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Subcritical Flow Regime of Tandem Interfering Cylinders

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Abstract. The subcritical flow regime of Reynolds number Re = 1.0 x 10^4 is the subject of the paper in the context of aerodynamic interference of tandem circle cylinders. The two cylinders are in distance of L/D equal to 2.0. The purpose of the paper is to determine aerodynamic drag and lift coefficients and the frequencies for upstream and downstream cylinders and also the distribution of the velocity around the cylinders and in their wakes. In numerical analysis the RNG $k-\varepsilon$ method was used.

1. Introduction
Wind flow around civil structures and their parts are the subject of detailed analysis due to practical applications in design and operations of constructions [1-2]. Wind load action on structures cause their vibrations. In addition, behind the structure the vortex street appears which may play significant role in the interaction with elements located in it. Cables of the suspended and stayed bridges and conductors of overhead transmission lines are often in the system of several elements so they may interact. The researchers presented in [3-7] show a high level of complexity of the problem of flow around cylinders with the changing distance between them.

Due to the flow around the elements, the pressure differences appear. Followed by separation, the boundary layer is located in the upstream and downstream cylinders. The boundary layer as a disturbance has the different level depending on inertia, viscosity of the media, surface roughness and distance between the bodies.

The study includes a numerical analysis of the mechanism of wind flow around two cylinders of circular cross-section, one behind the other, for a given Reynolds number Re = 1.0 x 10^4. The analysis allows to determine the drag and lift forces, frequencies and velocity distribution.

2. Mathematical formulation
In the paper Navier-Stokes equations of motion for description of incompressible and viscous air is used. In the analyzed state, the inertial force, pressure and viscosity forces are balanced.

The state of medium can be described by three parameters: temperature, pressure, density or volume. These three variables are independent and are connected by equation of state [8-14]:

$$\mathrm{f}\,(\mathrm{T},\mathrm{p},\varrho)=0 \tag{1}$$

For analysed case of the air the relationship between T, p, ρ is came from theoretical formulas based on the experimental molecular - kinetic theory. For ideal gas the equation of state has the form:

$$\mathrm{p} = \varrho \mathrm{R}'\mathrm{T} \tag{2}$$

R' is the gas constant:

$$R' = \frac{R}{M} \tag{3}$$

R - universal gas constant equal to 8314,3 J/(kmol*K), M - mol mass.

The second important equation is the equation of continuity, expressing the conservation of mass. Mass contained in the volume V at time t is calculated from:

$$m = \int_V \rho(x, y, z, t)\, dV \tag{4}$$

According to the law of mass conservation the volume integral of the formula (4) must have the constant value at any time t. The material derivative of this integral of the volume V should disappear i.e.:

$$\frac{Dm}{Dt} = \frac{D}{Dt}\int_V \rho(x, y, z, t)\, dV = \int_V \left(\frac{D\rho}{Dt} + \rho div\boldsymbol{v}\right) dV = 0 \tag{5}$$

This formula is valid at any volume V, so the integrand must vanish:

$$\frac{D\rho}{Dt} + \rho div\boldsymbol{v} = 0 \tag{6}$$

and in the scalar components:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y} + \frac{\partial(\rho v_z)}{\partial z} = 0 \tag{7}$$

where v_x, v_y, v_z are component of velocity vector.

In the fluid mechanics the constitutive equations are the quantitative expression of the relations found experimentally that occur between the stress field $\sigma_{ij}$and the velocity field v_i.
In the analysed problem, the equations of motion take the following form [15-20]:

$$\frac{Dv_i}{Dt} = f_i - \frac{1}{\rho}\frac{\partial p}{\partial x_i} + v\frac{\partial^2 v_i}{\partial x_i \partial x_i} + \frac{1}{3}v\frac{\partial^2 v_j}{\partial x_i \partial x_j} \tag{8}$$

Taking into consideration the static and dynamic components, equation of motion has the form [21-22]:

$$\frac{\partial v'_i}{\partial t} + (\bar{v}_j + v'_j)\frac{\partial}{\partial x_j}(\bar{v}_i + v'_i) = (\bar{f}_i + f'_i) - \frac{1}{\rho}\frac{\partial(\bar{p} + p')}{\partial x_i} + 2v\frac{\partial(\bar{s}_{ij} + s'_{ij})}{\partial x_j} \tag{9}$$

and:

$$\frac{D\bar{v}_i}{Dt} = \bar{f}_i - \frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + \frac{1}{\rho}\frac{\partial}{\partial x_j}(\bar{\tau}_{ij} + \tau^*_{ij}) \tag{10}$$

