Static Stiffness Optimization of Rubber Absorber Based on Taguchi Method

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Abstract. Static stiffness is an important property to ensure good bearings capacity performance of rubber absorber. Taking BE-300 type rubber absorber as an example, in order to obtain desirable static stiffness, Taguchi orthogonal design is proposed in this study based on the finite element method(FEM) replacing the practical test. Firstly, the FEM model of original rubber absorber which is consistent with practical test is established through constitutive test of rubber material. Dimensional parameters are one of the key factors affecting static stiffness, regarding bottom clearance height, rubber support layer thickness, width of support bearings and distance between large radius and small radius as controllable factors to design orthogonal FEM model by using Taguchi orthogonal methods. 25s FEM models with different level of controllable factors combinations are built based on the established FEM analysis technique. For each controllable factors combination scheme, the mean sensitivity parameters are calculated to evaluate the incidence of each controllable factor. The result show that the width of support bearings is a dominant factor affecting static stiffness. Finally, the optimization scheme can be proposed based on sensitivity analysis. Compared with the result from orthogonal optimization simulation and practical test, the static stiffness is much closer to the desired value. The study demonstrate that the orthogonal simulation is a useful approach for designing rubber absorber to specified static stiffness.

1. Introduction

Rubber absorbers have been widely used in modern ships because of its superior design and vibration reduction performance. BE-300 type rubber absorber, which are with features of low intrinsic frequency, excellent damping capacities, stable performance and compact construction, and can be installed in confined space, became one of superior absorber using in various ship. Therefore, it is essential to study its mechanical properties to design products that fulfil with performance requirements.

The structural optimization of rubber absorber includes topology optimization, shape optimization and dimension optimization[1-3]. Orthogonal design is a conventional technique method during the manufacture of rubber products. Taguchi orthogonal design method, with multiple controllable factors, treat sensitivity parameter as an indicator to evaluate which level of controllable factors are the key to the response results. The purpose of improving test efficiency and reducing cost can be achieved with high reliability. In this paper, the optimization design of rubber absorber is replaced by FEM. Using sensitivity parameters to evaluate the magnitude of the influence of the dimensional parameters on the static stiffness, Finally, an optimal scheme of controllable factor combination satisfying the desired

value is determined.

2. FEM simulation of rubber absorber

2.1 Material Parameters Obtain

The constitutive relation of rubber material is essential for the FEM simulation of rubber absorber[4], Usually, the constitutive equations are derived according to the differential energy function by the description of the strain energy function, The strain energy function of the 2 order polynomial model define as follows[5]:

$$W(I_1, I_2) = \sum_{i,j=0}^{2} C_{ij}(I_1-3)^i (I_2-3)^j + \sum_{i=1}^{N} \frac{1}{D_i}(J-1)^{2i} \qquad (1)$$

In the formula (1), C_{ij} is the Rivlin coefficient, $C_{00}=0$, I_1 and I_2 are the first and second Green strain invariants, and J is the volume ratio of the rubber before and after deformation, and the parameter of D_i determines whether the rubber material is compressible. 2 is the polynomial order, when treats the rubber material as incompressible material, $J=1$[6], so 2 order polynomial model define as follows:

$$W(I_1, I_2) = \sum_{i,j=0}^{2} C_{ij}(I_1-3)^i (I_2-3)^j \qquad (2)$$

Strain energy function of rubber materials generally tested by uniaxial tensile and uniaxial compression. As shown in figure 1, uniaxial tensile is refer with GB/T 528-2009[7], the rubber specimen is dumbbell shaped. The thickness of the specimen is 2±0.2mm, the length of the test is 25±0.5mm, and the width of the narrow part is 6±0.4mm. As shown in figure 2, the tensile test is carried out on the SANS testing machine. The elongation of large deformation part is measured with extensometer. Uniaxial tensile tests of rubber materials are designed to obtain complete tensile stress-strain data. Because of the Mullins effect and hysteresis effect in the filled rubber, the loading and displacement are not synchronized in the test. In order to reduce the experimental error caused by the material itself, the tensile test of the rubber specimen was carried out by quasi-static tensile rate. figure 3 is the stress-strain curve obtained by uniaxial tensile test fitting with the 2 order polynomial model.
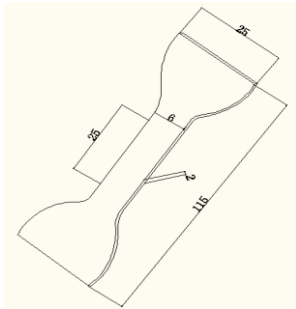

