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# Game Theory Analysis of Bidding for A Construction Contract

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**Abstract**. The authors are concerned with a bidding problem. There are two companies (P1 and P2) bidding for a highway construction project. In order to be more competitive, P1 considers buying a new gravel pit near the construction site. The basic cost of the pit is known to both companies. However, there is also an additional, hidden, cost (C) known only to P1. P2 is uncertain whether the hidden cost is C = 0 or C = x. P1 plans to bid for the job, but has to decide whether to buy the gravel pit. P2, not having a complete knowledge about C, thus not knowing the strategy choice of P1, has to decide if to bid for the job. In effect we have two payoff matrices, one for the additional cost C = 0, and the other one for C = x. If the probability of P2 bidding for the project can be estimated by propagating intelligence information through a Bayesian Belief Network, the best strategy for P1 can be readily determined.
Otherwise, the solution calls for changing this game of incomplete information (players may or may not know some information about the other players, e.g., their "type," their strategies, payoffs) into a game of imperfect information (players are simply unaware of the actions chosen by other players).
This is achieved by introducing an additional "Nature" node which for this problem determines with some probability "p" the additional cost C= 0 (thus, C = x with probability 1-p). The solution of this game turns out to depend on the probability "p". For some values of p the game is solved with pure strategies, whereas for other values the game is in equilibrium when the players randomly mix their strategies.

## 1. Introduction

In our daily life, there exist many a setting when one has to make decisions whose outcome depends on the actions of the opposing side [1, 2]. In addition, the decision-maker may often have incomplete information regarding nature, style, state, pay-offs, etc. about his adversary [3, 4]. Some of typical settings are given below:

1. Bidding for a project,
2. Political campaign of a candidate,
3. Prices charged by a firm for a particular product,
4. Art auctions.

A simple introductory example: Consider the following conflict game. In this game player 1 can be in two states: Strong and Weak; and payoffs for both players differ in each of the case.

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**Table 1.** Utilities, introductory example

| | Strong | Fight | Not |
|---|---|---|---|
| Player 1 | Fight | 1, -2 | 2, -1 |
| | Not | -1, 2 | 0, 0 |

| Player 2 | | |
|---|---|---|
| Weak | Fight | Not |
| Fight | -2, 1 | 2, -1 |
| Not | -1, 2 | 0, 0 |

Introductory example

Suppose both the players fight then the outcome depends whether the player 1 is "strong" or "weak". Consequently, Nash Equilibrium will also differ.

Suppose both the players know whether player 1 is going to be strong or weak. If player 1 is strong then he has a dominant strategy of fighting. Thus player 2 will not fight and Nash Equilibrium is {Fight, Not}. If the player 1 is weak then Nash equilibrium is {Not, Fight}.

Suppose player 2 does not know what player 1 is going to be. So he assigns a probability "p" player 1 is strong and probability "1-p" player 1 is weak.

Player 2 expected payoffs when he fights: $p(-2)+(1-p)(1)$, expected payoff when not fighting: -1.

Therefore, player 2 should fight if $p(-2)+(1-p)(1) > -1$, i.e. if $p<2/3$.

It is clear that in the case of incomplete information player 2 to make the best decision (fight or not) would have to collect relevant evidence that would enable him/her to estimate the probability of player 1's state.

Such situations are quite common when considering conflict problems that arise in the construction industry [5]. In the following paragraphs we are concerned with one typical conflict/competition situation, bidding for a construction project. In the bidding process each competitor's decision whether to bid and what bid to submit depends on the expected actions of the other players in the bidding game. Thus we are dealing with the problem of interactive decision-making, the kind of problem that is best addressed by the tools of Game Theory.

## 2. Analysis of bidding for a construction job from the perspective of Game Theory

Let us consider one interactive decision-making problem. The problem is an example of interactive decision-making situations that arise in the process of bidding for construction projects. The solution presented herein follows the Game Theory methodology described in [6, 7]. There are two companies, P1 and P2, that want to decide whether to submit bids for a highway construction project. P1 to be more competitive considers building a new facility (let it be a gravel pit). The basic cost of the gravel pit is known to both companies. However, there are additional cost of this facility that are known only to P1. P2 is uncertain if the additional cost is K=0 or K=1.0.

In this game P2 has to decide if to submit a bid, while P2 wants to choose between building or not the new facility. The payoffs for every decision combination and the two states (K=0 and K=1.0) are given in Tables 2 and 3.

**Table 2.** Utilities/payoffs for K=0

| | | P2 | |
|---|---|---|---|
| P1 | | Bid | Not |
| | Build | 1.5, -1 | 3.5, 0 |
| | Not | 2, 1 | 3, 0 |

**Table 3.** Utilities/payoffs for K=1.0

| | | P2 | |
|---|---|---|---|
| P1 | | Bid | Not |
| | Build | 0, -1 | 2, 0 |
| | Not | 2, 1 | 3, 0 |

Please note: the first number in each cell is the utility for P1, the second one for P2.
A question: what will P1 and P2 do for K=0 and K=1.0? Observations:
1. The utilities for P2 depends only on whether P1 builds the graves pit or not, and does not depend on the gravel pit's cost.
2. For P2 bidding makes sense only when P1 does not build the gravel pit.
3. Not to build is dominant strategy for K=1.0.
4. For K=0 the strategy choice for P1 depends on the prediction of what P2 will do.

