

Fitting Function for Flexural Strength of Cement Paste

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Abstract. There are many analytical expressions for compressive strength as functions of porosity of cement-based materials but only a few such expressions exist for flexural strength of these materials. In the present paper a new functional candidate for fitting the data of flexural strength of hydrated Portland cement paste has been tested. The functional candidate has been initially derived for porous polymeric materials on the basis of the percolation theory. The parameters of this function have been optimized for the cement paste by using the Levenberg-Marquardt iterative fitting procedure. The optimized function has been capable of accurate reproducing all the measured flexural data. This fact has been confirmed by a high value of the correlation coefficient and rather low values of statistical uncertainties. It has been shown that this modified fitting function is well applicable to the pastes of ordinary Portland cements and probably to other cementitious materials, too.

1. Introduction

Recently a detailed study of *compressive strength* as a function of the irregularities of fracture surfaces of hydrated ordinary Portland cement pastes has been published [1]. The specimens of hydrated pastes were characterized by different capillary porosities that caused different stages of irregularities with fracture surfaces. In introductory part of that study a large number of expressions for compressive strength as functions of porosities were discussed. However, when the *flexural (bending) strength* as a function of porosity is required, one may find only a very limited number of such expressions [2-5]. One of those few expressions was derived [2] on the basis of percolation theory for polymeric materials but it might be formally adjusted for cement pastes as well:

$$\sigma^{(flex)} = \sigma_o \sqrt{\left(1 - \frac{P}{P_c}\right)^\beta \cdot (1 - P^{2/3})}, \quad 0 \leq P \leq P_c \quad (1)$$

where σ_o is maximum strength at zero porosity ($P = 0$), the exponent β is a parameter that has to be optimized for the particular material, and P_c is a critical porosity for which the flexural strength approaches zero $\sigma_{por}^{(flex)}(P = P_c) = 0$. In practice, the symbols σ_o , β , and P_c are fitting parameters.

The goal of this paper is to verify applicability of function (1) to hydrated ordinary Portland cement pastes of different porosities and in this way to offer a new alternative for fitting the flexural data of cementitious materials in which capillary porosity plays an important role.



2. Experimental arrangement

These specimens were hydrated under standard conditions for 4 years. They were divided into seven groups according to their water-to-cement ratios ($w/c = 0.2, 0.3, 0.4, 0.5, 0.6, 0.7$, and 0.8). Each group contained 6 notched specimens whose dimensions were $b \times H \times D$, i.e. $3\text{ cm} \times 3\text{ cm} \times 12\text{ cm}$. The depth of the notches was $h = 3\text{ mm}$ and the width 1 mm . The porosity of each group of specimens was measured by the gravimetric method ($P = 0.207, 0.270, 0.334, 0.385, 0.471, 0.524$, and 0.582). The specimens were subjected to three-point bending tests to estimate their values of flexural strength $\sigma^{(flex)}$

$$\sigma^{(flex)} = \frac{3F_{\max}L}{2b(H-h)^2} \quad (2)$$

where $F_{\max}$ is a maximum loading force and $L = 8\text{ cm}$ represents a distance between two supports of the testing apparatus.