Figure 1. rubber specimen of uniaxial tensile
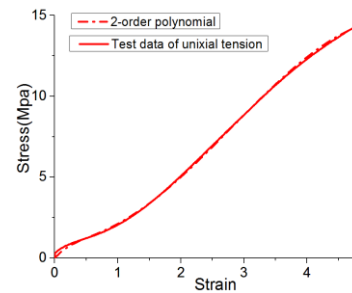

Figure 2. uniaxial tensile test

Figure 3. 2 order polynomial fitting with uniaxial tensile.

The tensile and compressive strain of rubber is very different, only have the tensile test data can not describe the constitutive model of rubber material well. Study shows that the relationship between uniaxial compression and equiaxial tension is describe as[8]:

$$\begin{cases} \sigma_{Eb} = \dfrac{\sigma_{Ec}}{\sqrt{(1+\varepsilon_{Ec})^3}} \\ \varepsilon_{Eb} = \sqrt{\varepsilon_{Ec}+1}-1 \end{cases} \qquad (3)$$

Where σ_{Ec} is compressive stress, ε_{Ec} is compressive strain. σ_{Eb} is equivalent tension stress , ε_{Eb} is equivalent equiaxial tension strain. Therefore, equiaxial tension can be replaced by uniaxial

compression to obtain pure compression stress and strain of rubber. With reference with GB/T 528-2009[9], the uniaxial compression specimen is cylinder and carried out on the SANS testing
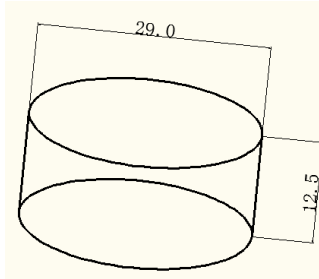

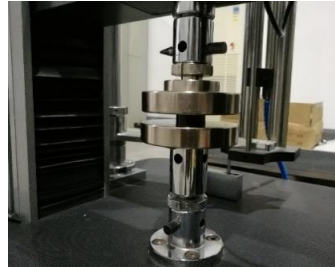Figure 4**.** rubber specimen of uniaxial compression
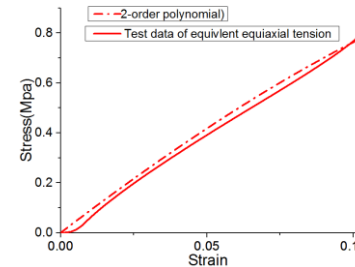

Figure 5. uniaxial compression test.

Figure 6. 2 order polynomial fitting with equivalent equiaxial tension result.

machine(as shown in figure 4& 5), firstly, compressing the specimen at 10mm/min. and then relaxing the specimen at the same speed, repeat 4 times, take the last compression data as valid compressing data of rubber, based on formula (3), the fitting curve of quivalent stress and strain with 2 order polynomial model are shown in figure 6.

As can be seen from figure3 and figure 6, 2 order polynomial model and experimental data are all fitting well, so it is reasonable to select the 2 order polynomial model as a hyperelastic constitutive model for the absorber rubber material. The fitting results are obtained with 2 order polynomial model parameters are:C_{10}=0.27832Mpa, C_{01}=0.46764Mpa, C_{20}=-5.1424E-02Mpa, C_{11}=0.38994Mpa, C_{02}=-0.46408Mpa, the metal part is carbon steel of Q235 which the modulus is E=2.1E6Mpa, the poisson's ratio is μ =0.3.

