

Thermal radiation heat transfer in participating media by finite volume discretization using collimated beam incidence

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Abstract. The main objective of this paper is to study the heat transfer rate of thermal radiation in participating media. For that, a generated collimated beam has been passed through a two dimensional slab model of flint glass with a refractive index 2. Both Polar and azimuthal angle have been varied to generate such a beam. The Temperature of the slab and Snells law has been validated by Radiation Transfer Equation (RTE) in OpenFOAM (Open Field Operation and Manipulation), a CFD software which is the major computational tool used in Industry and research applications where the source code is modified in which radiation heat transfer equation is added to the case and different radiation heat transfer models are utilized. This work concentrates on the numerical strategies involving both transparent and participating media. Since Radiation Transfer Equation (RTE) is difficult to solve, the purpose of this paper is to use existing solver buoyantSimpleFoam to solve radiation model in the participating media by compiling the source code to obtain the heat transfer rate inside the slab by varying the Intensity of radiation. The Finite Volume Method (FVM) is applied to solve the Radiation Transfer Equation (RTE) governing the above said physical phenomena.

1. Introduction

The availability of high performance computing hardware and the introduction of user friendly interface have incited the change of CFD packages. Among them OpenFOAM which is the open source CFD software is used in major researches and Industrial applications. The OpenFOAM contains numerous solvers and utilities, having license free covering a wide range of problems and utilities created by its users with basic knowledge of the underlying Numerical methods and programming techniques. Angular discretization scheme of the Finite Volume Method (FVM) of radiative heat transfer is applied to emitting and scattering media by Talukdar et al. [1] with different optical thickness in a rectangular geometry and figured out that the FVM performs better than other Discrete Ordinate Method (DOM). Radiative flux, the average incident radiation, and diffuse incidence were found out by Coelho et al. [2]. Moreover the computation of collimated beam radiation have been proposed by Tuba Okutucu et al. [3] for short pulse laser irradiation in scattering and absorbing media. It is observed that as optical thickness increases, the backscattering signal is observed for a long time interval and reflectance signals get closer to each other. Chai et al. [4] observed that fluxes near to the hot surface will not change with respect to the angular grids and the fluxes at the centre of the enclosure are more than those at the ends. S.C Mishra et al. [5] figured out the variation of optical parameters by irradiating 3D geometry with a laser beam. The refractive index of the particles strongly influence the transmitted pulse shape of collimated beam. Ben Abdallah et al.



[6] have used discrete transfer radiation method for the analysis of radiation transfer in varying refractive index medium. Ananda Krishnan et al. [7] extended Discrete Transfer method to radiative transfer in a variable refractive Index semi-transparent medium and inferred that emissive power and radiant flux vary with the square of refractive index. Venkateshwar et al. [8] modeled two dimensional geometry of a slab and investigated the temperature distribution, transmissivity and critical angle. A collimated beam is passed through two media of different refractive index separated by a semi-transparent boundary which is validated in ANSYS Fluent. It is observed that the thin collimated beam entering through the opening get thick as it travels through the medium. In the present work, we use the Finite Volume Method (FVM) to discretize the Radiation Transfer Equation (RTE) in OpenFOAM to validate temperature distribution, transmitted angle and generate the collimated beam to find the rate of heat transfer across the slab by validating the work of Venkateshwar et al. [8].

2. Methodology

2.1. Mathematical Modelling

A radiant beam entering the participating medium get attenuated by absorption and out scattering, which then regains energy due emission as well as by in-scattering from some direction into the direction of travel. The Radiation Transfer Equation (RTE) is governed by the integro-differential equation:

$$\frac{1}{c} \frac{\partial I}{\partial t} + \frac{1}{c} \frac{\partial I}{\partial s} = -kI(s, \hat{s}, t) - \sigma I(s, \hat{s}, t) + k(s, \hat{s}, t)I_b + \frac{\sigma}{4\pi} \int I(s, \hat{s}, t) \phi(s, \hat{s}, t) d\psi \quad (1)$$

$$\frac{1}{c} \frac{\partial I}{\partial t} + \frac{1}{c} \frac{\partial I}{\partial s} = -\beta I(s, \hat{s}, t) + k(s, \hat{s}, t)I_b + \frac{\sigma}{4\pi} \int I(s, \hat{s}, t) \phi(s, \hat{s}, t) d\psi \quad (2)$$

