

Ultimate Limit State Assessment of Timber Bolt Connection Subjected to Double Unequal Shears

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Abstract. Nowadays the problems occur when a structure engineer need to assess the ultimate limit state of timber bolt connection which is subjected to double unequal shears. This assessment of ultimate limit state shows the reliability of these connections. In assessing the reliability of this connection in ultimate limit state is a problem, because the formulas and equations that are currently available in design standards and available literature, describing only connections loaded symmetrically – this mean that they describe the timber bolt connection subjected to double equal shears. This fact causes problems because structural engineers have no available support, according to which they could assess reliability of the connection in terms of the ultimate limit state. They must therefore often report following an asymmetrically loaded connections carry about using formulas, which are primarily designed for checking connections loaded symmetrically. This leads logically to the fact that it is not respected by the actual behaviour of the connection in the ultimate limit state. Formulas derived in this paper provide the possibility to assess the ultimate limit state for such connection. The formulas derived in this article allow to carry out a reliability assessment of the ultimate limit state of timber bolt connection subjected to double shear. The using of the formulas derived in this paper leads to better description of the behaviour of this type of connection and also to the more economic design. An example of using these derived formulas is shown. There is shown in this example, how to assess the reliability of timber bolt connection subjected to double unequal shears in terms of ultimate limit states.

1. Introduction

For the ultimate limit state assessment of timber bolt connection subjected to double shears is possible in present to use formulas which were derived in [1]. These formulas can be also found in [2] or [3]. But these formulas deal with the connections subjected to equal shears only. In [5] the formula for load carrying capacity of timber structure bolt connection subjected to double unequal shears was derived, see equation (1). This paper shows how to get to the assessment of ultimate limit state.

$$R_{d1} = \frac{k_{mod}}{\gamma_M} \cdot f_{h,2,k} \cdot d \cdot \left(\sqrt{2 \cdot \left(t_2^2 + \frac{2 \cdot \gamma_M \cdot R_{d2} \cdot t_2}{k_{mod} \cdot f_{h,2,k} \cdot d} \right)} - t_2 \right) - R_{d2} \quad (1)$$



2. Assessment of the ultimate limit state

The design loads F_{d1} and F_{d2} of the expected connection can be illustrated in Figure 1 as a point with coordinates $[X=F_{d2} ; Y=F_{d1}]$. The red curve represents the equation (1). If this point is located under the red curve and the load F_{d2} doesn't exceed the load R_{d2} , the connection will be satisfied in terms of the ultimate limit state. But for the assessment of the connection reliability is advisable to derive a formula, which would give us a percentage measure of the reliability connection. This percentage would tell us "how far" the connection design is to the ultimate limit state. The equation (19) gives us only the relation between the R_{d1} and R_{d2} therefore it can't be used for this purpose. In the following part of the paper a formula for calculation of the connection reliability is derived.

Because there is assumed, that $F_{d1} \geq F_{d2}$, the points representing the design loads of connection $[X=F_{d2} ; Y=F_{d1}]$ will be always above a line going through the origin of the coordinates and the angle between this line and the axis X is 45° . Then there is possible to say, that the connection is satisfied, if the points representing the design loads $[X=F_{d2} ; Y=F_{d1}]$ are located in the hatched area.

If we want to calculate the reliability of the connection in terms of the ultimate limit state, we need to define, what way the connection will be loaded. We need to know a curve or a function, which define how the F_{d1} depends on the F_{d2} . These functions or curves are infinitely many. In Figure 1 there are presented two possibilities, which are the most likely ones to occur in practice. These two possibilities of the connection loading are marked as "A" and "B".