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where

$$\bar{\tau}_{ij} = \mu\left(\frac{\partial \bar{v}_i}{\partial x_j} + \frac{\partial \bar{v}_j}{\partial x_i}\right) \tag{11}$$

$$\tau^*_{ij} = \mu_t\,(\bar{v}_{i,j}+\bar{v}_{j,i}) - \frac{2}{3}\bar{\rho}k\delta_{ij} \tag{12}$$

$$\mu_t = \frac{\rho c_\mu k^2}{\varepsilon} \tag{13}$$

The $k - \varepsilon$ dependences are formulated in additional transport equations:

$$\frac{\partial}{\partial t}(\bar{\rho}k) + (\bar{\rho}k\bar{v}_i)_{,i} = \left(\bar{\tau}_{ij}\bar{v}_j\right)_{,i} - \bar{\rho}\varepsilon + \left(\mu_k k_{,i}\right)_{,i} \tag{14}$$

$$\frac{\partial}{\partial t}(\bar{\rho}\varepsilon) + (\bar{\rho}\varepsilon\bar{v}_i)_{,i} = c_{\varepsilon 1}\left(\bar{\tau}_{ij}\bar{v}_j\right)_{,i} - c_{\varepsilon 2}\bar{\rho}\frac{\varepsilon^2}{k} + \left(\mu_\varepsilon \varepsilon_{,i}\right)_{,i} \tag{15}$$

where: k - turbulence kinetic energy;ε - turbulence kinetic energy dissipation;$\mu_{k(\varepsilon)} = \mu + \frac{\mu_t}{\sigma_{k(\varepsilon)}}$ with the following assumption: $c_\mu = 0{,}09$, $c_{\varepsilon 1} = 1{,}45$, $c_{\varepsilon 2} = 1{,}92$, $\sigma_k = 1{,}00$, $\sigma_\varepsilon = 1{,}30$.

3. Analysed model, discussion and results

Analyzed model with the mesh of flow domain and general assumption of inflow direction and section of cable is shown in figure 1 and figure 2. The analysis is performed for Re = 1.0 x 10^4 for tandem cylinders of circle sections. The assumption that air is incompressible is made. Upstream cylinder is located in the distance from the flow inlet of 19D and 21D from the flow outlet, downstream cylinder is located 21D from the inlet and 19D from the outlet. The initial conditions are: uniform flow inlet with velocities $v_x = V_\infty$, $v_y=0$. No slip condition is applied on the cylinder surface and $v_y=v_y=0$. For improving accuracy of results, 27820 elements were applied. In the analysis the grid was required to be fine to capture the flow distribution near the cylinders and in their wakes.

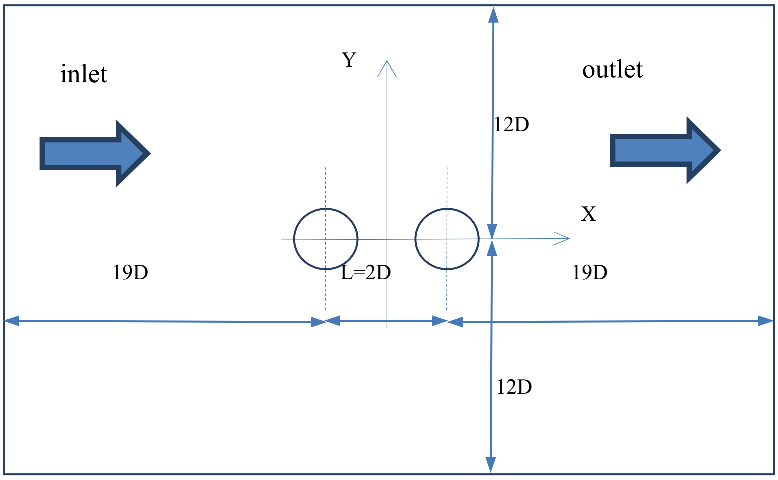

Figure 1. Geometry of the model

Figure 2 presents the element grid used in the simulations. Naiver Stokes equations were solved numerically with use of Semi Implicit Method for Pressure Linked Equations (SIMPLE) with sequenced calculations of velocity and pressure's components. The coefficients of sub relaxations were additionally used for stabilizing the calculations process. Additionally, second order upwind method for momentum equations was adopted.