Here C is velocity of light beam, I is Intensity of radiation in the direction s , k is Scattering coefficient, β is the extinction coefficient, σ is absorption coefficient, ϕ is Azimuthal angle and θ is Polar angle. The initial two terms on the right side of equation (1) represents the decrease in Intensity of radiation in a given direction as a result of absorption by the medium or it is scattered onto another direction. The third term with a positive sign implies an increase in the intensity of radiation. This term is due to thermal emission of radiant energy from the medium. I is the spectral radiation intensity at point that propagates along direction s , s is the coordinate along that direction.

The spectral radiance (I) of a body describes the amount of energy that it radiates with different wavelengths. It describes the variation of emission with temperature i.e. the emitted radiant power increases with temperature rise and the peak of the radiated spectrum is shifted towards shorter wavelengths with rising temperature.

$$T^4 = \frac{\int I d\psi}{4\pi\sigma} \quad (3)$$

The transmitted beam is governed by Snell's law which is given as follows:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad (4)$$

When a beam of light travels from a higher refractive index to lower refractive index, it deviates from its perpendicular axis at the boundary. At a particular angle of incidence, the beam reflects back to the same medium. This phenomenon is called total internal reflection and angle of incidence at which total internal reflection happens is called critical angle. The rate at which heat transferred is given by steffan-Boltzmann law which states that heat transferred in a particular direction s varies with the area perpendicular to the direction of heat flow and fourth power of radiative temperature. It is given as:

$$Q = \sigma AT^4 \quad (5)$$

The constant of proportionality is called steffan-Bolztmann's constant.

2.2. Initial and Boundary Condition

Initially, the source is kept at a position $x=0$ at the extreme left end of the slab. Initially the surface temperature is given as 300k. The Intensity of Radiation is set to zero such that when the radiation model is set to on in a different time step, the Intensity value become maximum initially and then gradually decreases. To give Intensity as initial boundary condition, there is Marshak Boundary condition where Intensity is given as a function of time, wavelength and position. The transmissivity is given as 0.8 and reflectivity as 0.152 as boundary condition initially. Another boundary condition is given in absorption-emission model where emissivity is designed as 0.2. The convergence criteria is set with gauss-seidel iteration of 10^{-4} tolerance where the solution converge by discretizing using Finite Volume Method at polar discretization =10 and azimuthal discretization=25.

2.3. Modelling and Computational Modelling

A two dimensional model of a slab model made of flint glass with refractive index 2 is designed as shown in Figure1. A light source is provided at a distance 80mm from the top surface. The length of the slab is taken as 100mm and height is 200mm such that light beam initiates from the source when the radiation model is on and get scattered along the whole length of the slab. At the semi-transparent layer at the extreme right end, both reflection and transmission occurs. The beam of light starts from the source and get transmitted through the boundary. The case was named as slabradiationFoam and the model was designed in blockMeshDict file of the solver. The solver used to model this two dimensional slab is buoyantSimpleFoam is a steady and incompressible solver. The solver buoyantSimpleFoam satisfies all the three basic mass, momentum and energy equations.