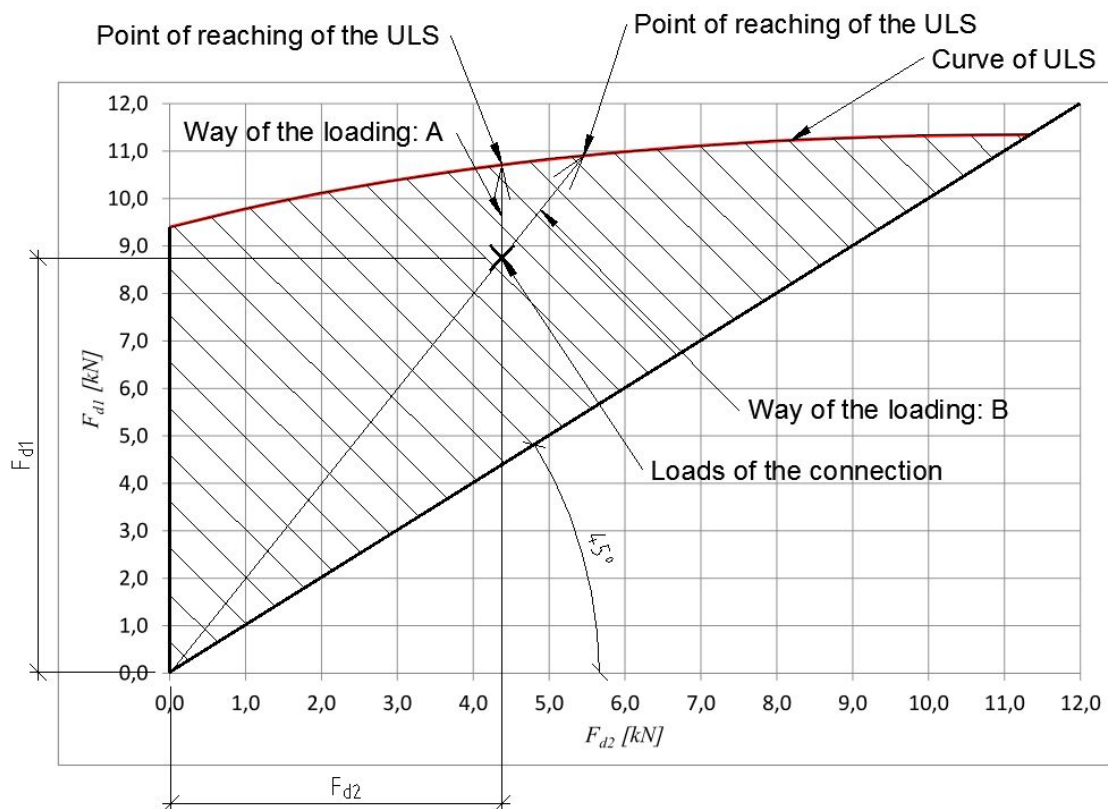


Figure 1. Assessment of the connection reliability in terms of the ultimate limit state

3. Assessment of the ultimate limit state for connection loading by the way "A"

In this way of loading the design load F_{d2} stays constant and the design load F_{d1} is increasing only. Then we can define $R_{d2} = F_{d2}$ and the previous definition we can substitute into the equation (1), which

we calculate the design load-carrying capacity R_{d1} from. The reliability condition of the connection in terms of the ultimate limit state is then:

$$\frac{F_{d1}}{R_{d1}} \leq 1 \quad (2)$$

The formula (2) is illustrated in

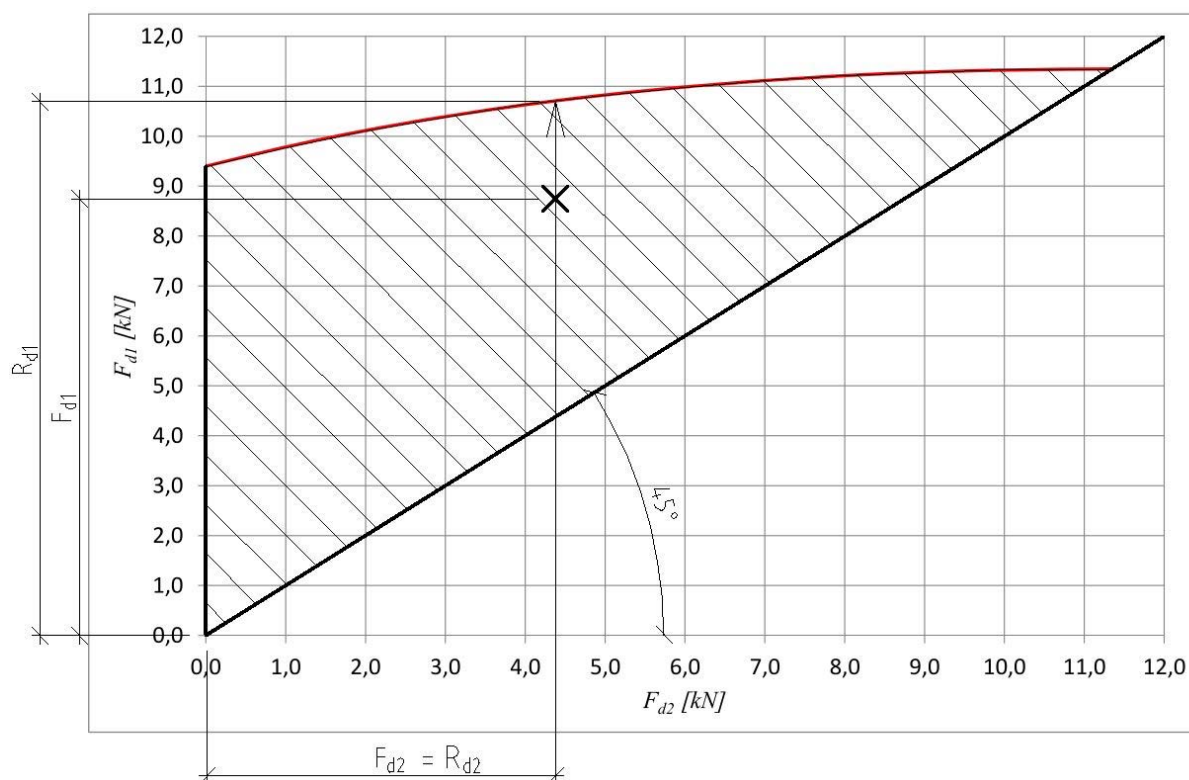


Figure 2. Assessment of the connection reliability in terms of the ultimate limit state – way of loading A

4. Assessment of the ultimate limit state for connection loading by the way “B”

In this way of loading, the both design loads are increasing and the ratio F_{d1}/F_{d2} remains constant. This way of loading is possible to expect in practice most often. The assessment of the ultimate limit state is graphically illustrated in Figure 3.

According to Figure 3 the assessment of the ultimate limit state can be expressed as:

$$\frac{a}{b} < 1 \quad (3)$$

From Figure 3 there is seen, that:

$$\frac{F_{d1}}{R_{d1}} = \frac{a}{b} \quad (4)$$

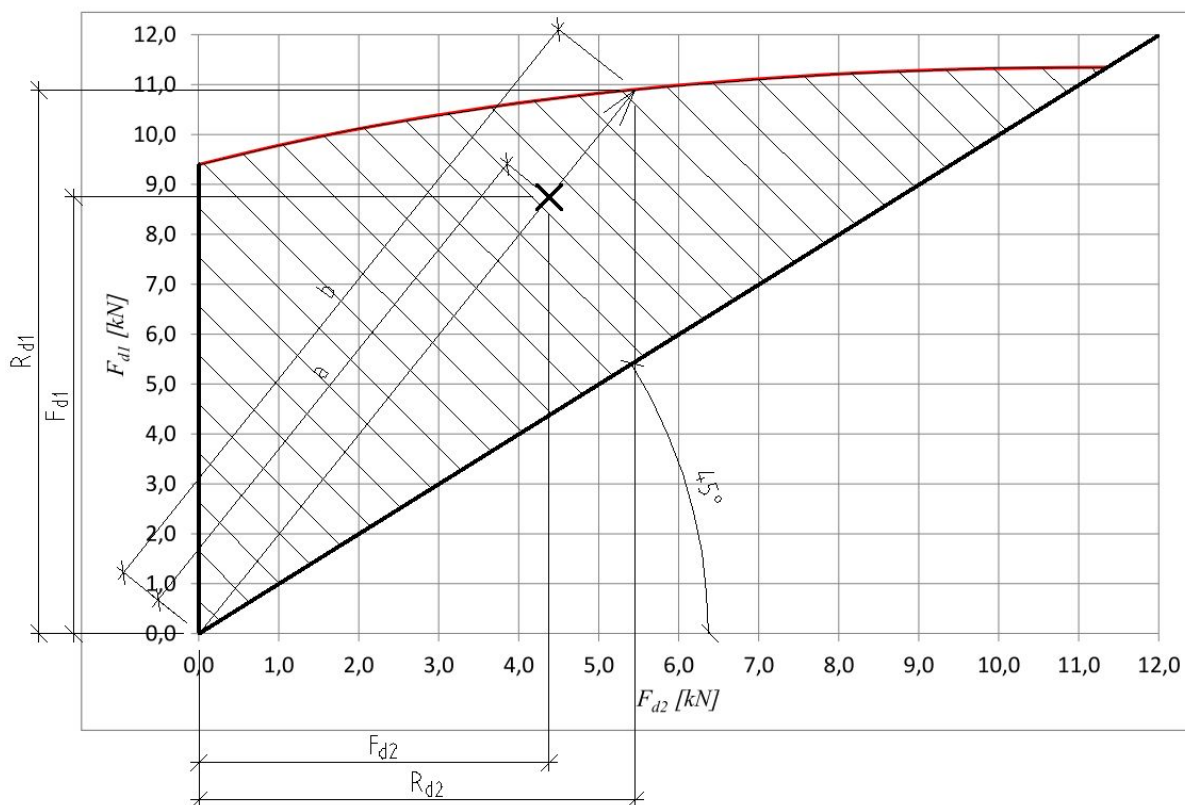


Figure 3. Assessment of the connection reliability in terms of the ultimate limit state – way of loading B.

If we substitute the equation (4) into the formula (3), we will get the reliability condition

$$\frac{F_{d1}}{R_{d1}} < 1 \quad (5)$$

Let's define a ratio:

$$n = \frac{F_{d2}}{F_{d1}} < 1 \quad (6)$$

From Figure 3 there is possible to see, that:

$$n = \frac{F_{d2}}{F_{d1}} = \frac{R_{d2}}{R_{d1}} \Rightarrow R_{d2} = n \cdot R_{d1} \quad (7)$$

If we substitute the equation (7) into formula (1) we will get the formula for R_{dl} :

$$R_{d1} = \frac{k_{\text{mod}}}{\gamma_M} \cdot f_{h,2,k} \cdot d \cdot \left(\sqrt{2 \cdot \left(t_2^2 + \frac{2 \cdot \gamma_M \cdot (n \cdot R_{d1}) \cdot t_2}{k_{\text{mod}} \cdot f_{h,2,k} \cdot d} \right)} - t_2 \right) - n \cdot R_{d1} \quad (8)$$

If we make some mathematical simplifications, we will get the design load-carrying capacity R_{dl} depending on n:

$$R_{d1} = \frac{k_{mod}}{\gamma_M} \cdot t_2 \cdot f_{h,2,k} \cdot d \cdot \frac{\sqrt{2 \cdot (1 + n^2)} + n - 1}{(1 + n)^2} \quad (9)$$

We can see in formula (9), that if we assume $n=0$, we will get the same formula like for the connection subjected to single shear. If we assume $n=1$, we will get the same formula like for the connection which is loaded by equal loads on each steel outer plate: $F_{d1} = F_{d2} = F_d$.