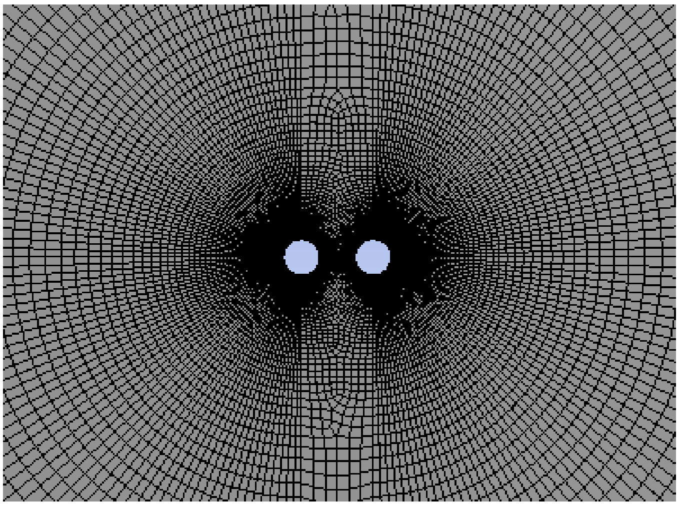

Figure 2. Mesh system of the cylinder

The results for Re = 1,0 x 10^4 and the circle tandem cylinders are shown in figures 3 to 8. The average drag coefficient is: C_{D1} = 1.182 for the upstream cylinder, C_{D2} = 0.154 for the downstream cylinder. The amplitude of the lift coefficient is ΔC_{L1}=1.48 for upstream cylinder and ΔC_{L2}= 1.92 for downstream (figure 4).

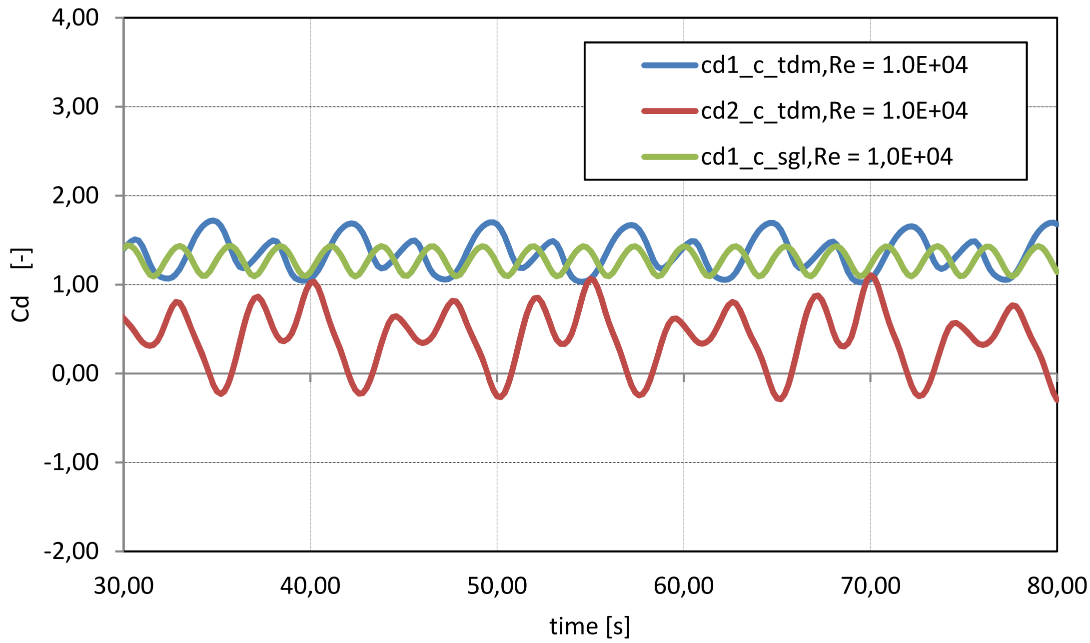

Figure 3. Variation of drag coefficients with time for single (sgl) circle cylinder and tandem (tdm,1-upstream,2-downstream) circle cylinders for Re = 1.0 x 10^4

The frequency of drag force is approximately twice that of lift force, i.e. for upstream cylinder f_{cd1}=0,352 Hz and f_{cl1}=0,207 Hz, for downstream cylinder f_{cd2}=0,391 Hz and f_{cl2}=0,201 Hz but for single cylinder f_{cds1}=0,381 Hz and f_{cls1}=0,215 Hz (figure 5 and figure 6).