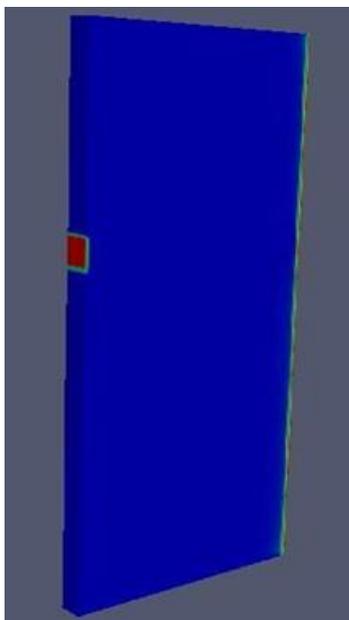


Figure 1. Slab model

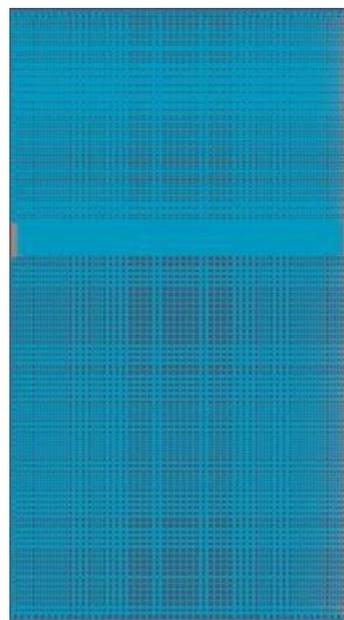


Figure 2. Rectangular Mesh

The generated mesh is a rectangular mesh which is made uniform throughout the whole domain. It consist of 180000 elements and 192000 cells. The rectangular mesh is the best suited for modeling a slab and will obtain accurate results with greater clarity. A fine mesh in Figure 2 150x400x1

consisting of 180000 elements is created with aspect ratio 10. Concerning the FVM, the angular domain is discretized by polar and azimuthal control angles respectively. It is also considered to have azimuthal symmetry with reflectivity, transmissivity and scattering coefficient. The base solver is compiled by adding radiation model into it by combining the source code such that a new solver solving RTE for a slab model is designed. The code is then run for finite number of iterations where the diffusion of rays happens for a minute interval of time gap.

2.4. Grid Independency

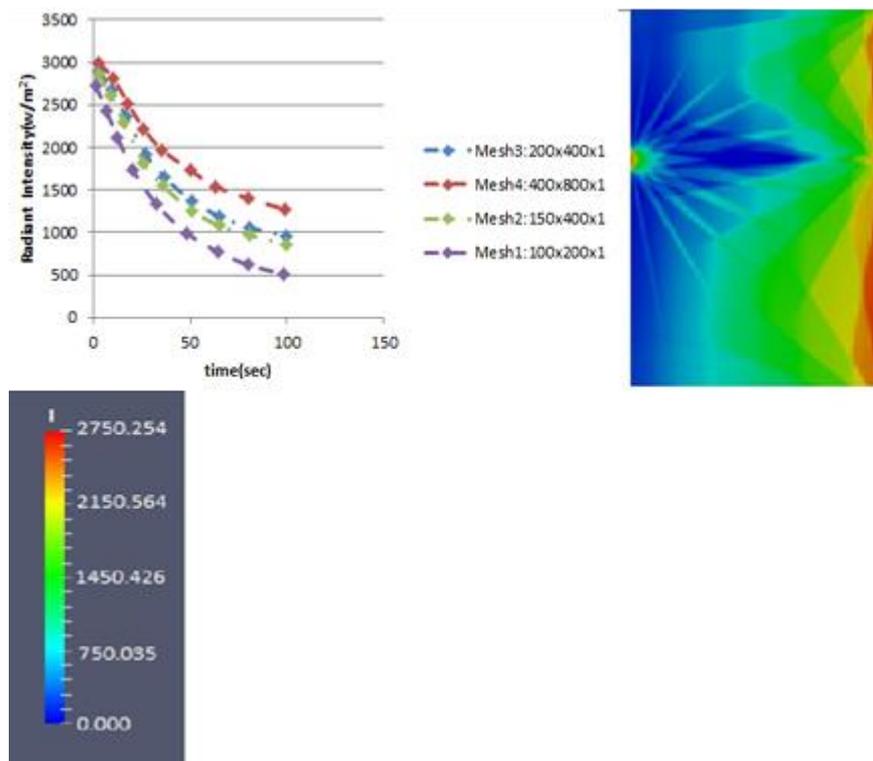


Figure 3. Variation of Intensity with time and contour plot

This paper considers the two dimensional steady state to reveal the effect of grid resolution on numerical results. Here we increase the grid resolution and then compare the results of two neighbouring results. If the results tend towards identical, the grid can be considered as grid independent. This will reduce the computational time and obtain reasonable result most efficiently. Mesh Independence is thus a way to find optimum grid size for a particular Intensity of radiation. As the number of grid points/mesh is increased, the error in the numerical solution would decrease and the agreement between the numerical and exact solution would be better. When the numerical solution obtained on different grid/mesh level agrees with a level of tolerance specified, solution is grid converged. The computational domain was of 100 mm x 200 mm in dimension. A grid independency test is performed to ensure the accuracy of the results obtained by the modified solver. Various mesh sizes of the order 100x200x1, 150x400x1, 200x400x1, 400x800x1 were used to compute the variation of Intensity along the axial distance. The result as shown in Figure 3 tend to be almost similar for 150x200x1 grid system in comparison with 200x400x1 grid system. Hence mesh size of 150x400x1 was selected as the optimum number of mesh elements for analysis of Radiation heat transfer in participating medium.