5. Example ultimate limit state assessment calculation of timber bolt connection subjected to double unequal shears

For the example the connection arrangement according to Figure 4 is assumed.

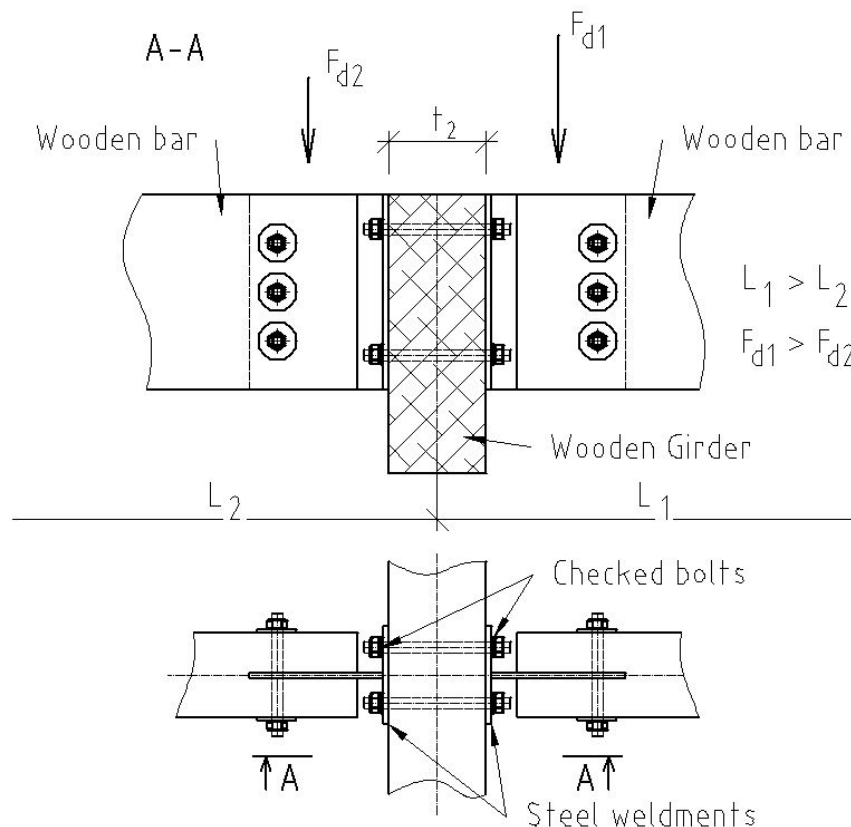


Figure 4. Connection arrangement for the numeric example

Figure 4 presents a connection of wooden bars with the wood girder in a roof structure. The connection between bars and girder is made by steel elements of T shape and by bolts. The spans of bars L_1 and L_2 are not equal, $L_1 > L_2$ so the design load F_{d1} is bigger than F_{d2} . The loads F_{d1} and F_{d2} strain the bolts in girder by shear.

In the example, there is only checked the shear resistance of the bolts in girder. These bolts are marked in Figure 4 as “Checked bolts”.

The input variables are as follows:

- design loads of the connection: $F_{d1} = 43kN$; $F_{d2} = 28kN$,
- width of the middle timber part: $t_2 = 140mm$,
- material of the middle timber part: Glued laminated timber, class GL24h,
- apparent density: $\rho_k = 380 \text{ kg/m}^3$,
- angle between the load directions and the grain of the middle timber part: $\alpha = 90^\circ$,

- characteristic tensile strength of the bolt: $f_{uk} = 800 \text{ MPa}$,
- diameter of the bolts: $d = 16 \text{ mm}$,
- tensile bolt area: $A_s = 157 \text{ mm}^2$,
- thickness of the outer steel plates: $t_{plate} = 8 \text{ mm}$,
- modification factor: $k_{mod} = 0,8$,
- partial factor for timber: $\gamma_M = 1,3$,
- partial factor for bolts: $\gamma_{M2} = 1,25$.