2.2 FEM Analysis and Test

The deformation of rubber absorber under 3000N downward force is analysed by FEM based on ABAQUS software[4]. The rubber part is simulated by hybrid C3D8H units, and the metal part is simulated by reduced C3D8R units. The static compression test was carry out on the MTS test machine (as shown in figure 7). Because of the typical hysteresis and Mullins effects of rubber materials, the loading and unloading paths do not coincide. Therefore, this test circularly loading and unloading 4 times, and takes the hysteretic line as an index to measure the static deformation of the rubber absorber. The testing result of static stiffness of the rubber absorber is 786.44N/mm. The static simulation results of rubber absorber are obtained under 3000N downward axial force, and the deformation nephogram as shown in figure 8.

From the FEM analysis results in figure 8, the deformation of the rubber absorber is 3.710mm, so the static stiffness is calculated to be 808.63N/mm, which is about 3% higher than the experimental result. The test calculation results and simulation results plotted in figure 9, it can be seen that the FEM analysis results and experimental results are quite close, which illustrate that the analytic results

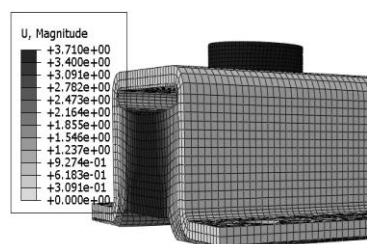Figure 7. Static test diagram of rubber absorber

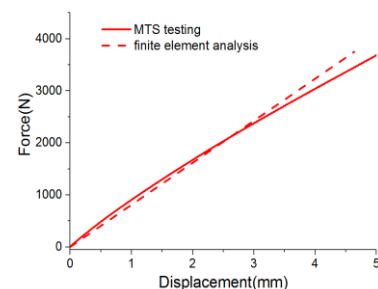Figure 8. Simulation results under 3000N vertical pressure

Figure 9. comparison of MTS testing and static simulation result rubber absorber

of FEM simulation instead of rubber absorber test is reasonable.

3. Orthogonal design and optimization

3.1 Taguchi Orthogonal Design theory

The basic idea of Taguchi parameter design method is to establish orthogonal test table which contain various controllable factors that affect properties[10]. By analysing each controllable factor combination of the products, the optimum combination scheme can be determined. Orthogonal design is an important means of Taguchi methods[11]. Mean sensitivity parameter of each level refers to the influence of controllable factors affect the properties[12]. The greater the sensitivity is, the greater the influence are. When determining the optimization scheme, the level of the controllable factor which with higher sensitivity is preferentially determined.

Assuming there exist adjustment factors in the multiple tests, the average of the experimental results can be adjusted to the target properties of m, with n test data:$y_1, y_2, \cdots, y_n$, the total fluctuation S_T is calculated as [13, 14]:

$$S_m = \frac{1}{n}\left(\sum_{i-1}^{n} y_i\right)^2 \tag{4}$$

and the error variance V_e is calculated as: $\quad V_e = \dfrac{\sum_{i-1}^{n} y_i^2 - \dfrac{1}{n}\left(\sum_{i-1}^{n} y_i\right)^2}{n-1} \tag{5}$

Sensitivity is an indicator of the average product quality response. the sensitivity is defined as:

$$S = 10\lg\frac{1}{n}\left(S_m - V_e\right) \tag{6}$$

Where S is sensitivity, S_m and V_e can be obtained by formula (4) and formula (5).

3.2 Orthogonal Simulation Design

According to the requirements of installation condition and BE-300 type rubber absorber existing design conclusion, 4 factors which essential to the static stiffness are selected as controllable factors of Taguchi orthogonal design plan. As shown in figure 10, this 4 factors respectively are bottom clearance height h_1 (factor A), rubber support layer thickness h_2 (factor B), the width of support bearings l (factor C), the distance between large radius and small radius axis of D_1 (factor D). Sensitivity is an indicator of the average product quality characteristics, so in a test orthogonal table, each scheme must have two or more experimental data, otherwise the sensitivity will be meaningless.