2.5. Radiation Modelling

In order to run the Radiation model, the base solver is compiled by discretizing the Radiation Transport Equation (RTE) using Finite Volume Method. Thus new solver named “buoyantSimpleradiationFOAM” is developed which will run the code for finite number of iterations. This is being done by compiling using Wmake code thereby including RTE into the base solver. The model is made in the 0, constant and system directory and the output can be seen in the paraFoam window.

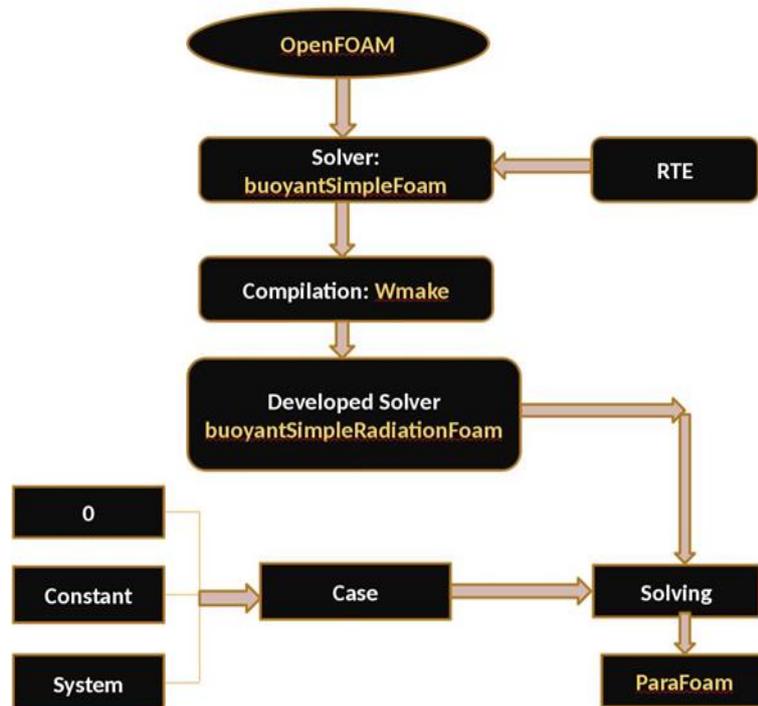


Figure 4. Flow chart of modelling radiation solver in OpenFOAM.

2.6. Material Specification

Table 1. Material Specification

Material Property	Numerical Value
Density	2.23g/cm ³
Refractive Index	2
Expansion coefficient	3x10-6
Thermal Conductivity	1.2 w/mk
Youngs Modulus	64Gpa
Viscosity of air	1.79x10-5kg/ms
Refractive Index of air	1

3. Result and Validation

3.1. Validation of Snells Law and Reflectivity

A collimated beam is passed through medium 1 through a source provided at the inlet of the slab and finally strikes the boundary separating the two media. Some amount of energy of the beam crosses the boundary and enters another medium of different refractive index. The accuracy of angle at which the beam incident on the boundary depends on angular discretization for Finite volume method of radiative transfer equation. From Table 2, it is seen that when polar angle is kept fixed as 5 and azimuthal angle is allowed to vary, the transmitted angle decrease in a consistent manner.

Table 2. Effect of Transmitted Angle With Various Angular Discretization

Polar Discretization	Azimuthal Discretization	Transmitted Angle	Error(%)
5	5	27.20	8.92
5	15	21.01	1.86
5	25	19.81	0.12
10	5	27.20	8.92
10	15	21.01	1.86
10	25	19.81	0.79

Also when polar angle is kept fixed as 10 and azimuthal angle is varied, the transmitted angle decrease upto 19.80. So the accuracy of strike angle depends on the azimuthal angle discretization rather than polar angle. According to snells law, transmitted angle for a beam emerging from medium 1 with refractive index 2 to medium 2 with refractive index 1 is 20.70 in Ansys Fluent. In the work of venkateswar et al., the transmitted angle varies from 27.022 to 19.70. Thus we can infer that the error percentage decreases gradually and is very less in OpenFOAM software as compared to the benchmarked result of venkateswar et al. [8].