At first we calculate some values needed for further calculation:

Characteristic tensile capacity of the bolt:

$$F_{ax,Rk} = \frac{0,9 \cdot f_{uk} \cdot A_s}{\gamma_{M2}} = \frac{0,9 \cdot 800 \cdot 157}{1,25} = 90,43 \text{ kN}$$

Characteristic embedment strength in the hole for the bolt in the girder $f_{h,2,k}$:

$$f_{h,0,k} = 0,082 \cdot (1 - 0,01 \cdot d) \cdot \rho_k = 0,082 \cdot (1 - 0,01 \cdot 16) \cdot 380 = 26,17 \text{ MPa}$$

$$k_{90} = 1,35 + 0,015 \cdot d = 1,35 + 0,015 \cdot 16 = 1,59$$

$$f_{h,2,k} = \frac{f_{h,0,k}}{k_{90} \cdot (\sin \alpha)^2 + (\cos \alpha)^2} = \frac{26,17}{1,59 \cdot (\sin 90)^2 + (\cos 90)^2} = 16,46 \text{ MPa}$$

Characteristic bolt yield moment

$$M_{y,Rk} = 0,3 \cdot f_{u,k} \cdot d^{2,6} = 0,3 \cdot 800 \cdot 16^{2,6} = 3,24 \cdot 10^5 \text{ Nmm}$$

In further calculations we will assume the way of loading B according to the Figure 3. At first we calculate the characteristic and design load-carrying capacity for the case, when the failure is caused by creating of the plastic hinge in the bolt:

$$\begin{aligned} R_{k1} &= 1,15 \cdot \sqrt{2 \cdot M_{y,Rk} \cdot f_{h,k} \cdot d} + \frac{F_{ax,Rk}}{4} = 1,15 \cdot \sqrt{2 \cdot 3,24 \cdot 10^5 \cdot 16,46 \cdot 16} + \frac{F_{ax,Rk}}{4} \\ &= 15,03 \text{ kN} + \frac{F_{ax,Rk}}{4} = 15,03 + 3,76 = 18,79 \text{ kN} \end{aligned}$$

$$R_{d1} = k_{mod} \cdot \frac{R_{k2}}{\gamma_M} = 0,8 \cdot \frac{18,79}{1,3} = 11,56 \text{ kN}$$

For the case of failure mode when no plastic hinge in the bolt will occur we will use the formulas (7) and (9):

$$n = \frac{F_{d2}}{F_{d1}} = \frac{28}{43} = 0,651$$

$$R_{d1} = \frac{k_{mod}}{\gamma_M} \cdot t_2 \cdot f_{h,2,k} \cdot d \cdot \frac{\sqrt{2 \cdot (1 + n^2)} + n - 1}{(1 + n)^2} =$$

$$= \frac{0,8}{1,3} \cdot 140 \cdot 16,46 \cdot 16 \cdot \frac{\sqrt{2 \cdot (1 + 0,651^2)} + 0,651 - 1}{(1 + 0,651)^2} = 11,14 kN$$

Resultant design load-carrying capacity

$$R_d = \min(11,56 kN, 11,14 kN) = 11,14 kN$$

Assessment of the ultimate limit state:

$$\frac{F_{d1}}{R_{d1}} = \frac{(43/4)}{11,14} = \frac{10,75}{11,14} = 0,96 < 1 \Rightarrow OK$$

6. Results and discussions

In present days, no formulas for ultimate limit state assessment of timber structure bolt connection subjected to double unequal shears are available. Structure engineers designing such connection presented here have only formulas describing the case of connections subjected to double equal shears and cannot describe the behaviour of these connections more precisely. Formulas derived here enable this description.

7. Conclusions

The formulas enabling assessment of timber structure bolt connection subjected to double unequal shears were derived in this paper.

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