration of building structures, or their components. Those are systems that have invariable geometry and conservative form of equilibrium [1]. Structure vibration consists in oscillations around the static equilibrium position, which is treated as a reference system in the dynamic analysis.

In structural mechanics, the eigenvalue problem is understood as a determination of quantities that are characteristic of a given system, i.e. eigenvalues and eigenvectors. Those quantities depend entirely on the parameters of the system under analysis, i.e. stiffness, type of support and dimensions.

In structural dynamics, eigenvalue problem concerns the determination of eigenvalues, which are eigen frequencies and eigenvectors corresponding to those, in other words the displacement distribution in the vibrating system.

3.1. Eigenvalue problems in structural dynamics with elastic subsoil taken into account

One of the basics issues in structural dynamics is to specify the conditions, under which the structure could perform motion around the equilibrium position, without any external excitation forces acting on it [5]. The equation of such a motion is as follows (1):

$$M\ddot{q} + Kq = 0 \quad (1)$$

where:

M – matrix of inertia of the system,

K – matrix of linear stiffness of the system,

q, \ddot{q} – vectors of displacements and the system accelerations.

If the structure lies on the Winkler base, equation (1) should be extended to include matrix K_p , which accounts for the effect produced by subsoil:

$$M\ddot{q} + (K + K_p)q = 0 \quad (2)$$

where:

M – matrix of inertia of the system,

K – matrix of linear stiffness of the system,

K_p – matrix of stiffness of the subsoil,

q, \ddot{q} – vectors of displacements and the system accelerations.

The solution of equation (2) is vector

$$q = a e^{i\omega t} \quad (3)$$

where:

$$a = a \{a_1 \ a_2 \ \dots \ a_n\} \quad (4)$$

Formula (3) represents a set of harmonic functions $q_j(t)$ having the same frequency ω and amplitudes a_j ($j=1,2, \dots n$). The motion described by formula (3) is termed eigenvibration of the system. That is not a physical phenomenon, but it results from the properties of the structure exposed to dynamic effects. When function (3) is introduced into equation (2), a homogeneous matrix equation (5) is obtained, which must be satisfied at every instant t .

$$(\tilde{K} - \omega^2 M)a = 0 \quad (5)$$

where:

$$\tilde{K} = K + K_p$$

The solution, in which $a \neq 0$, is sought, therefore the condition for the existence of non-zero solutions is equation (6), from which n values of ω_j are received, which are the eigenfrequencies of the system [2].

$$\det(\tilde{K} - \omega^2 M) = 0 \quad (6)$$

Roots ω_j are positive real numbers, multiple roots can also be found. Non-zero vector a_i , termed the eigenvector of the i -th vibration mode, corresponds to each eigenvalue ω_j ($i = 1, 2 \dots n$). The vector describes the distribution of displacements q in vibration at frequency ω_j . Eigenvectors are determined within an accuracy of a constant factor. The set of eigenvectors forms the eigenmatrix W .

4. Analysis of dynamic eigenvalue problem of concrete road slab

The analysis of the dynamic eigenvalue problem concerned a concrete road slab, 19cm in thickness. In conformity with recommendations [6], slab critical dimensions were determined, namely: 418x456cm. The slab dimensions adopted for computations were 400x300cm, which corresponds to a single lane of traffic. Subsoil layers were assumed for loads related to KR2 traffic category (light traffic). Underneath the 19cm-thick concrete slab, made from C30/37 class concrete, the following are found in succession: 15cm thick coarse aggregate layer stabilised with cement (secondary bulk modulus $E_2=100\text{MPa}$) and clay subgrade with a degree of plasticity of $I_L=0.2$.

The mean axle weight was assumed to be 100kN (buses and trucks), which gives mean characteristic load on the slab produced by vehicular movement: $100 \text{ kN} / (3 \text{ m} * 4 \text{ m}) = 8.3 \text{ kN/m}^2$. the slab deadweight (characteristic value) is: $25 \text{ kN/m}^3 * 0.19 \text{ m} = 4.75 \text{ kN/m}^2$. After assuming load coefficients: 1.35 for deadweight and 1.5 for loads from the movement of vehicles, the load on slab was estimated: $8.3 * 1.5 + 4.75 * 1.35 = 18.9 \text{ kN/m}$.

The input data above and listed loads provided a basis for the determination of the subsoil elasticity coefficient. The computations were carried out using a "Building Land" module, which constitutes a component of Autodesk Robot software. The value of the coefficient was $K = 41469 \text{ kN/m}^3$. Figure 3 shows a screenshot of the page with computations performed with the module of concern.