Table 3. Effect of reflectivity and transmissivity With Various Angular Discretization

Polar Discretization	Azimuthal Discretization	P	T	Error(%)
5	5	0.12493	0.87507	1.624
5	15	0.12390	0.87610	1.024
5	25	0.12386	0.87614	1.001
10	5	0.12378	0.87622	0.948
10	15	0.12310	0.78269	0.297
10	25	0.12289	0.77110	0.231

From Table 3, reflectivity(ρ) and transmissivity(τ) only vary with azimuthal angle discretization of FVM. Some amount of incident energy get transmitted and rest of them get reflected. The reflectivity and transmissivity at the boundary matches close to the calculated value of literature and error(%) is much less than 5 percentage.

3.2 Temperature distribution

The temperature variation is different for a collimated beam as compared to the diffused one. For a diffused beam of light, there is gradual decrease of temperature along the length of the slab. But for a collimated beam emerging from a source, there is an initial rise in temperature followed by a gradual

decrease. The temperature become maximum for $I=2200(\text{w}/\text{m}^2)$ at the extreme end of the 2D slab model. The variation in temperature is due to the effect of back scattering followed by reflection of light at the boundary which overcome the loss of intensity and give rise to a sudden increase of temperature. It is also dependent on the angular discretization for the Finite volume method of radiation heat transfer equation.