The orthogonal test in this study is completed by computer simulation, so there is no external interference, so the external environment influence can be excluded. Two kinds of mesh size of rubber material are selected as noise factors (factor Y), the one approximate global mesh size is 3mm and the another is 4mm, two attributes of mesh are C3D8H. The level of the 3 controllable factors is determined by referring to the magnitude parameters of the existing absorber. Interference factors and

Figure 10. The geometry parameters of rubber absorber

Table 1. Interference factors and its level

factors		level				
controllable factors	A($h1$/mm）	8	10	12	14	16
	B($h2$/mm）	5	7	9	11	13
	C(l/mm）	44	46	48	50	52
	D(d/mm）	1	2	3	4	5
noise factors	Y	mesh size is 3mm		mesh size is 4mm		

its level arrangement are as shown in table 1. According to the selection of controllable and noise factors of each level, orthogonal analysis table of L25 (5^4) is established. The orthogonal table ensure the times of repetition of one level is the same in each column, and each level collocation between two rows arbitrarily in horizontal is balanced. Completely FEM model is established in the *ABAQUS* software in accordance with the dimension combination scheme in table 2. For the same model, the rubber absorber model is divided into two mesh sizes referring to table 1. A total of 25×2 times FEM analysis were carried out, and the obtained simulation results as shown in table 2.

Table 2. Orthogonal arrangement with FEM analysis results

NO	Level			Static stiffness N/mm		Sensitivity S	NO	Level			Static stiffness N/mm		Sensitivity S
	A	B	C	Y_1	Y_2			A	B	C	Y_1	Y_2	
1	8	5	44	749.63	753.58	57.520	14	14	5	48	844.12	848.90	58.553
2	10	7	46	815.00	820.79	58.254	5	16	7	50	916.31	930.52	59.308
3	12	9	48	878.48	885.22	58.908	16	8	11	46	844.36	849.86	58.559
4	14	11	50	929.08	933.13	59.380	17	10	13	48	898.74	900.90	59.083
5	16	13	52	964.32	978.79	59.749	18	12	5	50	940.44	945.18	59.488
6	8	7	48	893.92	896.59	59.039	19	14	7	52	1024.24	1036.99	60.262
7	10	9	50	966.81	973.08	59.735	20	16	9	44	704.72	714.63	57.021
8	12	11	2	1044.20	1047.49	60.389	21	8	13	50	1000.33	1003.34	60.016
9	14	13	44	719.94	727.98	57.194	22	10	5	52	1034.13	1040.94	60.320
10	16	5	46	759.11	763.16	57.629	23	12	7	44	737.10	740.38	57.370
11	8	9	52	1076.43	1085.38	60.676	24	14	9	46	788.85	792.39	57.959
12	10	11	44	768.25	774.39	57.745	25	16	11	48	826.22	834.26	58.384
13	12	13	46	803.00	809.50	58.129							

3.3 Optimization Analysis

The absorber specification usage request that the vertical deformation of BE-300 rubber absorber under rated load (3000N) is 3.5~5.0*mm*. Taking the vertical deformation of 3.5*mm* as an example, the static stiffness is 857.1N/mm. The simulation results of BE-300 rubber absorber are about 3% higher than the experimental results, therefore, the static stiffness targets of the simulation result should be *m*=857.1×(1+0.03) =882.8*N/mm*.

The mean sensitivity of one controllable factor for a level are respectively calculated. Taking the

Table 3. Mean sensitivity analysis results

level	A/*N/mm*	B/*N/mm*	C/*N/mm*	D/*N/mm*
M1	59.162	58.702	57.370	58.852
M2	59.027	58.846	58.106	58.799
M3	58.857	58.860	58.793	58.912
M4	58.670	58.891	59.585	58.778
M5	58.418	58.834	60.279	58.793
Δ	0.744	0.189	2.909	0.134
Rand	2	3	1	4
$\overline{M}$				46.85

Table 4. Mean static stiffness analysis results

level	A/*N/mm*	B/*N/mm*	C/*N/mm*	D/*N/mm*
M1	915.3	867.9	739.1	882.3
M2	899.3	881.2	804.6	879.2
M3	883.1	886.6	870.7	889.4
M4	864.6	885.1	953.8	876
M5	839.2	880.7	1033.3	874.8
$\overline{M}$		880.3		

controllable factor of A's 1 level as an example, the simulation scheme contains A's 1 level include No.1, No.6, No.11, No.16 and No.21. Then the mean sensitivity of A's 1 level can be calculated by

$$M1_A = \frac{S_1 + S_6 + S_{11} + S_{16} + S_{21}}{5} \tag{7}$$

$M1_A$ is the mean sensitivity of A's 1 level, S_1 is sensitivity of No.1. The mean sensitivity of the remaining controllable factors can be obtained in turn, and the results are shown in table 3.