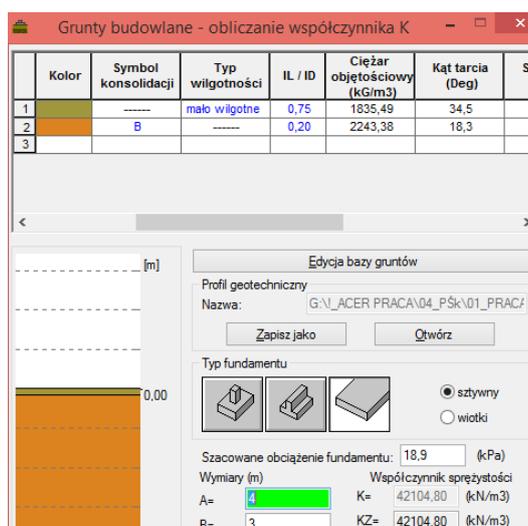


Figure 3 Computations of the elasticity coefficient of the subsoil

Figure 4 shows the axonometric view of the road slab together with its division into finite elements, and also basic input data.

The analysis was performed using Autodesk Robot Structural Analysis Professional 2014 software, based on the Finite Element Method. The adopted structural arrangement was that of a slab supported by elastic subsoil. To find out how the degree of discretisation of the slab model affects the results of computations, different dimensions of the square finite element were assumed, namely: 20 cm, 10 cm and 5 cm. Modal analysis with continuous mass distribution was applied.

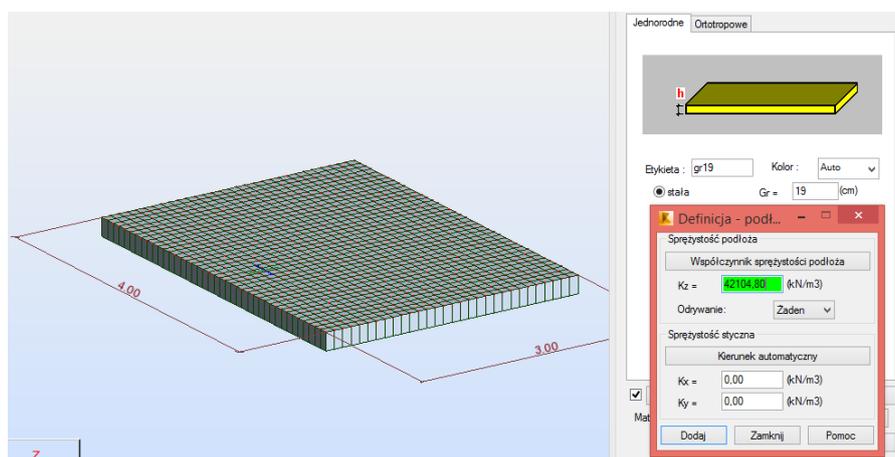


Figure 4. Axonometric view of the road slab

4.1. Eigenfrequencies of the road slab

Table 1 shows the results of vibration frequency for the slab under analysis. Because the division into finite elements did not produce practically any effect on the values of eigenfrequencies obtained, the results were given only for the 5 cm element. It should be noted that a few successive eigenfrequencies (termed pulsations in the Robot program) have very close values. This fact must be given particular attention when the road slab is designed.

Table 1. Values of eigenfrequencies of the road slab.

Case	Form	Frequency	Period	Pulsation
3	1	47.277	0.021	297.051
3	2	47.319	0.021	297.311
3	3	47.372	0.021	297.646
3	4	59.988	0.017	376.914
3	5	64.017	0.016	402.231
3	6	91.062	0.011	572.160
3	7	97.709	0.010	613.924
3	8	114.607	0.009	720.096
3	9	129.670	0.008	814.741
3	10	166.581	0.006	1046.658

4.2. Slab vibration modes and eigenvectors

Figures 5 to 9 show representative modes of eigenvibration of the road slab. For each mode, eigenvector was also given, as the maximum vertical nodal displacement in centimetres. To make displacements more apparent, each mode of slab vibration was shown in *different axonometric projections*, and the displacements are not to scale, when referred to the actual dimensions of the slab.

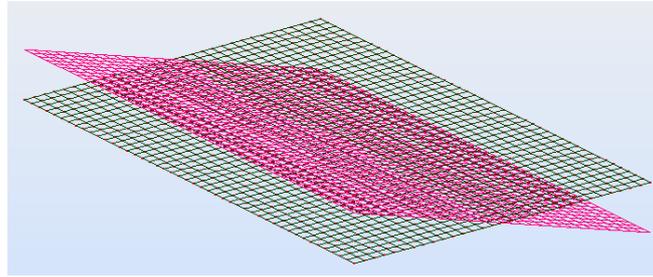


Figure 5. Vibration mode for vibration frequency 1, maximum displacement: 0.023 cm