Let us denote by y the probability that P2 is going to bid. Then for P1 to build is better than not to if:

$$1.5y + 3.5(1 - y) \geq 2y + 3(1 - y) \tag{1}$$

$$y \leq ½$$

In this situation P1 is well advised to collect relevant evidence that could be used in a problem-specific Bayesian Belief Network (BBN) [4] to estimate posteriori probability "y" via evidence propagation, and then select the best strategy depending on the value of "y".
If "y" cannot be estimated we have a problem of mutual dependence, i.e, P1 has to "guess" the strategy of P2 to select his/her optimal strategy. In turn, P2's selection depends on the strategy choice of P1. So we have mutual, two-way, feedback.

For a long time it was not known how to undo this Gordian knot. Harsanyi (1967-68) proposed a method that allowed to transform a game of incomplete information to a game of imperfect information [6]. In incomplete information games, players may or may not know some information about the other players, e.g. their "type", their strategies, payoffs or their preferences. In a game of imperfect information, players are simply unaware of the actions chosen by other players. However they know who the other players are, what their possible strategies/actions are, and the preferences/payoffs of these other players. Hence, information about the other players in imperfect information is complete [8].
The Harsanyi transformation is performed by adding a nature node which determines, in our example, the type of P1 (the additional cost of gravel pit). Let us denote by "p" the probability that K = 1.0.
Figure 1 shows the transformation from incomplete to imperfect game [6, 8].

B = bid, D = don't

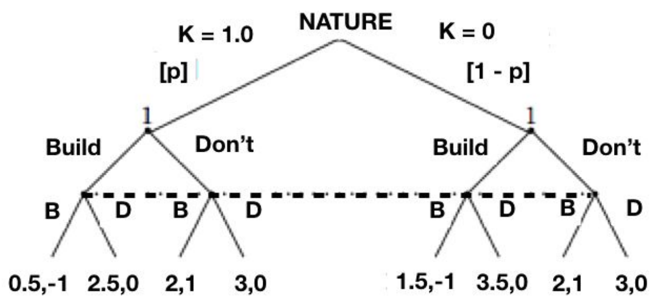


**Figure 1**. Imperfect game

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Please note that the left-hand branch is concerned with K=1.0, the right-hand one with K=0, and the utilities listed at the bottom of the tree are related to the pay-offs shown in Tables 2 and 3.

Assumptions:

1. The nature node makes the first move and randomly selects type of P1, K=1.0 with probability p, and K=0 with probability “1-p”. P1 and P2 have the same prior information about p.
2. P1 “observes” his type, i.e., he has private information about it.
3. However P2 knows only probability p.
4. Important: P1 will optimise based on what he thinks P2 will do, thus his choice of strategy depends on his beliefs about P2.
5. P2 has one large information set, and his strategy choice has but one component: what to do in this set.

Interesting question: P1 knows his type, say K=0, so why he has to consider the strategy choice for (not existing) K=1.0? Answer: to choose optimal strategy P1 has to know what P2 will do. But P2 does not know the type of P1, and he will optimise based on his expectations of P1 will do for K=0 and K=1.0.

Strategy sets: Nash equilibrium will consist of a strategy triplet:

1. one for P1 when K=0,
2. one for P1 when K=1.0, and
3. one for P2.

Now we can encode our bidding problem using the strategy form. Let’s recall that P1 has the following pure strategies: S1 = (Bb, Bd, Db, Dd). The first letter refers to the strategy for K=1.0, the second for K=0. P2 has just two pure strategies: B and D. The utility table for these strategies is given below (Table 4).

**Table 4.** Utilities - imperfect game

| | | Player 2 | |
|---|---|---|---|
| Player 1 | Bb | 1.5 - 1p, -1 | 3.5 - 1p, 0 |
| | Bd | 2 - 1.5p, 1 | 3 - 0.5p, 0 |
| | Db | 1.5 + 0.5p, 2p - 1 | 3.5 - 0.5p, 0 |
| | Dd | 2, 1 | 3, 0 |

Please note that in this matrix Dd dominates Bd, and Db dominates Bb, for any value of p>0. Thus we can remove the dominated strategies to obtain.

**Table 5.** Reduced utility matrix

| | | Player 2 | |
|---|---|---|---|
| Player 1 | Db | 1.5 + 0.5p, 2p - 1 | 3.5 - 0.5p, 0 |
| | Dd | 2, 1 | 3, 0 |

Please observe:

1. If P2 selects B, then the best unique response of P1 is Dd, for any p<1.
2. Hence (B, Dd) is one Nash equilibrium for p lying in (0.1).
3. B exactly dominates D for 2p-1 > 0 => p >1/2.

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It can be shown that p≤1/2 the solution of this game calls for randomly mixing their strategies by either player. Such solution does not, however, provide practical recommendations to the players with respect to selecting their optimal strategies. Therefore, we suggest that in this case P1 uses the above-presented solution for p>1/2. It may be achieved by P1 creating evidence that leads P2 to believe that p>1/2. Base on this belief P2 will select B. P1 responds accordingly by selecting Dd, as indicated in the introductory analysis of this interactive game.

## 3. Conclusions

In this paper we have demonstrated how the tools of Game Theory could be used to select optimal strategies while bidding for a construction project. We have not attempted to provide one general solution to such a problem. Instead a methodological framework that could be employed to optimise strategies in the bidding process was presented. In particular, it has been showed how the problems of incomplete information could be dealt with by transforming them to the ones of imperfect information. Finally, we recommend that whenever probabilities of the opponent's state and/or utilities are of importance in interactive decision-making a combination of Game Theory and Bayesian Belief Networks might be an effective architecture for modelling such situations and seeking optimal but still practical solutions.

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