For comparison the plots for single circle cylinder is shown. The typical velocity contours around the tandem circle cylinders and in their wakes are shown in the figure 7. Figure 8 shows the distribution of pressure coefficient C_p along the surface of the upstream and downstream cylinders. For upstream cylinder the magnitude of pressure is zero at $\phi = 37^0$ and is minimum C_p=-1 at $\phi = 71^0$. For downstream cylinder the magnitude of pressure is maximum C_p=-0.25 at $\phi = 60^0$ and is minimum at $\phi = 0^0$.

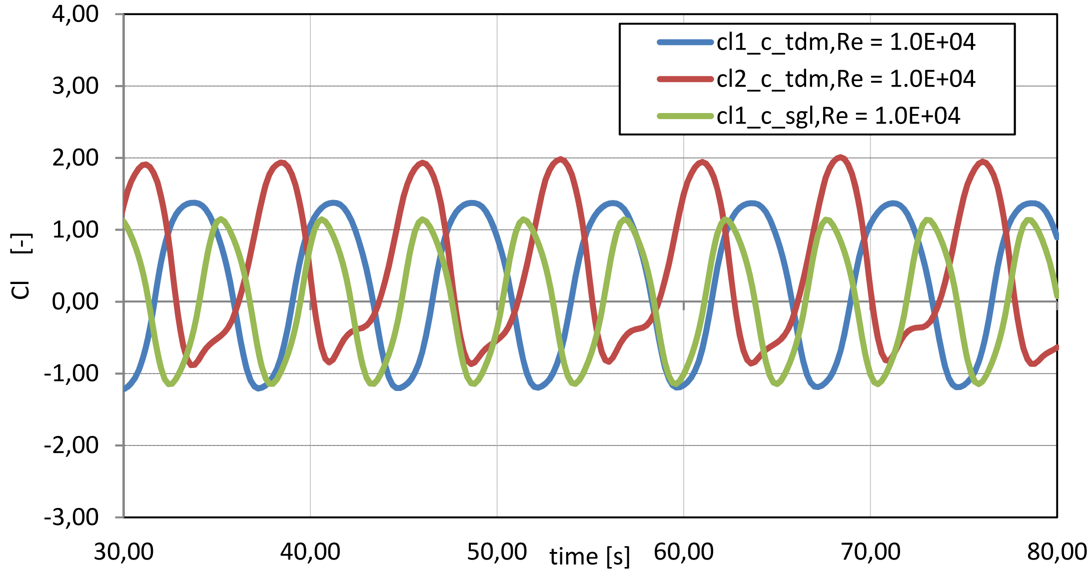

Figure 4. Variation of lift coefficients with time for single (sgl) circle cylinder and tandem (tdm,1-upstream,2-downstream) circle cylinders for Re = 1.0 x 10^4