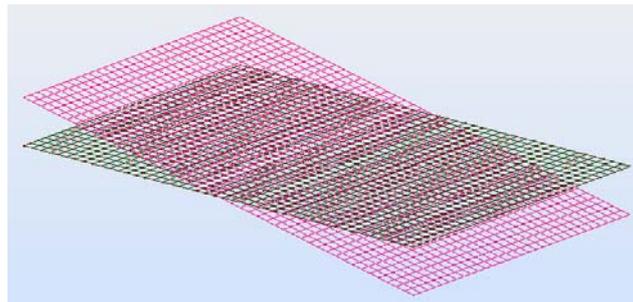


Figure 6. Vibration mode for vibration frequency 2, maximum displacement: 0.023 cm

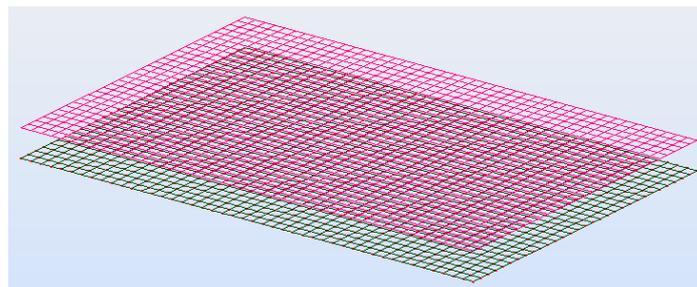


Figure 7. Vibration mode for vibration frequency 3, maximum displacement: 0.023 cm

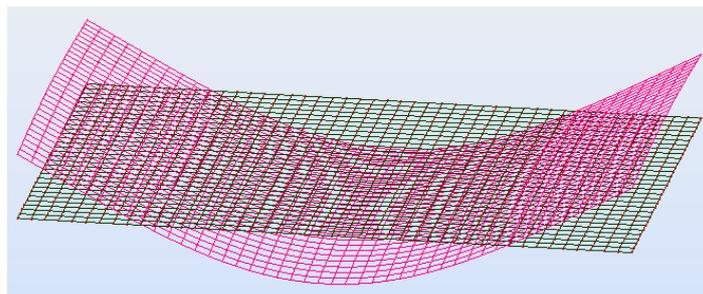


Figure 8. Vibration mode for vibration frequency 5, maximum displacement: 0.023 cm

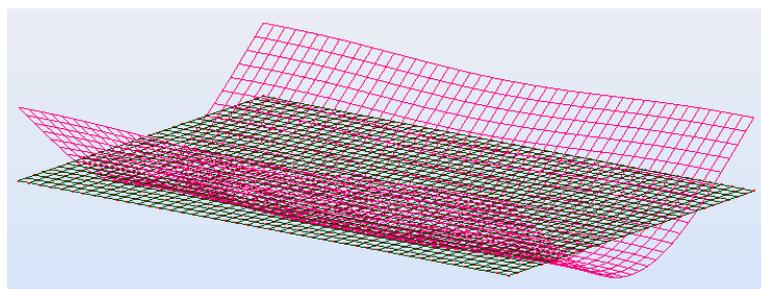


Figure 9. Vibration mode for vibration frequency 6, maximum displacement: 0.023 cm

5. Conclusions

Design of any structure to which variable loads are applied entails the necessity of performing dynamic analysis. As regards road surfaces, that is one of the most crucial analyses on account of the fact that such surfaces undergo dynamic and cyclic loads. Consequently, the occurrence of mechanic resonance phenomenon must be taken into consideration. The results of computations conducted for the study show that a few successive vibration frequencies have very close values. In addition, the first of the vibration modes presented in the study seems surprising, as it goes perpendicular to the direction of vehicle movement, unlike the second vibration mode which looks typical. Thus, the results indicate that eigenvalue problems in the dynamics of road surface concrete slab are of key importance, and should rather not be tackled intuitively. Additionally, it must be remembered that the road surface vibration may be transferred not only to the subsoil, but also to another engineering structure, like flyover or a bridge, and also induce resonance.

References

- [1] T. Chmielewski, Z. Zembaty, Fundamentals of structure dynamics (in Polish), *Arkady* 1998
- [2] A. Gomuliński, M. Witkowski, Advanced course on structural mechanics (in Polish), *OWPW*, Warszawa 1993.
- [3] K. Gradkowski, Bearing capacity of automobile road subsoils strengthened with geosynthetics (in Polish), *OWPW*, Warszawa 2008, Z. 151
- [4] <http://notes.pl/types-road-paving>, (on line: 19.05.2016 r.).
- [5] P. Obara, W. Gilewski, Dynamic stability of moderately thick beams and frames with the use of harmonic balance and perturbation methods, *Bulletin of The Polish Academy of Sciences: Technical Sciences*, Vol: 64, Notebook: 4, 2016, Pages: 739-750.
- [6] www.concrete.roads.pl, (on line 13.02.2017 r.).
- [7] www.polishcement.pl, (on line: 19.05.2016 r.).