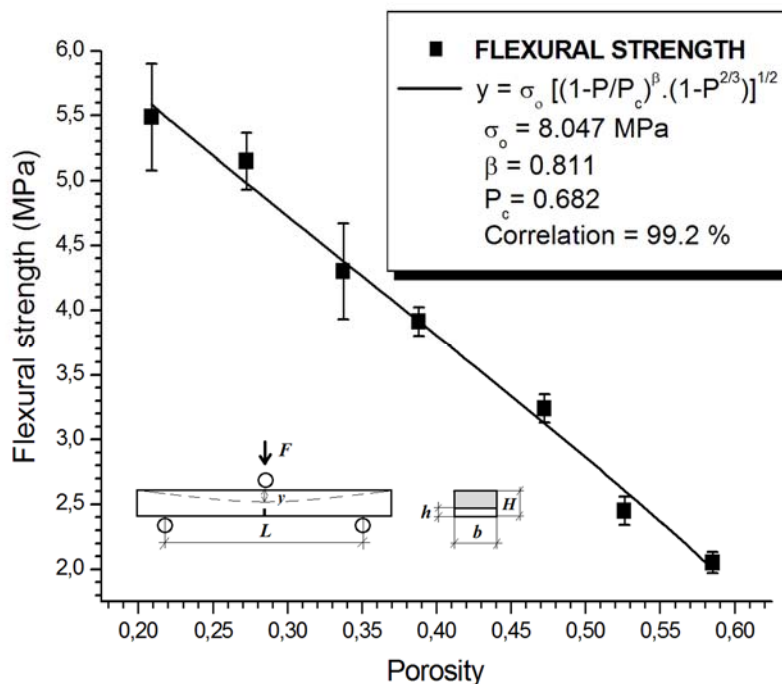


Figure 1. Results of the flexural tests of ordinary Portland cement pastes

3. Results and discussions

The measured values of $\sigma^{(flex)}$ together with the porosities P are shown in figure 1. The flexural data were fitted by the tested function (1) whose three parameters σ_o , β and P_c were optimized by the Levenberg-Marquardt iterative procedure [6], [7]. The graph drawn by the solid line represents the result of the fitting procedure and illustrates a clear dependence of flexural strength on porosity. The statistical uncertainties of the flexural data calculated as the mean probability errors of the repetitive measurements are marked by error bars. The first three data points associated with the porosities 0.207, 0.270, and 0.334 show a bit larger statistical uncertainties exceeding 5 % whereas the uncertainties of remaining data points indicate more accurate measurements whose statistical uncertainties do not exceed 5 %, which is a normal accuracy of laboratory measurements of this type.

The global trend of the measured dependence $\sigma^{(flex)}(P)$ is well approximated by function (1) which is documented by a very high value of the correlation coefficient (99.2 %). All the aforementioned facts lead us to the conclusion that the tested function (1) seems to be a good candidate for fitting flexural data of hydrated cement pastes. In addition, the non-linear functional form (1) may be a good prerequisite for modelling possible non-linear behaviour of flexural data of other cementitious materials like concretes. Nevertheless, the flexural strength of plain concretes is influenced not only by the capillary porosity of the cement matrix (hydrated cement paste) but also by the stability of the transition zones between the matrix and the aggregates (sand grains and gravels) which may be a complicated factor. However, in case of stable transition zones the influence of capillary porosity may dominate and the functional fitting pattern (1) may again show a good performance.