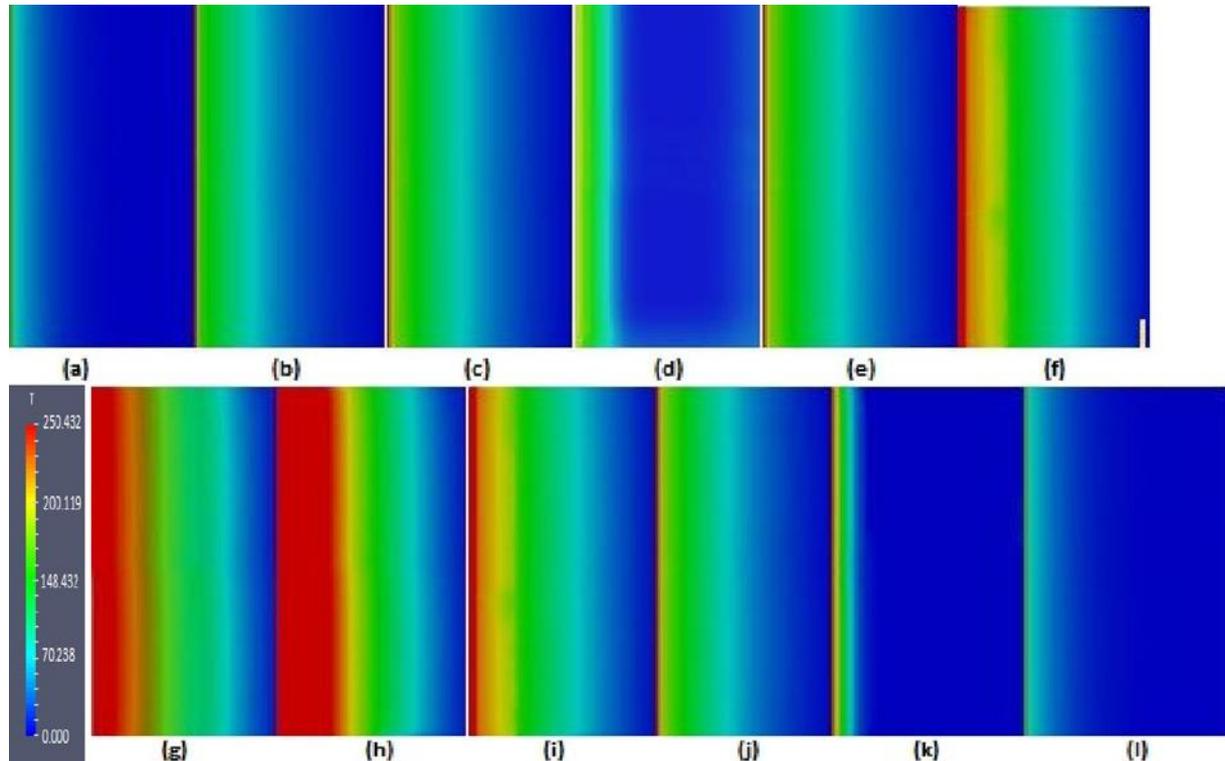


Figure 5. Variation of Temperature(T) along slab (a) T at $t=9\text{s}$ (b) T at $t=18\text{s}$ (c) T at $t=25\text{s}$ (d) T at $t=36\text{s}$ (e) T at $t=48\text{s}$ (f) T at $t=54\text{s}$ (g) T at $t=66\text{s}$ (h) T at $t=72\text{s}$ (i) T at $t=81\text{s}$ (j) T at $t=90$ (k) T at $t=98\text{s}$ (l) T at $t=106\text{s}$

3.3. Rate of Heat Transfer

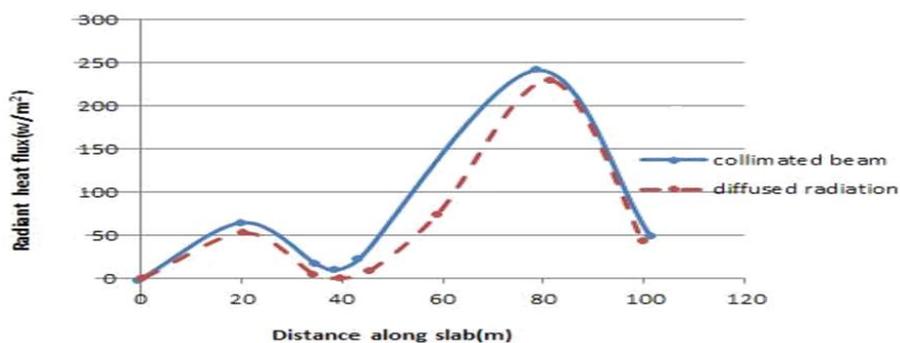


Figure 6. Variation of heat transfer rate.

From Figure 6 the heat transfer initially increases, reaches a maximum value and then decreases along the length of the slab for a diffused beam. But for a collimated beam, the decrease of heat energy happens very slowly and linearly until it reaches $50(\text{w}/\text{m}^2)$. The maximum rate of heat transfer occurs corresponding to the peak intensity of the collimated beam. At an intensity of $2750\text{w}/\text{m}^2$, maximum value of heat transferred is $248\text{w}/\text{m}^2$ for collimated incidence and is $232\text{w}/\text{m}^2$ for diffused beam. This is in accordance with steffan- boltzman law. The rate of heat transfer varies with the fourth power of the temperature with constant area of slab. There is a sudden reduction of heat energy from 200w to 50w for a collimated beam because there is no backscattering along its direction as compared to the diffused beam when polar angle= 5 and azimuthal angle= 25 .

4. Conclusion

The transmissivity and reflectivity matches the calculated value which does not influence the angular discretization. Thus modified solver is validated against the existing work and it is obtained that accuracy is much high for the modified solver. The temperature and heat flux first increases, reaches a maximum and then decreases across the length of the slab. This variation is due to the effect of backscattering followed by reflection of light at the boundary which overcomes the loss of intensity and give rise to a sudden increase of temperature. The rate of heat transfer at the end of the slab is very low. For a diffused beam, it varies linearly but for a collimated beam heat transfer decreases very slowly but linearly. The research can be done for different mediums with different refractive index thereby finding the heat transfer rate across each medium. The variation can be made by changing the position of the light source and incident angle by discretization of polar and azimuthal angle in the participating medium. The variation of emissivity with scattering albedo for a particular transmittance and reflectance are the further researches that can be done. Also one of the semi-transparent layer can be given additional heating to incorporate in the slab model which is used for the design of heat transfer mechanism in the medium.

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