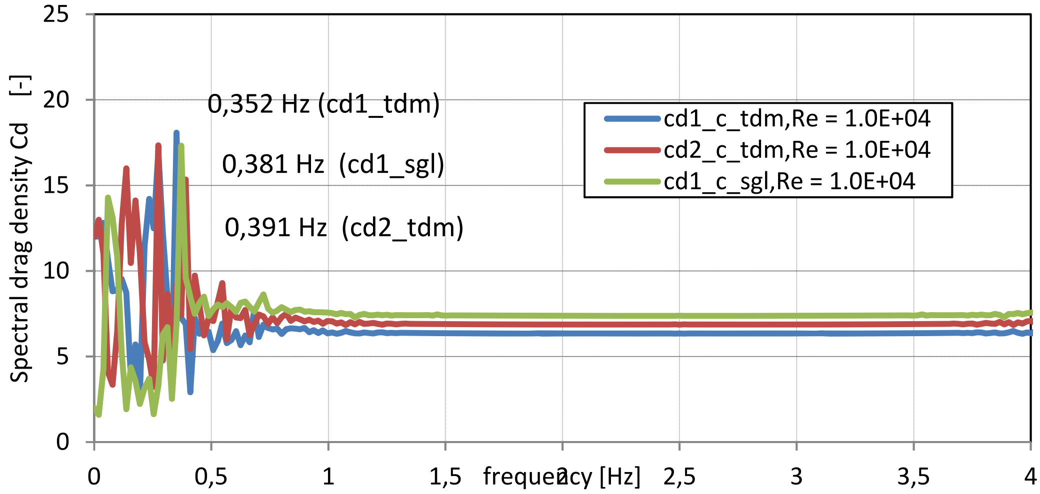

Figure 5. Spectral drag density for single (sgl) circle cylinder and tandem (tdm,1-upstream,2-downstream) circle cylinders for Re = 1.0 x 10^4

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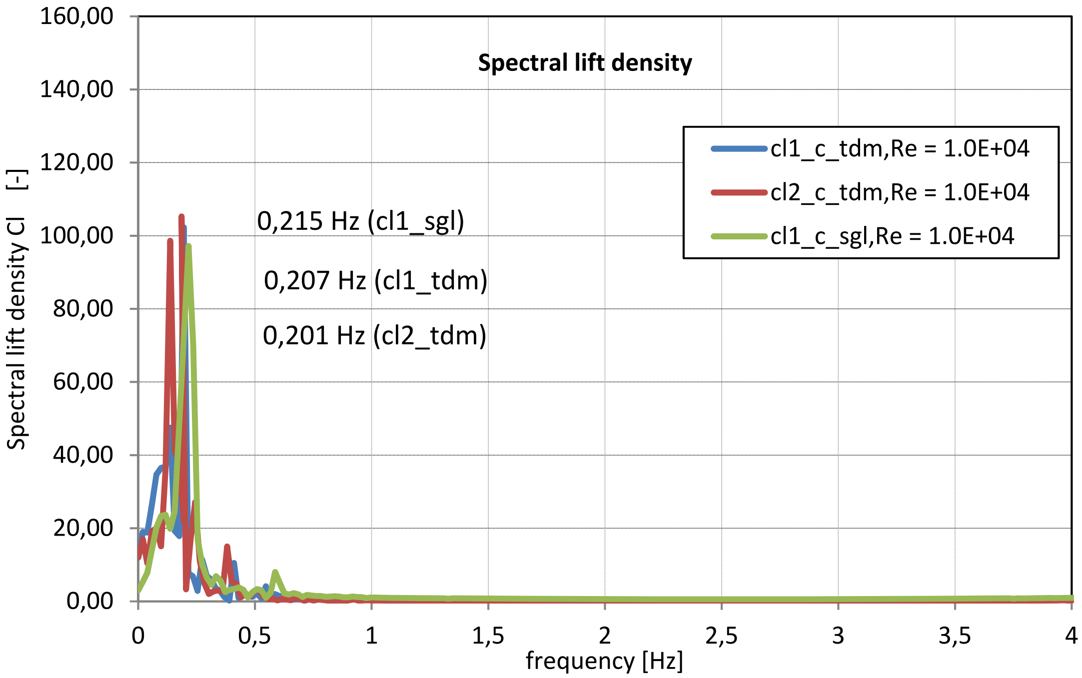

Figure 6. Spectral lift density for single (sgl) circle cylinder and tandem (tdm,1-upstream,2-downstream) circle cylinders for Re = 1.0 x 10^4