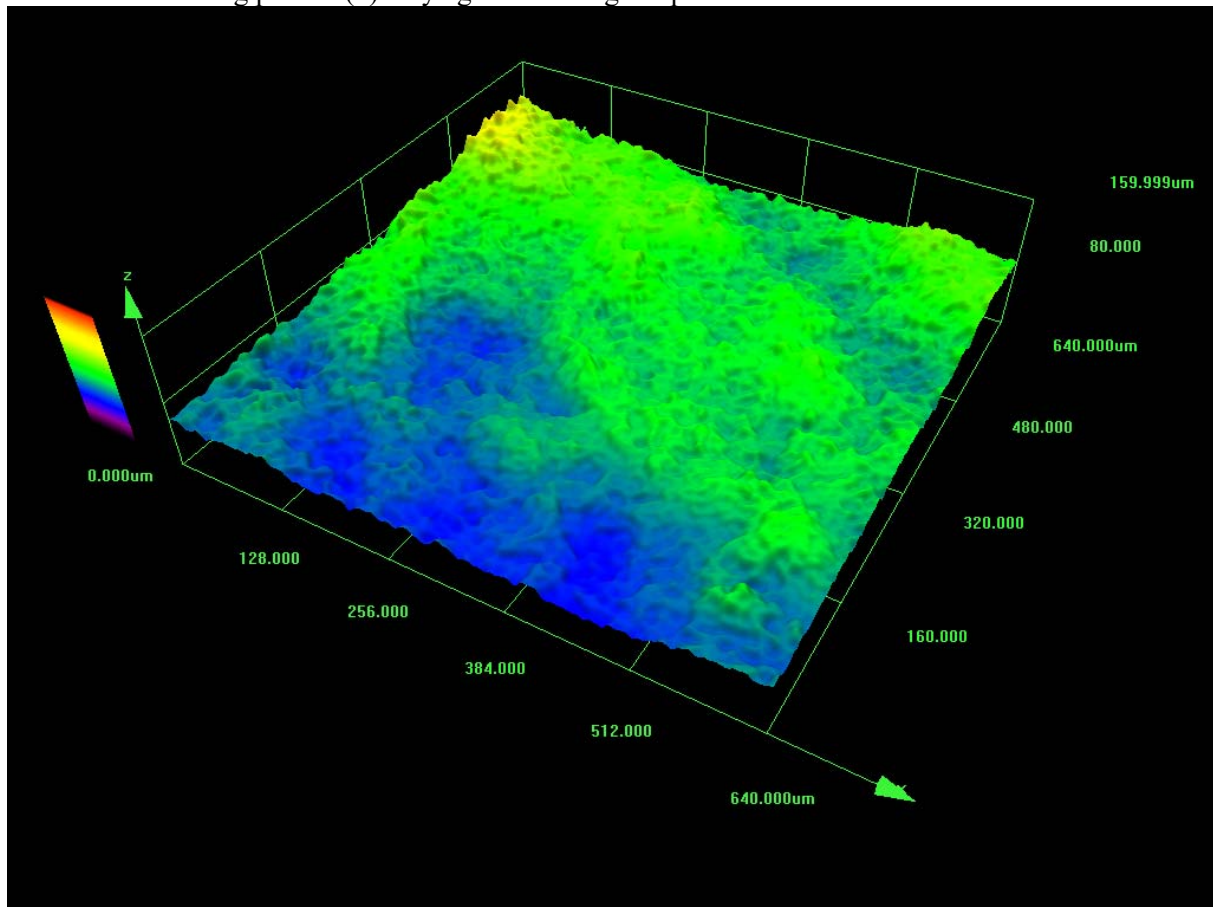


Figure 2. The three-dimensional relief of the fracture surface of the low-porosity cement paste ($P = 0.270$)

The discussion of the measured results should also tackle the problem why the first three data points of the graph in figure 1 show a higher statistical errors. All the specimens that were subjected to flexural tests were prepared in the same way and from the same cement material. They were hydrated under the same conditions and flexural tests were carried out on the same mechanical equipment within the same time interval. Yet, the first three data points which are associated with the specimens of low porosities show higher statistical errors, i.e. a larger statistical scatter of the repetitive flexural measurements.