The mean static stiffness of one controllable factor for a level also can be calculated. Firstly,

calculating the mean static stiffness of each experiments, then based on the calculation methods which similar with mean sensitivity to obtain the mean static stiffness of each level of controllable factors. The results shown in table 4.

As can be seen in table 4, *M1~M5* is 5 different levels of controllable factors, $\overline{M}$ is the mean value of 25 times FEM analysis results. Δ is the maximum mean static stiffness minus the minimum of one controllable factor at different levels, and Δ indicate the impact significance of controllable factors. As can be seen from table 3, the impact significance of sensitivity of S is $\Delta_C > \Delta_A > \Delta_B > \Delta_D$, show that the most effective factor is the width of support bearings, the distance between large radius and small radius only have slightly influence on the static stiffness. Therefore, the first step of the optimization scheme should select the level of C which closest to the desired value *m*. As can be seen from table 4, the mean static stiffness of C3 equal to 870.7*N/mm*, closer to the desired value m=882.8*N/mm*. Then selecting the level of controllable factor A, from the table 4, A3 is closer to the desired value. After the level of C and A is determined, according to table 4, the difference between optimization value and the desired value *m* is ∇_1=(870.7+883.1)- 2×882.8=-11.8 *N/mm*. Therefore, the mean static stiffness of B and D should slightly higher than the desired value *m*, so that the overall static stiffness can be approached to the desired value *m* as possible. The mean static stiffness of B3 and D3 combinations is slightly higher than the desired value *m*, the difference ∇_2=(889.4+886.6)- 2×882.8=10.4 *N/mm*, and the total difference $\nabla=\nabla_1+\nabla_2$=-1.4N/mm. So the optimization scheme can be determined as A3, C3, B3, and D3. This combination is the scheme No.3 which in the orthogonal table 2.

4. The optimization results
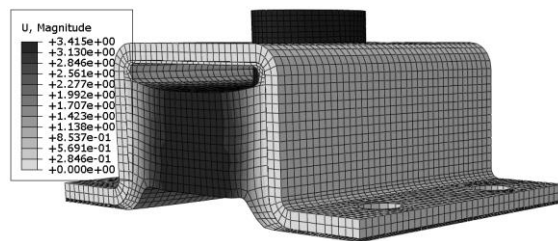The FEM analysis results of optimization scheme of No.3 are shown in figure 11 and in figure 12.

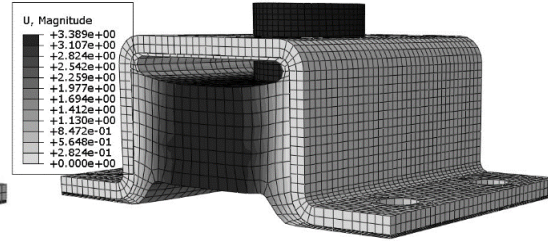Figure 11. Optimization result of mesh size is 3*mm*

Figure 12. Optimization result of mesh size is 4*mm*

Figure 11 and figure 12 are the static stiffness FEM analysis results of approximate global mesh size is 3*mm* and 4*mm*. It can be seen that the calculated mean static stiffness is 881.85N/mm, which is closer to the desired value m=882.8*N/mm*, this indicate that the optimization scheme can improve the static stiffness of rubber absorber effectively.

5. Conclusion
The performance of static stiffness is the key to ensure bearing capacity of rubber absorber. Four factors which are essential to the static stiffness are selected as controllable factors to optimize the static stiffness. Optimization scheme indicated that the combination of A3, C3, B3, and D3 can improve the static stiffness to the desired value effectively. This calculation results show that using sensitivity parameter as evaluation index in Taguchi orthogonal methods is reasonable, and the Taguchi orthogonal methods can help to design static stiffness of rubber absorber approach to the desired value utmost.

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