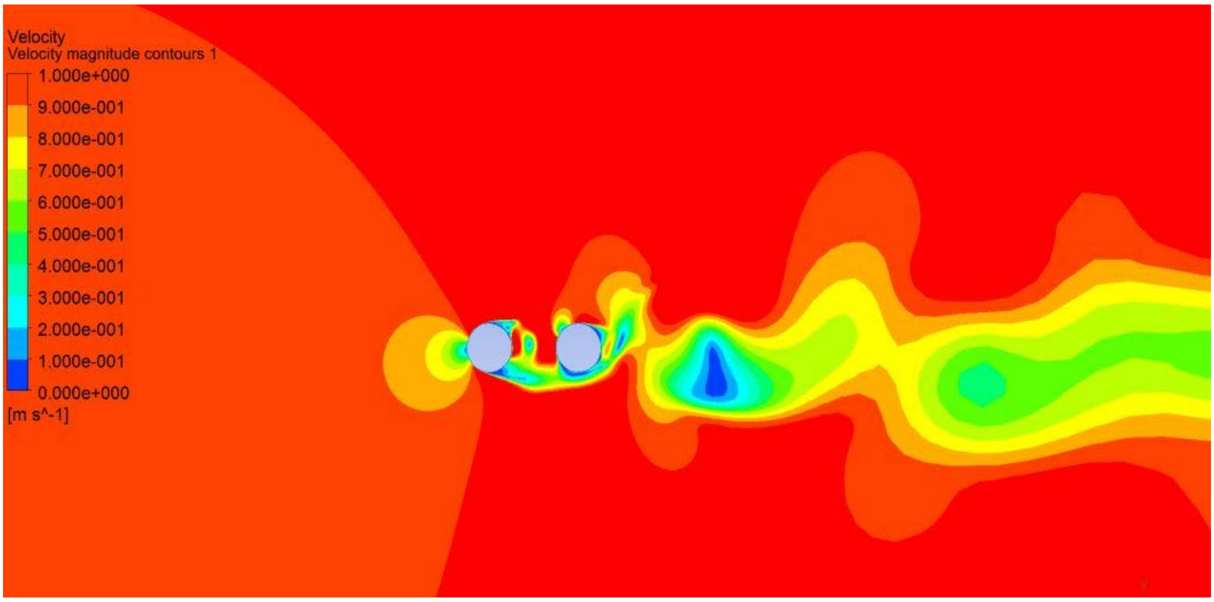

Figure 7. Velocity contour for tandem circle cylinders for Re = 1.0 x 10^4

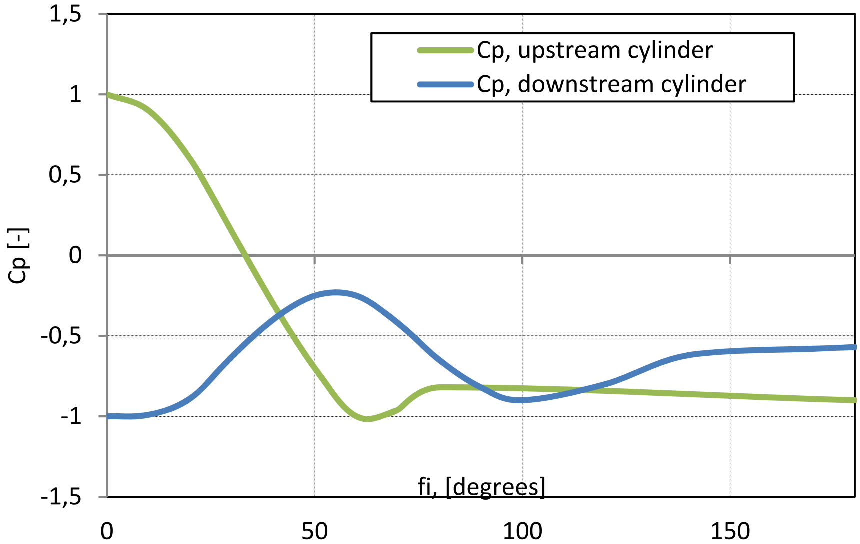

Figure 8. Distribution of pressure coefficients along the surface of upstream and downstream cylinders for Re = 1.0 x 10^4

4. Conclusions

In the paper the wind flow past tandem cylinders with circle cross-section was discussed. Drag and lift coefficients for Reynolds number Re = 1.0 x 10^4 was analyzed. These coefficients were analyzed in the time domain and also power spectra analysis was performed.
The average drag coefficient for the upstream cylinder is bigger than for the downstream cylinder. The amplitude of the lift coefficient for upstream is smaller than for downstream cylinder. The frequency of drag force is approximately twice that of lift force. In the paper the distribution of pressure coefficients along the surface of upstream and downstream cylinders is also presented. The received results are the good background for future study on flow past cables with control of the vortex shedding what is especially important for design and minimization of vibrations of civil engineering structures.

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