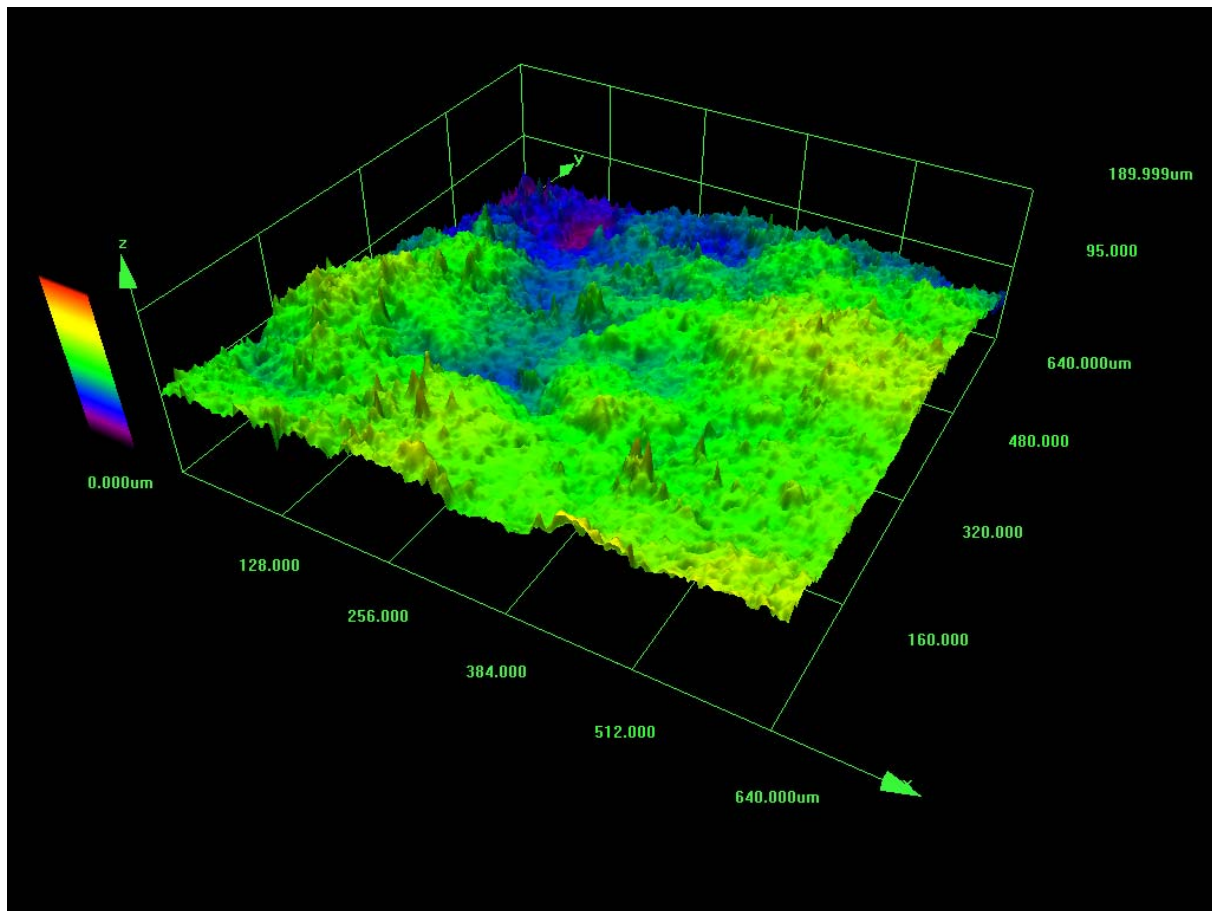


Figure 3. The three-dimensional relief of the fracture surface of the high-porosity cement paste ($P = 0.270$)

Since the preparation procedure and the subsequent processing were identical for all the used specimens, the only reason for their different mechanical (flexural) behaviour may consist in their final structural arrangement. The first three mentioned graphical points belong to the specimens with low porosities, i.e. these specimens are more compact and probably more brittle as compared with the remaining specimens that are associated with higher porosities. We can only *speculate* that the larger variability of flexural strength of the low-porosity specimens is connected with their higher brittleness. The fact is that more brittle specimens show smoother fracture surfaces and this can also be observed in our case. Figures 2 and 3 illustrate fracture surfaces of the specimens with a low porosity 0.270 and a high porosity 0.582, respectively. The specimen of low porosity (figure 2) shows a smoother fracture surface in comparison with the specimen of higher porosity (figure 3). This may also be quantified numerically by means of the *average absolute height* A of the surface relief discretely scanned in the three dimensions $z = f(x, y)$

$$A = \frac{1}{C \cdot D} \int_0^C \int_0^D |f(x, y)| dx dy \quad (3)$$

where C and D are length and width of the relief. The discrete reliefs depicted in figures 2 and 3 were created by means of the laser confocal microscope Olympus Lext 3100. The commercial software that accompanies this microscope is capable of computing average absolute heights A of both these reliefs.

The average absolute height A of the low-porosity specimen shown in figure 2 has been characterized by the value $1.9\text{ }\mu\text{m}$ whereas the relief of the high-porosity specimen shown in figure 3 has generated the value $5.1\text{ }\mu\text{m}$. As can be seen, the low-porosity specimen shows considerably smoother fracture surface with much smaller height irregularities in agreement with the assumption of its higher brittleness.

4. Conclusions

The tested function (1) has proved to be a good candidate for fitting pattern capable of reproducing flexural data of hydrated cement pastes. The multi-parametric character of this function is a good prerequisite for successful fitting performance in a variety of materials where porosity is a dominant factor influencing flexural strength. The general form of this function facilitates its applicability to both the linear and non-linear flexural data, which stands a good chance for acceptable performance also in the case of plain concretes containing sound transition zones between cement matrix and aggregates.

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